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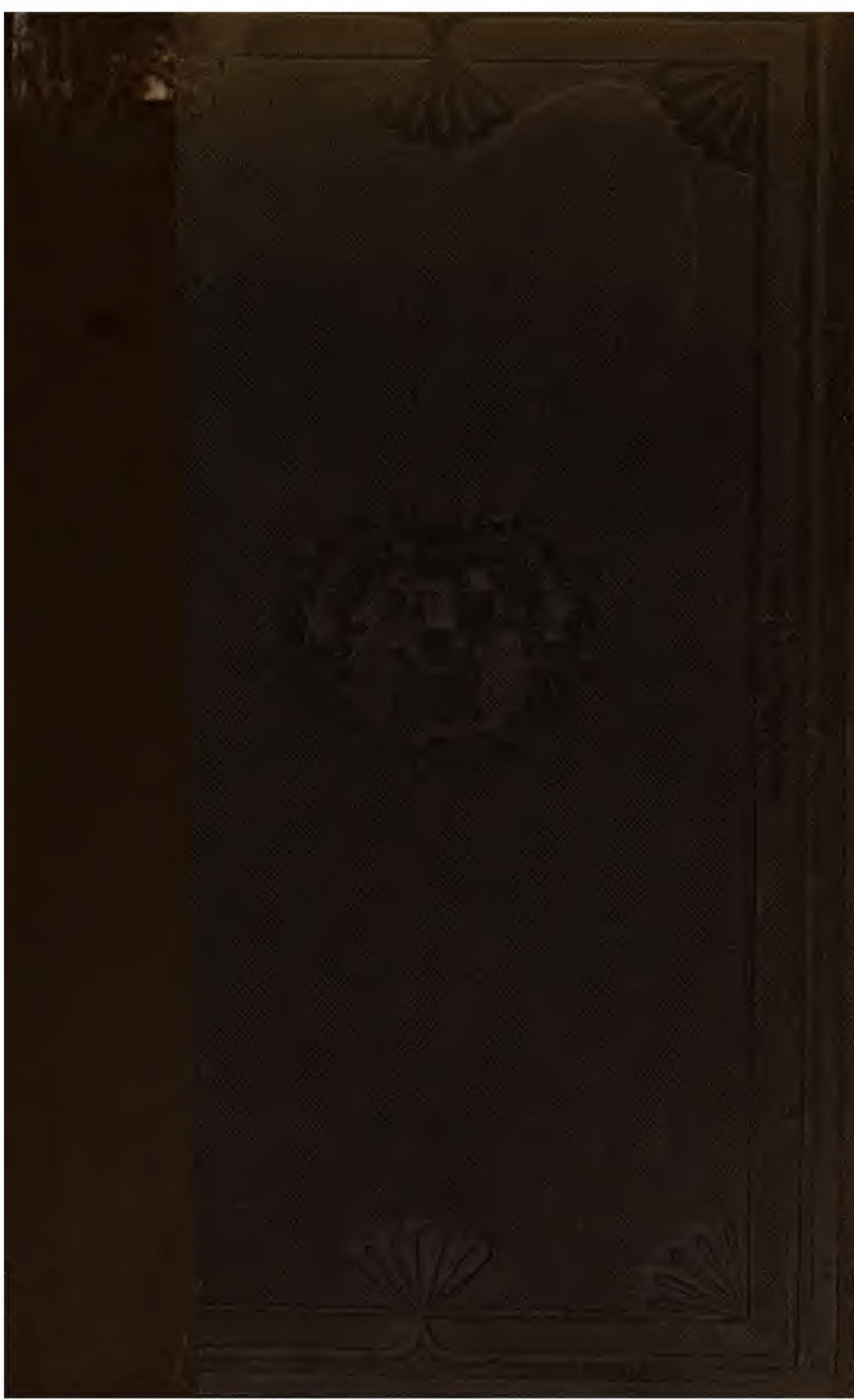
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A TREATISE ON HEAT.

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TREATISE ON HEAT.

PART I.

THE THERMOMETER; DILATATION; CHANGE OF STATE; AND LAWS
OF VAPOURS.

BY THE

REV. ROBERT V. DIXON, A. M.,

FELLOW AND TUTOR, TRINITY COLLEGE, DUBLIN; AND ERASMUS SMITH'S PROFESSOR OF NATURAL AND
EXPERIMENTAL PHILOSOPHY.

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TO THE

REV. HUMPHREY LLOYD, D. D., S. F. T. C. D.,

PRESIDENT OF THE ROYAL IRISH ACADEMY,

FORMERLY PROFESSOR OF NATURAL AND EXPERIMENTAL PHILOSOPHY IN THE
UNIVERSITY OF DUBLIN,

THE FOLLOWING PAGES ARE RESPECTFULLY DEDICATED,

BY HIS SINCERE AND OBLIGED FRIEND,

THE AUTHOR.

P R E F A C E.

THE following work has been compiled at the instance of the Board of Trinity College, and published at their expense, for the use of the Students in the School of Engineering connected with the University of Dublin, and also of such students in Arts as may take up the subject of Experimental Physics for their degree.

The object of these pages, therefore, is not only to convey a knowledge of the leading facts and general laws of the Science of which they treat, but also to assist in the task of mental training and discipline, which forms an important feature of University education. With this object in view, the author has described, as fully as the limits of the work permitted, the details of the principal experimental methods which have been employed in the examination of the phenomena of heat, has pointed out the disturbing causes complicating their results, and, where their object has been to obtain numerical values, he has explained the mode of investigating the formulæ which connect such values with the data of experiment.

And here it may be remarked, that the introduction of the experimental sciences into the University Course

has supplied an important omission which formerly existed in that system, by furnishing examples of the application of the rules and principles of the inductive philosophy. For, to adopt the quaint but expressive image of Bacon, the human mind is so devoted to the worship of various intellectual idols, that it requires not merely to have the true object of its allegiance brought before its view, but also to have its erroneous tendencies counteracted, and its evil habits reformed, by a system of discipline adapted to promote the formation of correcter habits.

It is true that, until a comparatively recent period, few of the Experimental Sciences were sufficiently advanced to admit of their introduction into the University Course, with the object above referred to. The fundamental laws of most of them were very imperfectly understood, many were disturbed and obscured by conflicting theories, and all were in a state of rapid progression and change. Owing to the exertions of the distinguished physicists, however, who have rendered the past years of the present century for ever memorable in the annals of experimental philosophy, the leading principles of the more important of the Physical Sciences are now established by such evidence as entitles them to rank, in point of certainty, with the deductions of the Abstract Sciences, and to justify their being referred to as the authentic results of a correct system of physical investigation.

Although the present work, however, has been specially designed for a particular class of students, the author trusts that it will not be devoid of interest to the general reader. No pains have been spared to collect the latest and most accurate information on the subjects

of which it treats, and much of the matter which it contains is now for the first time presented to the public in a connected form.

Several of the Tables in the following pages will be regarded with interest by experimental physicists in different departments, especially those which exhibit the elastic force of aqueous vapour derived from M. Regnault's experiments, and expressed in English measures. There can be little doubt that the experiments on which those tables are based are the most accurate which have hitherto been made in reference to this subject, and it is probable that a considerable time will elapse before a similar series will be undertaken by an experimenter combining so much acquired skill and natural aptitude for physical investigation, with the facilities and advantages that M. Regnault had at his command.

The author cannot omit this opportunity of acknowledging his obligations to Dr. Apjohn, Professor of Chemistry and Mineralogy to the School of Engineering, for much useful advice and many valuable suggestions kindly furnished by him during the progress of the following sheets through the Press. The author owes a similar acknowledgment to S. Downing, Esq., Assistant Professor of Engineering in the same School, in reference to the concluding paragraphs of Book I. chap. i. sect. 6.

TRINITY COLLEGE,
October, 1849.

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N. B. —All students presenting themselves for examination in Experimental Physics are required to be acquainted with the subjects contained in the paragraphs marked with an asterisk; candidates for Honors alone are expected to be prepared in the remainder.

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WE give the following account of the French system of weights and measures, in consequence of their frequent occurrence in the present work.

Modern French System.

In this system the *Metre* is the unit of length. Its successive subdivisions, according to the decimal scale, are designated by the *Latin* prefixes, *Deci-*, *Centi-*, *Milli-*, and its multiples by the Greek prefixes, *Deca-*, *Hecto-*, *Kilo-*, *Myria-*; thus a Decimetre is the tenth, a Centimetre the hundredth part of a metre, and a Decametre and Hectometre are equal to ten and one hundred metres, respectively. This mode of expressing the divisions and multiples of the respective units is carried out through the whole of the modern French system.

The *Are* is the unit of superficial measure; it is a square of a decametre on the side.

The *Stere* is the unit of solid measure; it is a cube of a metre on the side.

The *Litre* is the unit of measures of capacity; it is a cube of a decimetre on the side.

The *Gramme* is the unit of weight. It represents the weight of a cubic centimetre of distilled water at its greatest density. These quantities may be reduced to their equivalent values in the English system by the following relations:

1 Metre	= 39.37079	English inches.
1 Litre	= 1.760759	„ pint.
1 Gramme	= 15.433	„ grains.

Old French System.

In this system the *Toise* was the unit of length; it was divided into six *feet*, each foot into twelve *inches*, and each inch into twelve *lines*.

The toise equalled 76.73336 English inches, and consequently the old French inch was equal to 1.06574 English inch.

TREATISE ON HEAT.

INTRODUCTION.

SECT. I.—DEFINITIONS.—DIVISION OF THE SUBJECT.

1. *Definition of Heat.*—The word Heat is used in the English language to denote both a well-known corporeal sensation and also the external cause to which we refer that sensation and various physical phenomena. Some writers have proposed to employ the word Caloric in the latter sense, and to restrict the word Heat to the former; but as such practice does not appear to have met with general approbation, and as no ambiguity seems likely to arise from using the word Heat in both senses, we will throughout the following Treatise employ this word to express the cause of the corporeal sensation, as well as that sensation itself.

The physical phenomena referable to the agency of this cause are striking and numerous. Before proceeding to describe them in detail, it appears advisable to lay before the reader a sketch of the method which we intend to adopt in treating of them, and also to explain some terms of more frequent occurrence.

2. *Definition of Quantity of Heat.*—The existence of any physical cause is indicated by the existence of the effects which we attribute to it, and its energy is measured by the intensity of those effects. In the case of heat, the sensation produced by it is its most obvious and striking effect, and the intensity of that sensation is its first and simplest measure. Experience proves to us that this effect is produced by the transfer of heat, from some

source in which it resides, to our organs, and that the intensity of the sensation depends on the force with which the transfer is made. If, moreover, we observe two bodies, placed in the immediate vicinity of each other, in which the force of heat, thus measured, is found to be different, we uniformly find that the hotter body transfers or communicates to the colder a portion of its force, and at the same time decreases in volume. We are hence led to speak of heat as of something which may exist in bodies in different *quantities*, which may be subtracted from, or added to them; and the *same* body is said to possess a greater or less *amount of heat* according as it possesses the power of producing the sensation of heat with greater or less intensity.

3. *Definition of Temperature.*—What the actual amount of heat, thus understood, may be, which a body possesses under ordinary circumstances, or what amount it is capable of containing, we have no means of determining with certainty. Experiments prove that this amount is very considerable, compared with the quantities added and subtracted by the ordinary operations of heating and cooling. We are able, however, to measure and compare the quantities by which, under various circumstances, this unknown total amount is increased or diminished; and we learn from experiment that the addition of equal quantities of heat to different bodies affects in very different degrees the energy with which they effect the transfer of a portion of the force residing in them to other bodies. This is obvious in the case of *unequal* masses of the same material. The same quantity of heat which sensibly increases this energy in a small mass scarcely at all affects it in a larger, and it is also true in *equal* masses of different materials. The quantity of heat, therefore, in *different* masses, which produce the sensation of heat in the same degree of intensity, may be very different. We are to distinguish then between the quantity of heat in *different* masses, and the energy with which this quantity seeks to effect a transfer of a portion of itself to our organs or other neighbouring bodies. To this energy we give the name of *Temperature*; and two bodies are said to possess the same temperature when the quantity of heat in one of them acts with the same energy or power of transfer and communication as the quantity of heat in the other, although those quantities may be

very different in amount. Thus if two bodies *A* and *B* are placed in the vicinity of each other, and if *A* transfers in a given space of time to *B* the same quantity of heat as *B* transfers to *A*, which is known by the heat in *A* and *B* respectively retaining its original intensity, the *temperature* of *A* is said to be equal to that of *B*, although the amount of heat in each may be very different. And if *A* transfers more heat to *B* than *B* to *A*, which is known by the intensity of the heat in *B* increasing, then the temperature of *A* is said to be greater than that of *B*, although the *amount of heat* in *B* may be equal to or greater than that in *A*. To speak popularly, it appears that a given quantity of heat acts *outwardly* with different degrees of intensity when placed in different bodies; and as we must suppose that the total effect of a given quantity of heat is always the same, we are led to conclude that in different bodies different proportions of this force are absorbed in overcoming internal resistances, and therefore that the balance free to act *outwardly* varies in different cases. The temperature of a body, therefore, may be defined to be *the energy with which the heat in a body acts in the way of transferring or communicating a portion of itself to other bodies*.

Instruments used for the determination of temperature are called *thermometers*, and those for measuring the quantities of heat added to or subtracted from bodies, *calorimeters*. The construction of the latter will be described in the course of the work, when occasion requires; but as no progress can be made in the study of the phenomena of heat without a knowledge of the means employed for determining the temperature of bodies, we will devote the next section to a description of the principles on which thermometers are constructed, and the method of their application to the measure of temperatures, although in doing so we shall be unavoidably compelled to anticipate results developed in subsequent portions of the work.

4. *Division of the Subject. Two Effects of Heat noticed in the following Work: Change of Temperature and Change of Volume and State.*—Of the various effects referable to the agency of heat, we purpose to direct the attention of the student in the following pages to two, namely, the changes produced by its action in the temperature of bodies, and in the state of aggregation of their

integrant molecules. In most cases these two effects are co-ordinate. When in a solid body, for instance, the quantity of heat which it contains is increased, its temperature rises, and the mutual distance of its molecules being at the same time increased, its volume is enlarged; and this gradual increase of volume and of temperature accompany each other, until a certain limit is attained, constant in the same, but varying in different bodies, when a change in its molecular structure commences, known by the name of a change of state. While this change from the solid to the liquid state is in progress, the temperature of the body remains unaltered, and the total action of the increased quantity of heat communicated to the body is exhibited in the alteration produced in its molecular constitution. The change to the liquid state having been completed, the action of heat again exhibits itself in the double effect of increased temperature and increased volume, until a second limit is attained, at which the transition from the liquid to the vaporous or gaseous state takes place. The phenomena accompanying this change are similar to those attending the former. Between and beyond those limits the change of volume which, as we have remarked, always accompanies change of temperature, is gradual and slow, but at those two points a brusque and abrupt change occurs. At the former limit, owing to certain disturbing forces, to which we will subsequently refer, this change is sometimes in the direction contrary to the general analogy, which, as we have said, connects increase of temperature with increase of volume; but in the transition from the liquid to the vaporous state, the change of volume is always in accordance with the general law, and is, moreover, very considerable in amount. So that if we draw a base line *xy* (Fig. 1), the distance of successive points of which from any fixed point *a*, shall represent the temperatures of a body counted from some given temperature as origin, and erect ordinates *aA*, *bB*, *cC*, &c., representing the volumes occupied by the body at the origin, and at the temperatures corresponding to *ab*, *ac*, &c., the progress of the increase of volume, in relation to the increase of temperature, will be represented in most cases by a figure similar to *ABCDEFGH*. The temperatures *ad* and *af* are those at which change of state occurs; and the abrupt rise in the ordi-

nates from D to E , and from F to G , marks the sudden increase of volume unaccompanied by change of temperature, which, as we have remarked, characterizes those transitions.

The object of the First Book of the following Treatise will be to investigate the relations which exist, in various bodies, between their temperature and volume, between and beyond the points at which change of state occurs (Chap. I.); and secondly, to ascertain the temperatures at which these changes take place in different bodies, the phenomena which accompany them, and the relations existing between the temperatures of vapours and their elastic force and density (Chap. II.) So that the subject of this Book, in fact, will be, first, the relation existing between those *two effects* of heat above referred to, namely, change of temperature and change in the state of aggregation of the molecules of bodies, when they occur together; and secondly, the phenomena which attend, under peculiar circumstances, the exhibition of *one* of those effects alone.

5. Having in the First Book considered these effects in relation to one another, we will proceed, in the Second, to consider them in relation to their common cause. Experiments prove that the quantity of heat necessary to produce a given change of temperature in a given mass, varies according to the material of which it is composed. The ratio of this quantity to the quantity required to produce the same change of temperature in an equal mass of some material selected as the standard, is called the *specific heat* of the substance under examination. The subject of specific heat will occupy the first chapter of this Book

We have observed (4) that while a body is undergoing a change of state its temperature remains unaltered; the quantity of heat, therefore, added to a body during the progress of this change, not producing any effect outwardly, is said to be *latent* in the body; and as its presence in it is necessary to its new constitution, it is also called its *constituent* heat. The second chapter of Book II. will be occupied with the subject of latent or constituent heat.

The subject of the Second Book, therefore, will be, the relation existing between the quantity of heat communicated to a given body and the resulting effect; first, when that effect is twofold, exhibited both by change of temperature and change of molecular

arrangement; and secondly, when that effect is exhibited by the latter change alone.

6. Having then treated of these two effects of heat, in relation to one another (Book I.), and in relation to their common cause (Book II.), we purpose in the Third Book to inquire into the laws which govern the transmission of this force, whether through pure space or intervening bodies, by the way of radiation (Chap. I.), or along the particles of bodies by conduction (Chap. II.); and in the Fourth Book to give a brief sketch of the theory of the transmission of heat. A short account of the principal sources of heat and cold, and of the application of the laws of heat to Hygrometry, will occupy the Fifth and last Book.

7. *Extent and Importance of the Subject.*—It will be seen from this sketch that we do not propose to refer to any of the chemical, optical, or electrical properties of Heat. We intend to confine our attention solely to the action of this force on the integrant molecules of bodies, leaving the consideration of the manner in which it affects their chemical affinities, and modifies the action of light and electricity, to treatises on those subjects. Its connexion also with the sciences of meteorology and physical geography will be but slightly touched upon in Book V. Even with these limitations, however, the subject is one of considerable extent, and of great interest and importance. As we proceed in our investigations, we will be led to regard heat as the great antagonist of the force of cohesion,—capable, if its amount were increased, of converting the solid earth into a limitless mass of vapour,—and by its presence alone preventing the atmosphere which surrounds it from being condensed into a crust upon its surface, and the globe itself from being compressed into an atom. It is the strong repulsive force which it communicates to the particles of matter, which renders heat so effective as a motive power, and has enabled modern ingenuity to obtain such a mastery over the elements of matter, as virtually lengthens life, and gives men more than the fabled power of giants. To the Engineer, its study, then, is of the utmost importance. Heat is, so to speak, the life that animates the machines which are his agents in the accomplishment of the stupendous feats he is now daily called upon to perform. To the Physicist the subject

is one of no ordinary interest, as it presents him with some most beautiful experimental investigations of the laws of Nature, and with results of the utmost elegance and simplicity; while the theory which has been advanced to account for the phenomena accompanying the transmission of heat opens a most interesting and profitable field for the investigations of the Mathematician, and, like the theories of the other physical sciences, has led to some of the most important discoveries in pure mathematics.

SECT. II.—CONSTRUCTION AND USE OF THE THERMOMETER.

8. *Measure of Temperature arbitrary; Measure adopted.*—

The temperature of a body is, as we have stated, the force with which the heat present in it seeks to transfer a portion of itself to neighbouring bodies. In *different* masses of matter this force is a function of their weight, of their specific heat, and of the total quantity of heat present in them; in the *same* mass, therefore, it simply varies with the quantity of heat which it contains.

We have no direct means of measuring the energy or intensity of this force in a given body; failing this, our simplest hypothesis would be to suppose it to be *proportional* to the quantity of heat which it contains, and therefore the *least arbitrary* measure of changes of temperature in any body would be the corresponding changes in the quantity of heat present in it. Such a measure, however, would be highly inconvenient in most cases, and impracticable in many, and therefore we are compelled to look out for one more convenient in its application, although, perhaps, more arbitrary in its nature; and this we find in the change of volume, which, as we have remarked, is always co-ordinate with change of temperature. For as experience proves that the same body always occupies the same volume at the same temperature, provided that during the alterations of temperature to which it is exposed it undergoes no loss of substance or peculiar change of molecular arrangement, it follows that the volume of a body is the exponent of its temperature, and accordingly we may adopt as an *arbitrary* measure of the latter the corresponding changes in the former. Hence any mass of matter fulfilling the above conditions, and whose changes of volume, moreover, for the changes of tempera-

ture which it is required to measure, are either sufficiently considerable to be easily observed, or are capable of being rendered so by some mechanical contrivance, will serve as a thermometer, which may be described as *an instrument employed for the determination of temperature and the measure of its changes.*

9. *Mode of determining the Temperature of a Body.*—To understand the use of this instrument it is to be remarked, that if two bodies of unequal temperature be placed in contact, the body whose temperature is higher communicates more heat to the other than it receives from it in a given time, and that thus, after a short time, the temperatures of both become the same. If, therefore, we place a thermometer in contact with the body whose temperature we desire to ascertain, after a certain space of time has elapsed, it acquires the same temperature with the latter, and indicates that temperature by the volume which it occupies.

It is to be remarked, however, that the temperature thus determined is not the original temperature of the body under examination, but its temperature after, by mutual transfer of different quantities of heat, it and the thermometer have acquired a common temperature; it is, in fact, this common temperature which we ascertain, which will be greater or less than the original, according as the body examined has received or parted with more heat than the thermometer during the process of assimilation, that is, according as it had originally a lower or higher temperature than the thermometer. This resulting temperature, however, will differ the less from the original, the smaller the mass of the thermometer is, as compared with the mass of the body examined; and we shall accordingly assume, for the present, that the mass of the thermometer is so small that the quantity of heat necessary to be transferred to or from the body under examination, to equalize the temperatures, does not sensibly affect the temperature of the latter. We shall see subsequently (Book II. Chap. I.) how we can altogether remove the inaccuracy arising from this source.

10. *Principles on which the Comparability of Thermometers is founded.*—For the purposes of individual research, therefore, any body fulfilling the conditions above referred to will answer as a thermometer, and any scale applied to it, indicating its volume at a given time, will serve to determine the corresponding tem-

perature, and to measure its changes. But in order that different observers should be able to compare their results and experiments, it is necessary that the indications of different thermometers should be comparable. How this is effected we proceed now to explain.

Experiments prove that if different masses of the same material have a common temperature, the same change of temperature produces in them changes of volume proportional to their original volumes. Thus if, at a common temperature, which we shall denote by the symbol τ , we have two masses of the same material, whose volumes are v and v' , and if, on raising them both to a higher temperature τ' , the increments of volume are δv and $\delta v'$, we have always $\delta v : \delta v' :: v : v'$, or $\frac{\delta v}{v} = \frac{\delta v'}{v'}$; and conversely, if this analogy holds, and if the temperatures corresponding to v and v' are the same, then the temperature corresponding to the volume $v + \delta v$ is the same as the temperature corresponding to the volume $v' + \delta v'$. In order, therefore, that two instruments should be comparable, it is sufficient to construct them of the same material, to note on the scales attached to them their volumes at some common temperature τ , and to divide the scales into equal parts, each of which shall correspond to an increment of volume bearing some given proportion to the original volumes. Suppose, for instance, that we take two masses of the same material, whose volumes at some common temperature are v and v' , and attach to them scales, whose divisions shall correspond to increments of $\frac{1}{r}$ th part of those volumes respectively, so that each division on one scale shall be equivalent to $\frac{v}{r}$, and on the other to $\frac{v'}{r}$, then the temperature indicated by one of those masses occupying n divisions or degrees upon its scale, will be the same as that indicated by the other occupying the same number of degrees upon its scale, for the increments of volume being $\frac{nv}{r}$ and $\frac{nv'}{r}$ are obviously in the ratio of $v : v'$.

This graduation may either be effected by direct measurement, or more conveniently as follows:—Let the volumes v and v' , occupied by two masses at a common temperature τ , be marked

on scales applied to them; let the volumes V and V' , occupied by the same masses at a higher common temperature, be also noted on the same scales; from what we have said it follows that the intervals on the scales between those two points correspond to increments $V-v$ and $V'-v'$, which are in the ratio $v:v'$; and those intervals, or any convenient aliquot parts of them, may be assumed as the units of division. Suppose we divide the space occupied on the respective scales by the increments $V-v$ and $V'-v'$ into s parts, and assume one of these as the unit, then since $V-v : V'-v' :: v:v'$, we have $\frac{V-v}{s} : \frac{V'-v'}{s} :: v:v'$, and also $n \frac{V-v}{s} : n \frac{V'-v'}{s} :: v:v'$, and consequently the temperature indicated by one of the masses occupying n degrees on its scale, thus divided, is the same as that indicated by the same number of degrees on the other.

The conditions requisite to be fulfilled, therefore, in order to render two thermometers comparable, are, first, to construct them of the same material; secondly, to note on the scales attached to them the respective volumes corresponding to two constant temperatures; and thirdly, to divide the intervening portions of the scale into the same number of equal parts. Sir Isaac Newton first pointed out the advantage to be obtained by this method of graduating thermometer scales, and at the same time proposed as the fixed points in the graduation the temperatures of melting ice and boiling water. His suggestion has since been universally followed, with some diversity, as will be mentioned hereafter, in the number of degrees into which the scale is divided between these two points.

11. *Material of Thermometers.*—In our choice of the material of which to construct a thermometer, we must be determined, among other considerations, by the temperatures required to be measured by it. Thus if it be required to measure very high temperatures, a refractory metal, as platina, is best suited for the purpose. Again, if our object be to measure with extreme accuracy small changes, air or some fixed gas, corrected in its dilatations for alterations of barometric pressure, is most suitable; while for ordinary purposes some of the liquids which expand more than solids, and do not require the corrections necessary in the case of gases, are found

to answer best. Of all the thermometers of this class, that constructed with mercury contained in a glass envelope is most generally used; we will accordingly select it as an example, for the purpose of describing the mode of construction, graduation, and use of this important instrument.

12. *Construction of Mercurial Thermometers.*—The mercurial thermometer is formed of a glass tube with a fine cylindrical bore, sealed at one end, and terminating at the other in a ball or reservoir of large capacity, containing the mercury. By this arrangement one of the requisites above referred to is satisfied, namely, that small changes in volume of the thermometric substance should be rendered sensible; for the volume of the ball being very considerable compared with that of any length of the bore, a small increase in the volume of the liquid contained in the former fills a considerable length in the latter. This effect is increased in some instruments by using a bore of elliptical, instead of circular, section; the minor axis being almost evanescent, the area of the bore is greatly diminished, while the plane passing through the major axis being perpendicular to the line of vision, the mercury is even more perceptible than in a tube of circular section with greater capacity. It is to be remarked, that as the glass envelope expands with an increase of temperature, as well as the mercury it contains, the expansion of the latter which is observed, or, as it is called, the apparent expansion of the mercury, is the difference between its actual expansion and the expansion of the glass envelope, as will be explained more fully in the next chapter.

13. *Method of dividing the Tube into Portions of equal Volume.*—It has been stated that the mode of graduating the thermometer is to divide the space between the volumes it occupies, at two constant temperatures, into a given number of portions of *equal volume*; this would be effected, in the case of mercurial thermometers, by simply dividing the portion of the tube between the points at which the mercury stands at the two temperatures referred to, into the given number of parts of *equal length*, if the bore were truly cylindrical, and this is the way in which thermometers in ordinary use are graduated. But more care must be used in the graduation of instruments employed for purposes

where extreme accuracy is required. For these the tube should be divided into portions of equal volume by the following method, due to M. Gay-Lussac.

A tube having been selected as nearly uniform in bore as possible, a quantity of mercury is introduced into it, so small that the space it occupies may be considered, without sensible error, as perfectly cylindrical. The extremities of this space, as 1, 2 (Fig. 2), are marked on the tube with a diamond point; the index of mercury is now moved forward until its left hand extremity, m , coincides with 2; its right hand extremity, m' , now marks a point 3, such that the space 2, 3 is exactly equal in volume to 1, 2; and this process is repeated until the whole tube is divided into portions of equal volume. But as it is extremely difficult to move the column of mercury so that its left end in one position shall coincide exactly with the point occupied by its right end in a previous one, the following method is to be preferred. Draw a line AB (Fig. 3) on a plane surface, and having moved the column from the position mm' to $m''m'''$, so that m'' is as near to m' as possible, mark the corresponding length $\mu\mu'$ on the line AB , next bring the point 2 on the tube to coincide with μ , and μ' will then mark the point 3. The further subdivision of the portions 1, 2; 2, 3; &c., may be effected by simply dividing them into portions of equal length, as they are supposed to be perfectly cylindrical.

These divisions of equal volume will probably not agree in number with the degrees into which, as we have said, the space between the points at which the mercury stands at the two fixed temperatures is to be divided; it is easy, however, to ascertain the number of degrees to which any given number of divisions corresponds. Thus let N be the number of degrees between the fixed points, N' the number of divisions on the tube between the same points, then the number of divisions in a degree is $\frac{N'}{N}$, and therefore any number n' of divisions corresponds to $n' \frac{N}{N'}$ degrees, and a number n of degrees is equivalent to $n \frac{N'}{N}$ divisions.

14. *Method of filling and sealing the Tube.*—The tube having been thus divided, the ball is to be blown at its extremity, not, it may be remarked, by the breath, lest moisture should be deposited in the tube, which it would be extremely difficult to remove, but by means of an elastic ball of caoutchouc tied on one end, and compressed, when the glass at the other end has been rendered sufficiently plastic by heat, care being taken to prevent a collapse of the bulb, by admitting air into the caoutchouc bag, through an aperture in itself, on removing the pressure. The bulb being formed, the thermometer is ready to be filled with mercury, which should be perfectly pure, and freed from all moisture and air by recent boiling. The mode of introducing the mercury into the bulb is as follows:—The bulb being carefully warmed over the flame of a spirit-lamp, the air within is expanded, and a portion expelled; if the end of the tube be now plunged into the prepared mercury, when the air in the bulb cools it will contract in volume, and its place will be supplied by mercury, which will rapidly rise through the tube and enter the bulb: this operation is to be repeated until a sufficient quantity of mercury has been introduced. More frequently, however, the mercury is placed in an enlargement formed at the top of the tube, and the air forced through it by the heat of the lamp is replaced on cooling by the descent of the mercury. The instrument having been filled, the mercury is boiled for a considerable length of time, to remove all air and moisture, care being taken to keep all parts of the instrument at the same temperature, and not to press the operation too rapidly. The end of the tube is now sealed, to prevent any accidental loss of mercury, and when sealing, it is desirable to exclude all air, as its presence in the sealed tube is frequently a source of considerable inconvenience, by breaking up the mercurial column. The exclusion of air is effected by expanding the mercury by heat, so as to fill the whole of the tube while the process of sealing is taking place. A thermometer with which this precaution has been observed is easily recognised, since, on inverting the tube, the mercury will flow to its very extremity; whereas, if the least bubble of air has been left, it will form an elastic cushion resisting its motion.

15. *Method of obtaining the fixed Points on the Scale.*—The ther-

mometer is now prepared for graduation, the first step in which process is the determination of the fixed points, corresponding to the two given temperatures. Of these the lowest is that of *melting ice*. This temperature is absolutely constant, provided only that the water from which the ice is formed is free from salts; and the only precaution to be observed in ascertaining the point corresponding to it on the thermometer tube is to take care that all the mercury—that in the tube as well as that in the bulb—is brought to the required temperature, by keeping the thermometer plunged in the ice up to the point at which the mercury then stands. The point thus ascertained is to be marked on the tube.

The second constant temperature is, as we have said, that of boiling water. More correctly, it is the temperature of the vapour of pure water, boiling under a normal atmospheric pressure. For the amount of the pressure under which water is boiled, the nature of the vessel, and the presence of salts, all affect the temperature of its boiling point, as will be more fully explained in the second chapter of Book I. The second fixed point on the scale, therefore, is not so easily determined as the first; for as the pressure under which water is boiled affects the temperature of its boiling point, and as the water at the bottom of a deep vessel is subjected to a greater pressure than that at the top, it follows that the temperature of boiling water varies through the several horizontal strata of the same vessel; and accordingly, if, with the object of submitting all the mercury to the required temperature, we plunge the thermometer vertically into boiling water, the mercury in the different strata will, in fact, be at different temperatures. It has been observed, however, that the temperature of steam produced from pure water under a constant barometric pressure is absolutely constant, and hence an easy and simple method is obtained of fixing the corresponding point on our thermometer scale. A cylindrical tin vessel (Fig. 4), of suitable length, is furnished with a lid containing two apertures; through one of these (*a*), fitted with a cork, the thermometer tube is passed; the steam, supplied by a small quantity of water (*ef*) at the bottom of the vessel, which, when used, is heated by a spirit lamp placed underneath, escapes through the other (*b*). The air having been all ex-

pelled and replaced by steam, the thermometer is pushed down into the vessel according as the mercury rises, the top of the mercury being always kept on a level with the upper part of the lid, until it has attained its highest elevation in the tube. By this means, if the process be conducted when the barometer stands at the proper height, the second fixed point will be correctly determined. If the operation be conducted when the barometer stands at a different height, the correction to be applied to the point determined by the preceding method will be pointed out in Book I. Chap. II.

English physicists have adopted, as the normal pressure for the graduation of thermometers, that equivalent to a column of 30 inches of mercury, at the temperature of 0° Cent.; the French have selected, for the same purpose, the pressure corresponding to a column of 760 millimetres, or 29.922 inches, at the same temperature.

16. *Methods of Graduation; Fahrenheit's, Celsius', and Reaumur's.*—Independent of the slight difference in the determination of the second fixed point, arising from the cause just referred to, there exists also a diversity in the manner of graduating thermometer scales *between* the two fixed points, and in the selection of the zero. Thus, in the scale generally adopted in England, and called Fahrenheit's scale, the interval between the two points is divided into 180 degrees, and the zero is placed at 32° *below* the lowest fixed point. According to this scale, then, ice melts at 32° , and water boils at 212° . Again, Celsius, a Swede, proposed a division of the fixed interval into 100° , and placed the zero at the lowest point. This mode of graduation is called the Centigrade; it is universally adopted in France, and recommends itself strongly, by its simplicity, for scientific purposes. A third method is Reaumur's, who divided the interval into 80° , and fixed the zero, as in the Centigrade, at the point of melting ice.

17. *Method of determining corresponding Temperatures on the different Scales.*—It is easy to determine the point on one of those scales which corresponds to a given degree upon another, especially in the case of the two last mentioned, which both count from the same zero. For as the same space in the Centigrade is divided into 100° , which in Reaumur's is divided into 80° , it

follows that 1° of the latter = 1.25° of the former, and, accordingly, that any number of Reaumur's degrees is changed into the corresponding number of Centigrade, by multiplying them by 1.25, or by $5 \div 4$, or by adding one-fourth of their number to them. Conversely, Centigrade are converted into Reaumur's by multiplying them by .8, or by $4 \div 5$, or by taking one-fifth of their number from them. Again, 180° Fahrenheit equal 100° Centigrade, hence 1° Fahr. = $\frac{5}{9}^{\circ}$ Cent., and therefore any number of Fahrenheit's degrees is converted into Centigrade by multiplying them by $5 \div 9$; and conversely, Centigrade are converted into Fahrenheit's by multiplying them by $9 \div 5$. But if we wish to ascertain the point on one scale corresponding to a given point on the other, we must take notice of the difference of the zero on those scales. Thus 50° Fahr. is only 18° Fahr. above the point of melting ice, which is the zero of the Cent. scale, and accordingly the point of the latter corresponding to 50° Fahr. is obtained by multiplying 18° (or $50^{\circ} - 32^{\circ}$) by $5 \div 9$, and therefore 50° Fahr. = 10° Cent.; and similarly, to find the point on Fahrenheit's scale corresponding to a given degree on the Centigrade, after reducing the number of Centigrade degrees to Fahrenheit's by multiplying them by $9 \div 5$, and thus obtaining the number of Fahrenheit's degrees above the point of melting ice, that is, above 32° Fahr., we must add this number 32° to the number so obtained, to get the required degree on Fahrenheit's scale. Thus the temperature 15° Cent. = $15^{\circ} \times 9 \div 5 = 27^{\circ}$ Fahr. above melting ice, or above 32° Fahr., and therefore corresponds to 59° Fahr. Hence the following practical rules:

To convert any Number of Fahrenheit to Centigrade Degrees.—Multiply them by $5 \div 9$.

To convert Centigrade to Fahrenheit.—Multiply them by $9 \div 5$.

To find the Point on Fahrenheit's Scale corresponding to a given Degree on the Centigrade.—Multiply the number expressing the degree Centigrade by $9 \div 5$, and add 32.

To find the Point on the Centigrade Scale corresponding to a given Degree on Fahrenheit's.—Subtract 32 from the number expressing the latter, and multiply by $5 \div 9$.

18. *Correction for imperfect Assimilation in the Determination of Temperatures.*—In order to ascertain with accuracy the tem-

perature of a body by a comparison with the mercurial thermometer, it is obviously necessary to bring *all* the mercury in the latter to the same temperature with the body under examination. Otherwise, if but a part of the mercury is brought to this temperature, and the remainder continues at the temperature of the surrounding medium, the observed temperature will be greater or less than the true, according as the temperature of the surrounding medium is greater or less than that of the body examined. Where this condition cannot be fulfilled, and extreme accuracy is required, a correction can be applied by means of which we may approximate to the true temperature. We will now proceed to investigate this correction, and first show how the volume of the bulb of a thermometer, relatively to that of any portion of its tube, may be determined, as the knowledge of this volume is necessary in the subsequent investigation. There are two methods of determining this volume; one is by finding the weight of mercury which fills the bulb, and then the weight which fills, in addition, a known portion of the tube at the same temperature. Suppose the tube divided into portions of equal volume, and take one of these as the unit of volume; call V the volume of the bulb, including the small portion of the end of the tube, where the bore is conical at its entrance, and let W be the weight of mercury filling V and n divisions of the tube, $W + w$ the weight filling V and n' divisions; then w is the weight of $(n' - n)$, and the volumes being as their weights, we have $V + n : n' - n :: W : w$, and hence $V = \frac{W}{w}(n' - n) - n$.

The second method of determining V depends on the knowledge of the quantity by which mercury apparently expands in glass, between the temperatures of melting ice and boiling water, or 0° and 100° Cent. This quantity, as we shall see in the next chapter, equals $1 \div 64.8^{\text{th}}$ * part of the volume at the lower tempe-

* This is the value assigned by MM. Du-
long and Petit (*Ann. de Chim. et de Phys.*,
tome vii. p. 138), but as it depends on
the nature of the glass of which the ther-
mometer is constructed, it is not a constant
quantity, even when all precautions are
taken to procure glass of a description as

nearly identical as possible. Thus M. I.
Pierre (*Ann. de Chim. et de Phys.*, tome xv.
p. 335, 3^{me} Serie) found that, even when
the thermometers which he employed in
his experiments were composed of the same
description of glass (*crystal*, a kind of flint
glass, see Dumas, *Traité de Chimie*, tome

ture. Now suppose the mercury at 0° to fill the volume V and n divisions, and at 100° , $V+n'$, then $(n'-n) = \frac{V+n}{64.8}$, or

$$V = 64.8 (n' - n) - n.$$

Now to ascertain the correction to be applied to the observed temperature in the case of imperfect assimilation, as, for instance, where the thermometer is but partially immersed in a liquid whose temperature we desire to know, let AB (Fig. 5) represent a thermometer under such circumstances, immersed to a in a fluid; suppose that before immersion the mercury stood at b , marking the temperature t of the surrounding medium, then, if the fluid be of a higher temperature, the mercury will rise, suppose to c , marking the temperature t' ; suppose further, that, if totally immersed, the mercury would rise to d , marking the true temperature t_x , we require the correction cd . Let V represent the volume of the part immersed, expressed, as well as the portions ab , bc , &c., in the units already referred to. Now bd is the volume by which $V+ab$ expands when the temperature is raised from t to t_x , and cd is evidently the quantity by which ac , the portion retained at the temperature of the surrounding medium, would expand for the same change of temperature; and therefore, according to the law already referred to, namely, that the quantities by which bodies of the same material expand for the same change of temperature are proportional to their original volumes, we have $V+ab:ac::bd:cd \therefore cd = \frac{bc \cdot ac}{V-bc}$. A similar process shows that if the temperature of the fluid be lower than that of the medium, $c'd' = \frac{bc' \cdot ac'}{V+bc'}$, in the former case cd is additive, in the latter subtractive; hence the two formulæ may be comprehended in the one expression,

$$cd = \frac{bc \cdot ac}{V-bc},$$

by giving the proper sign to bc . In this formula cd is the correction in volume, bc the volume by which the mercury rises or

ii p. 614), in as perfect a state of homogeneity as could possibly be obtained, still the above quantity varied between the li-

mits 0.0161154 and 0.0154155, or, in vulgar fractions, $\frac{1}{64.87}$ and $\frac{1}{62.05}$.

falls, and ac the volume remaining at the temperature of the medium. If λ represent the volume occupied by a degree, $\frac{cd}{\lambda}$ will be the number of *degrees* corresponding to the volume cd ; and dividing both sides of the preceding expression for cd by λ , and the numerator and denominator of the right hand member by the same quantity, we get

$$\delta t = \frac{(t' - t)\nu}{V_1 - (t' - t)},$$

where δt is the correction in degrees, ν the number of *degrees* remaining unassimilated, V_1 the volume expressed in degrees as units. And as $V_1 = 6480 - n$, n being the number of degrees between a and the zero of the scale, we have

$$\delta t = \frac{(t' - t)\nu}{6480 - [n + (t' - t)]}.$$

This correction may be graphically represented in a manner which may assist the student's memory. Let ac (Fig. 6) represent the length of the column of mercury unassimilated in temperature, an the length of a similar column equal in volume to V ; take $nm = bc$, join mc , and draw nd parallel to mc ; cd is the correction required. Fig. 6 represents the case in which cd is additive; Fig. 6, *bis*, that in which it is subtractive.

If, when graduating a thermometer, we are unable to plunge the whole of the mercury into the ice or boiling water, the correction of the points apparently corresponding to 0° and 100° may be obtained by the same formula. In this case, referring to Fig. 5, if c' be the apparent, d' will be the true zero, and c and d will be the apparent and true boiling points.

It may be remarked that the correction for imperfect assimilation of temperature, which is never applied unless extreme accuracy is required, and perfect instruments employed, is rendered still less necessary in ordinary cases, if the instrument in use happens to have been graduated by incomplete immersion. Thus if c be the point assumed, from such a method of graduation, to be 100° , and d the true 100° , suppose that the mercury rises to c' , on incomplete immersion, instead of to d' , the error is $dd'' - cc''$, or $cd - c'd''$, instead of $c'd''$.

19. *Determination of Length of Stem necessary for a given Range.*—By a similar method of investigation we can determine the length of stem to give a thermometer, in order that it may serve to measure temperatures ranging from t_1° below, to t° above, zero, and also the height at which the mercury should stand at the time of filling, to allow of this range. Let ad (Fig. 7) represent the stem of the thermometer, the divisions commencing from a ; call the volume below a , V ; suppose the mercury at 0° to stand at c ; at the time of filling, when the temperature is t° , let it stand at b ; and at the upper limit t° at d ; we are required to determine the lengths ab and ad . As the volume of mercury at $0^\circ = V + ac$, the length of one degree upon the scale equals $\frac{V+ac}{6480}$, and therefore the length of $t_1^\circ = \frac{t_1(V+ac)}{6480}$; hence $ac = \frac{t_1(V+ac)}{6480}$, and therefore $ac = \frac{Vt_1}{6480-t_1}$. Similarly as cb contains t° , $cb = \frac{t(V+ac)}{6480}$, and thus we obtain for $ab = ac + cb$, the value

$$ab = \frac{V(t+t_1)}{6480-t_1}$$

also $cd = \frac{t(V+ac)}{6480}$; hence as $ad = ac + cd$

$$ad = \frac{V(t+t_1)}{6480-t_1}.$$

It is advisable to leave the stem somewhat longer than the length thus determined, as the dimensions are rendered irregular near its extremity by the process of sealing. It may be remarked that carefully constructed instruments are generally finished with a slight enlargement, to allow of the expansion of the mercury beyond the temperature which they are intended to measure; without this precaution, if they were accidentally exposed to a much higher temperature, the expansive force of the mercury would fracture the envelope.

20. *Displacement of Zero.*—We will conclude for the present our remarks on the subject of the mercurial thermometer, by noticing the discovery of M. Flaugergues,* relative to a change which takes

* Annales de Chimie et de Physique, tome xxi. p. 333 (1822).

place in the position of the zero point. It appears from his experiments, since confirmed by M. Legrand,* that all thermometers are liable to a displacement of their zero after they have been some time made; that this displacement takes place gradually, proceeding more rapidly at first, and continuing during a period of from four to six months, until at last the zero stands from $0^{\circ}3$ C. to $0^{\circ}5$ C. higher than at first. This displacement appears to be owing to the circumstance that the glass envelope requires the length of time above mentioned to contract to the volume which it finally occupies, after the high degree of heat to which it was exposed during the boiling of the mercury. This explanation is confirmed by the fact, that if, after the displacement of the zero, the mercury be again boiled, the zero returns to its original position, and the phenomenon is clearly connected in some manner with the nature of the envelope, as it does not occur, at least to the same extent, in the case of instruments made with crystal glass.†

21. *Spirit Thermometer*.—The range of the mercurial thermometer extends from about 350° Cent., or 662° Fahr., the boiling point of mercury, to -39° Cent., or -38.2° Fahr., at which temperature this metal congeals. When approaching this point, however, its indications become very irregular, owing to the change of state then beginning to take place in the mercury. For low temperatures, therefore, we must have recourse to the spirit thermometer, by means of which we can estimate the lowest attainable, as the liquid with which it is filled, pure alcohol, has never been frozen. It does not, however, answer so well to measure high temperatures, in consequence of the low position of its boiling point; although, if the stem be carefully sealed, and all air excluded, this, as well as other liquid thermometers, may be applied to measure temperatures considerably above those at which the liquid filling them boils in the open air. With respect to the graduation of the spirit thermometer it must be observed, that if it be intended to compare its indications with those of the mercurial thermometer, it should be graduated by a comparison

* Annales de Chimie et de Physique, tome lxiii. p. 368 (1836).

† For some further remarks on the effect produced on the indications of mercurial

thermometers by the irregularity in the expansion of their envelopes, and on the comparison of different thermometers, see Book I. chap. i. sect. 5.

with a carefully constructed instrument of the latter description, since, for the comparability of instruments graduated independently, it is not only necessary that their scales should be divided into a given number of equal parts between fixed points, but also that they should be all formed of the same material, or at least of materials for which the proportion referred to in (10) holds true mutually; but this is not the case with mercury and alcohol, or any other known liquid. This method of graduation cannot be extended, of course, below the inferior limit of the scale of the mercurial thermometer. Beyond that point the spirit thermometer should be graduated by comparison with an air thermometer, to be described hereafter, whose indications are almost identical with the mercurial for a very considerable range of temperatures.

We will reserve the description of other kinds of thermometers until we shall have investigated the laws of dilatation of the bodies of which they are composed.

BOOK I.

ON THE RELATION BETWEEN THE TEMPERATURE OF BODIES AND THE STATE OF AGGREGATION OF THEIR INTEGRANT MOLECULES.

CHAPTER I.

ON THE RELATION BETWEEN THE TEMPERATURE OF BODIES AND THEIR VOLUME.

22. *Definitions.*—It is our object, in the present chapter, to investigate the relation between the changes of temperature to which bodies are exposed, as measured by the mercurial thermometer, and the corresponding changes which they undergo in volume. We purpose to examine this relation in the three classes of bodies, solids, liquids, and gases, successively; first describing the principal methods of experimenting, with their results; next stating the general laws derived from such researches; and finally pointing out some of the more important practical applications of those laws.

It has been remarked, that, in general, when a body receives an increase of temperature, its volume is enlarged, and at the same time the area of any portion of its surface, and the length of any of its edges, is increased. The increase of a body in volume is called its *cubical* expansion or dilatation, its increase in surface, its *superficial*, and its increase in length, its *linear* expansion. If we denote the value of any of those quantities, namely, the volume of a body, the extent of any portion of its surface, or the length of any of its edges, by the symbol Q , and by q the increment which this quantity receives on increase of temperature, its new value will be $Q + q$, which may be put under the form $Q(1 + \delta)$, q being equal to δQ . In this expression δ denotes a number representing the fractional part of the original quantity

by which it increases for a given change of temperature, and is called the *coefficient* of expansion corresponding to the *original quantity* and the *given change of temperature*. The coefficient of expansion, therefore, may be defined to be the *expansion, corresponding to a given change of temperature, of the unit of volume taken at a determined temperature*. The value of this coefficient varies in different substances; it also varies in the same substance with the temperature of the original volume, &c., to which it is referred, and the extent of the change of temperature to which it corresponds. The change of temperature is generally expressed by an inferior index; thus δ_r denotes the coefficient of expansion corresponding to a change of temperature measured by r degrees. In like manner, if we denote the volume, surface, and length of a body by V , S , and L , respectively, these symbols are frequently distinguished by indices expressing the temperature to which they correspond. Thus V_0 , V_t , $V_{t'}$ represent the volumes of a body at the temperatures 0° , t° , t'° . Following this system of notation, therefore, we may express the volume, surface, or length of a body at the temperature t' , in terms of its volume, &c., at the lower temperature t , and of its corresponding coefficient of expansion, as follows:

$$V_{t'} = V_t(1 + \delta_r);$$

$$S_{t'} = S_t(1 + \delta_r'),$$

$$L_{t'} = L_t(1 + \delta_r''),$$

δ_r , δ_r' , δ_r'' being the coefficients of *cubical*, *superficial*, and *linear expansion*, corresponding to the original temperature t and to the change of temperature $(t' - t) = r$. We now proceed to explain the method of determining by experiment the values of the coefficients of expansion in different bodies, and first in the case of solid bodies.

SECT. I.—ON THE DILATATION OF SOLIDS.

23. *Mr. Ramsden's Method of determining the linear Dilatation of solid Bodies.*—The subject of the linear dilatation of solid bodies by heat was first investigated, with the degree of accuracy due to its importance, at the close of the last century. In the year 1782 the eminent French physicists, Lavoisier and Laplace, instituted an

extensive series of experiments in relation to this subject, the results of which, however, owing to the melancholy death of Lavoisier at the commencement of the French Revolution, were not published, we believe, until they appeared, in 1816, in M. Biot's *Traité de Physique*. In the year 1783, Mr. Ramsden, whose name is well known in connexion with the improvements introduced by him in the construction of various instruments of scientific research, made some experiments on this subject at the request of General Roy, who was at that time engaged in the trigonometrical survey of Great Britain. These experiments were made chiefly with the view of determining the variations of length to which the rods, employed in measuring the base on Hounslow Heath, were liable from changes of atmospheric temperature; and accordingly only seven bars, three of brass, one of steel, one of cast iron, and two of glass, were examined. Mr. Ramsden's method of experimenting, however, appears to us to possess such peculiar advantages, and his apparatus is characterized by such elegance and simplicity, combined with accuracy, that we purpose to give a brief description of its construction, referring the student for further details to General Roy's memoir in the seventy-fifth volume of the Philosophical Transactions.

On a strongly-framed table, of about five feet in length by three in breadth, were placed three troughs, parallel to one another, represented in their general plan in Fig. 8. The two exterior were formed of wood coated internally with pitch, the central one of copper. The exterior troughs contained each a bar of cast iron, permanently secured at one end, and moveable through a collar at the other; the nearer bar, *AB*, carried at its extremities two vertical frames bearing the eye-pieces, *E, E'*, of two microscopes represented in elevation in Fig. 10, with their axes horizontal, and at right angles to the direction of the bar; of these, *E* had in its focus two fixed parallel wires, *E'* two similar wires capable of being moved by a micrometer screw, *H*, in a plane at right angles to the axis. The microscope *E'* was hence called the micrometer microscope. The farther bar, *CD*, carried, on similar vertical frames, marks, *m, m'*, consisting each of two fine crossed wires, represented in elevation in Fig. 9. The central trough was placed upon rollers, and was capable of being moved

between guides in a direction parallel to the exterior troughs, by means of a milled-headed screw attached to the top of the table. In this trough was placed the bar, rs , whose expansion was to be measured, bearing at one end, r , against a frame which carried the object-glass, o , of the microscope, \mathcal{E} , and at the other against a moveable frame carrying the object-glass, o' , of the micrometer microscope. τ and τ' represent portions of the tubes of the microscopes, \mathcal{E} and \mathcal{E}' . At the commencement of the experiment the troughs were all filled with melting ice, and at the expiration of about a quarter of an hour, which was found sufficient to reduce the bars to the length corresponding to 0° Cent., the marks m, m' were brought to the centre of the parallel wires, in the fields of the microscopes, $\mathcal{E}, \mathcal{E}'$. The ice was now removed from the central trough, which was filled with hot water, and lamps, the handles of which are represented projecting under the bar rs , were lighted under it, which speedily brought the water to the boiling point, and maintained it at that temperature until the bar showed, by ceasing to expand, that it had also attained the same temperature. During the progress of the experiment the microscope \mathcal{E} was carefully watched, to ascertain if any motion took place at that extremity of the bar, and to correct it by means of the screw, in order that the whole amount of the expansion might be rendered sensible on the moveable frame at the end s ; the exterior troughs were meanwhile kept full of melting ice, to preserve unchanged the distances of the eye-pieces, and also of the marks, from one another. These precautions having been observed, the turns and parts of a turn of the micrometer screw, necessary to bring the wires of the micrometer microscope to their original position with respect to the mark, gave, by an easy computation, the displacements of the centre of its object-glass, that is, the amount of expansion of the bar rs . Let a (Fig. 11) represent the mark, o the centre of the object-glass, and c the middle point of the image of a , at the beginning of the experiment. Now suppose the centre of the object-glass transferred to p , the middle point of the image will be transferred to e , and the parallel wires must be moved through a space ce to bring them to their original position with respect to the image of a , and we are required from ce to compute op . Draw through o bd parallel to ae , then $ce = cd + de = cd + op$; but cd is the image

of $ab = op$, formed by the object-glass, $\therefore cd = m \cdot op$, m being the ratio of the image to the object, a ratio easily determined by experiment; hence $ce = (1 + m) op$ and $op = \frac{ce}{1 + m}$.*

The following Table contains the results of the experiments made with this apparatus.

TABLE of *Expansions of Metals, from Experiments made in April, 1785*.*

Description of Rods.	Revolutions and Parts of the Micrometer for the Expansion of 5 Feet by 180°.	Actual Expansion in Parts of an Inch on 5 Feet by 180°.	Length of a Rod at 212°, whose Length at 32° = 1.
Standard brass scale, . (Supposed to be Ham- burgh plate brass,—length 3.568 feet. Its expan- sion was measured by 25.47 revolutions, there- fore that on five feet would be measured by 35.69.)	35.69	0.111 323	1.001 855 4
English plate brass, in form of a rod,—length five feet,	36.41	0.113 568	1.001 892 8
English plate brass, in form of a trough,— length five feet, . . .	36.45	0.113 693	1.001 894 9
Steel rod,—length five feet,	22.02	0.068 684	1.001 144 7
Cast iron prism,—length five feet,	21.34	0.066 563	1.001 109 4
Glass tube,—five feet, Solid glass rod,—length 3.37 feet,	14.93	0.046 569	1.000 776 1
(Expansion measured by 10.46 revolutions, there- fore that of five feet would be measured by 15.54.)	15.54	0.048 472	1.000 807 8

* In Mr. Ramsden's apparatus, m equalled 3.4, therefore $(1 + m)$, which is the measure of the multiplying power of the apparatus, = 4.4; hence $op = \frac{ce}{4.4}$, and consequently the least value of op , or the smallest amount of expansion which could be distinctly observed and measured, was equal

to the least value of ce capable of being observed and measured by the micrometer, divided by 4.4. Now as 71.27 revolutions of the micrometer screw advanced the wire one inch, one revolution advanced it $1 \div 71.27$ inch; and as, by means of the graduation of the head of the screw, $\frac{1}{100}$ -th part of a

* See Philosophical Transactions, vol. lxxv. p. 480.

24. *MM. Lavoisier and Laplace's Method.*—We proceed now to explain the construction of the apparatus employed by MM. Lavoisier and Laplace in their investigations.

M, M', N, N' (Fig. 11), represent four piers of solid masonry, each about two feet by one in its horizontal section, and five feet in height. Between these lay the trough, GH, destined to contain the bar to be examined, under which was placed a furnace built of bricks, to raise the water or fixed oil surrounding the bar to the requisite temperature. The bar abutted at one end against a vertical glass rod FF, maintained in a rigidly fixed position by its connexion with strong iron bars TT, firmly bedded in the piers. The bar rested on rollers, gg, fixed at the extremity of glass stirrups, ff, and at the other end, L, acted on the arm of a lever, turning on the axis, c, whose other arm gave motion to a telescope six feet in length, directed to a graduated staff at the distance of 100 toises. The vertical bar, FF, being supposed to be absolutely fixed, and the system of levers, LC, L'C, inflexible, the whole expansion of the bar was represented by the angular movement of the telescope, oo', and the proportion of the several parts being such that an expansion of the unit of length in the bar caused the cross wires of the telescope to traverse 744 such units on the staff, the apparatus appeared capable of estimating changes of length equal to $1 \div 744$ th part of the smallest division legible on the staff.*

The results of MM. Lavoisier and Laplace's experiments are contained in the following Table:

revolution could be estimated, it follows that the effect of the micrometer was the same as if a scale divided into $\frac{1}{7127}$ ths of an inch was placed in the focus of the eye-piece; and as the divisions of such a scale would be distinctly observable with an eye-piece of sufficient power, it follows that the least value of *op* capable of being observed and measured was the $\frac{1}{7127 \times 4.4}$ th = $\frac{1}{31350.8}$ th of an inch. The bars whose expansion was examined being in general five feet in length, this minimum value equalled 0.0000053, or a little more than

the $\frac{1}{2,000,000}$ th part of the total length of the bar.

* In fact in this case the image of the staff in the focus of the eye-piece formed a finely graduated scale which was traversed by the cross wire. As the staff was divided into lines, each line being the twelfth part of a French inch, and placed at the distance of 600 feet, the actual value of each division of its image was about $\frac{1}{100}$ th

of a line, or $\frac{1}{1200}$ th of a French inch, = $\frac{1}{1126}$ th of an English inch. And since a

TABLE of linear Dilatation of Glass and Metals, from Experiments made in 1782 by MM. Laplace and Lavoisier^a.

Name of Substance.	Length at 100° C. of a Rod whose Length at 0° = 1.000000.	Dilatation from 0° to 100° expressed in fractional Parts of the Length at 0°.
Glass from St. Gobain,	1.000 890 89	1 ÷ 1122
Tube of glass without lead,	1.000 875 72	1 ÷ 1142
Do. do.	1.000 897 60	1 ÷ 1114
Do. do.	1.000 917 51	1 ÷ 1090
English flint glass,	1.000 811 66	1 ÷ 1248
French glass with lead,	1.000 871 99	1 ÷ 1147
Copper,	1.001 722 44	1 ÷ 581
Do.	1.001 712 22	1 ÷ 584
Brass,	1.001 866 71	1 ÷ 535
Do.	1.001 889 71	1 ÷ 529
Soft iron (forged),	1.001 220 45	1 ÷ 819
Do. (drawn),	1.001 235 04	1 ÷ 812
Steel (untempered),	1.001 078 75	1 ÷ 927
Do. do.	1.001 079 56	1 ÷ 926
Steel, tempered yellow (annealed at 65°),	1.001 239 56	1 ÷ 807
Lead,	1.002 848 36	1 ÷ 351
Tin (from East Indies),	1.001 937 65	1 ÷ 516
Tin (from Falmouth),	1.002 172 98	1 ÷ 462
Silver (fine),	1.001 909 74	1 ÷ 524
Silver (Standard of Paris),	1.001 908 68	1 ÷ 524
Gold (de départ),	1.001 466 06	1 ÷ 682
Gold (standard of Paris, not annealed),	1.001 551 55	1 ÷ 645
Gold (do. annealed),	1.001 513 61	1 ÷ 661
Platina (according to Borda),	1.000 856 55	1 ÷ 1167

25. *M. Pouillet's Method.*—We next proceed to notice an instrument, designed, as the preceding, to measure directly the

motion of a line at the end of the expanding bar caused the cross wire to traverse 744 divisions of the image, = 7.44 lines, it follows that the multiplying power of the system of levers was equal to 7.44; the corresponding quantity in Mr. Ramsden's apparatus was, as we have seen, 4.4. The

motion of the cross wire over one division of the scale corresponded to an expansion of $\frac{1}{100 \times 7.44} = \frac{1}{744}$ th of a line = $\frac{1}{8928}$ th of a French inch, = $\frac{1}{8377}$ th of an English inch. This was the smallest quantity which could be measured, on the preceding sup-

^a Biot, Traite de Physique, vol. i. p. 158.

linear expansion of solid bodies, invented by M. Pouillet, which possesses some considerable advantages, particularly that of enabling us to estimate the expansion of bars raised to very high temperatures, which is impossible by either of the methods previously described. M. Pouillet's apparatus* consists of a solid plate of metal, f (Figs. 12, 13), on which is placed a radius, ob , turning on the centre, o , and traversing a graduated arc, wv , whose divisions are read off by a microscope, xy . This radius carries a telescope, g , of short focal length, fixed at right angles to its direction, and a similar telescope, h , is fixed to the plate itself, allowing the radius to traverse under it. The bar under examination being placed in the copper trough (Fig. 14) furnished with plates, m , n , of parallel glass, through which its extremities may be seen, if one extremity, n , be kept opposite the fixed telescope, h , and the moveable telescope, g , be directed to the extremity, m , at the commencement of the experiment, then any expansion which the bar undergoes, by the elevation of its temperature, may be estimated by the arc through which the radius must be turned to bring the telescope, g , to bear on the other extremity, m , in its new position, the distance of the radius from the bar being accurately known. For very high temperatures the bar may be placed in a furnace, and when raised to the temperature required, apertures may be opened in the furnace walls, giving a view of the ends of the bar, and allowing its expansion to be measured in the manner described.†

positions with respect to the distance of the staff and its graduation; had the staff been divided into half lines, or placed at the distance of 200 toises = 1200 feet from the object-glass of the telescope, the value of each division of the image would have been $\frac{1}{200}$ th of a line, and the smallest expansion capable of measurement would have been $\frac{1}{17856}$ th of a French inch; but MM. Lavoisier and Laplace state, that on removing the staff to that distance, the distinctness of the image was sensibly impaired, owing to local currents in the air, and irregular refraction. Although the multiplying power of Mr. Ramsden's ap-

paratus, therefore, was inferior to that of MM. Lavoisier and Laplace's, its measuring power appears to have been far more sensitive, as well as more accurate.

* *Elemens de Physique*, vol. i. p. 234 (fourth edition).

† In the apparatus, as employed by M. Pouillet, the ratio $ob : om$ (Fig. 13), was about 3 : 1, and as the microscope was capable of reading a motion of $\frac{1}{660}$ th of a millimetre at b , the apparatus could appreciate an expansion in the bar of $\frac{1}{1980}$ th of a millimetre, or about $\frac{1}{50,000}$ th part of an inch.

26. *Mr. Daniell's Method.*—In the Philosophical Transactions for the years 1830, 1831, is contained an account of an instrument called “a register pyrometer,” for the measurement of the linear dilatation of solids, by Mr. Daniell. This instrument, represented in Figs. 16, 17, on a scale of one-half the full size, consists of two parts, of which one is called by the author the *register*, the other the *scale*. The former (Fig. 16) consists of a solid piece of black-lead earthenware, eight inches long, and seven-tenths of an inch in width and depth, cut out of a common black-lead crucible. In this a hole is drilled, about three-tenths of an inch in diameter, to the depth of seven inches and a half. The upper end of this piece is cut half through for the length of about six-tenths of an inch. A rod of the material whose expansion is to be measured, six inches and a half long, is dropped into the hole in the bar of black-lead, and presses at one end against its extremity; a piece of well baked porcelain, of the same diameter, and an inch and a half long, is placed against its upper end, and serves as an index of the expansion of the metal rod. This index is confined in its place by a platina ring and a wedge of porcelain, which so far constrain its motion as to admit of its being pushed outwardly by the expansion of the metal bar, but retain it in that position on the contraction of the latter.

The scale (Fig. 17) is constructed of two rules of brass, accurately joined together at a right angle by their edges, and fitting square upon two sides of the black-lead bar, and of about half its length. At one end of this double rule a small plate of brass, *k*, projects at a right angle, which plate, when the two sides of the former are applied to the two sides of the register, is brought down upon the shoulder formed by the notch cut away at its upper end, and the whole may be then firmly adjusted to the black-lead bar by three planes of contact.

On the outside of this frame another brass rule, *gh*, is firmly screwed down, which carries at its extremity a lever moving round the centre, *o*. When the scale is applied to the register, the index of the latter acts on a pin, *p*, in the shorter arm of this lever, at the distance of exactly half an inch from the centre of motion; the longer arm, which is five inches in length, tra-

verses a graduated arc divided into degrees and thirds, and read off by a vernier on the radius to minutes.

This instrument is thus used. The metallic rod is introduced into the hole in the register at the temperature of the surrounding medium, and the porcelain index pressed home against it, and secured in its place; the scale is next attached to the register in a determined position, by means of a guide, k , which rests on the shoulder, l , and the indication of the radius on the arc noted, or brought by means of a spring to the zero of the graduation. The scale is then removed, and the register exposed to the temperature to be measured. After it has been withdrawn and allowed to cool, the porcelain index retains the position corresponding to the maximum expansion of the rod, and the scale being again applied to the register in exactly the same position as before, the arc through which the radius must be moved to bring the point p in the arm of the lever to bear against the index, measures the quantity by which the latter has been protruded. This quantity is clearly the excess of the expansion of the metallic rod over the black lead envelope, or rather it is that excess diminished by the contraction of the index due to cooling from the higher temperature to that of the surrounding medium; the total quantity is, however, so small in all cases, that this correction of it is quite inappreciable.

On comparing the *apparent* expansion of platina and iron rods in this apparatus by immersing the register in boiling mercury, or in boiling water, with their *real* expansion, as determined by MM. Dulong and Petit, Mr. Daniell obtained the expansion of the black-lead envelope; he found this to vary in different specimens, but to be perfectly constant for the same change of temperature in the same piece.

The expansion of the envelope having been thus determined, it was only necessary to add it to the apparent expansion of any metallic rod operated on, to obtain the real expansion of the latter. By this means Mr. Daniell obtained the following table of the expansion of the metals enumerated in the first column, from the temperature of the surrounding medium, 62° F., to those of boiling water and boiling mercury.

Dimensions which a Bar takes whose Length at 62° F. is 1.000000.

Substance observed.	Length at 212° F. (Change of 150°).	Length at 662° F. (Change of 600°).	Length at Point of Fusion.
Black lead ware, .	1.000 244	1.000 703
Wedgwood ware, .	1.000 735	1.002 995
Platina,	1.000 735	1.002 995	(1.009 926 max. but not fused.)
Iron (wrought), .	1.000 984	1.004 483	(1.018 378 to fusing point of cast iron.)
Iron (cast),	1.000 893	1.003 943	1.016 389
Gold,	1.001 025	1.004 238
Copper,	1.001 430	1.006 347	1.024 376
Silver,	1.001 626	1.006 886	1.020 640
Zinc,	1.002 480	1.008 527	1.012 621
Lead,	1.002 323	1.009 072
Tin,	1.001 472	1.003 798
Brass, Zinc, $\frac{1}{2}$, . .	1.001 787	1.007 207	1.021 841
Bronze, Tin, $\frac{1}{2}$, . .	1.001 541	1.007 053	1.016 336
Pewter, Tin, $\frac{1}{3}$, . .	1.001 696	1.003 776
Type metal, . . .	1.001 696	1.004 830

27. *Mr. Adie's Method.*—Mr. Adie* read a paper in April, 1835, before the Royal Society of Edinburgh, on the expansion of different kinds of stone from increase of temperature. His pyrometer, which is admirably suited for the examination of such substances as are affected by moisture, consisted of a vertical metal cylinder, about two inches in diameter, and twenty-seven or twenty-eight inches long, which contained the rod whose expansion was to be measured. This cylinder was surrounded by a steam-tight case, through which a current of steam was passed, for the purpose of raising the temperature of the inner cylinder and contained rod. Fig. 18 represents a horizontal, and Fig. 19 a vertical section of the cylinder and case; sss is the space for the steam; w, a window of plate glass, giving a view of the end of the rod c. The rod aa (Fig. 19) rested on a support which was capable of being adjusted in height by means of a screw passing through the bottom of the cylinder, and was steadied

* Transactions of the Royal Society of Edinburgh, vol. xiii. p. 354.

against its sides by a couple of springs and friction rollers. Two silver studs were fixed in the rod, at the exact distance of twenty-three inches at the ordinary temperature. The cylinder and case were secured to a vertical beam of well-seasoned oak, to which were attached two microscopes \mathcal{E} , \mathcal{E}' , having their axes horizontal, directed towards the studs in the bar. The lower microscope enabled the observer to keep the lower stud perfectly stationary, and the upper one, by means of a micrometer, measured the expansion of the bar. The oak beam to which the microscopes were attached was protected by a screen of polished metal from the radiation of the heated case ; and as the expansion of the wood of which it was composed was ascertained, by direct observation, to amount to only .000062 of its length for a change of 180° F., the distance between the microscopes may be assumed to have continued invariable. A current of steam being passed through the case, regulated in quantity by a valve on the supply-pipe, a constant temperature could be maintained in the inner cylinder for any length of time ; and when the rod ceased to increase in length its expansion was measured. In Mr. Adie's experiments, the temperature to which he raised the rods was generally about 207° or 208° F., and it required about four hours to bring a rod, whose section was a square from half an inch to an inch on its side, to this temperature from 50° F., the ordinary temperature at the time of the experiments. From the amount of the expansion for the observed change of temperature, about 157° F., Mr. Adie calculated the following table of expansions for 180° F.

In the case of greenstone and some descriptions of marble, the effect of moisture was to increase the amount of expansion ; in other instances no effect of this kind was perceptible. Mr. Adie also found that in white Sicilian marble a permanent increase in length was produced every time that its temperature was raised, the amount of increase diminishing each time.

TABLE of *Expansion of Stone, &c.**

Name of Substance.	Decimals of an Inch on 23 Inches for 180° F.	Length at 212° of a Rod whose Length at 32° = 1.000000.	Observations.
1. Roman cement,	.033 004 3	1.001 434 9	When the rod contained more moisture it expanded more.
2. Sicilian white marble,	{ .032 539 2 .025 394 6	1.001 414 7 1.001 104 1	From first experiment when moist. Mean of three experiments when dry.
3. Carrara Marble,	{ .027 434 4 .015 040 5	1.001 192 8 1.000 653 9	From first experiment when moist. Mean of two experiments when dry.
4. Sandstone from the lime rock of Craigleith quarry,027 009 3	1.001 174 3	Mean of four ditto.
5. Cast iron from a rod cut from a bar cast two inches square, .	.026 375 5	1.001 146 7	Mean of two ditto.
6. Cast iron from a rod cast half an inch square,	.025 349 8	1.001 102 2	Mean of two ditto.
7. Slate from Penrhyn quarry, Wales,023 865 9	1.001 037 6	Mean of three ditto.
8. Peterhead red granite,	{ .022 041 6 .020 626 6	1.000 958 3 1.000 896 8	From first experiment when moist. Mean of two experiments when dry.
9. Arbroath pavement,020 665 2	1.000 898 5	Mean of four ditto.
10. Caithness pavement,020 578 8	1.000 894 7	Mean of three ditto.
11. Greenstone from Ratho,018 604 3	1.000 808 9	Mean of three ditto.
12. Aberdeen grey granite,018 156 9	1.000 789 4	Mean of two ditto.
13. Beststock brick	.012 654 2	1.000 550 2	Mean of two ditto.

* See Transactions of the Royal Society of Edinburgh, vol. xiii. p. 366.

Name of Substance.	Decimals of an Inch on 23 Inches for 180° F.	Length at 212° of a Rod whose Length at 32° = 1.000000.	Observations.
14. Fire brick,011 333 4	1.000 492 8	Mean of two experiments when dry.
15. Stalk of a Dutch tobacco pipe, . .	.010 517 7	1.000 457 3	Mean of three ditto.
16. Round rod of wedgwood ware (11 inches long)	1.000 452 9	Mean of two ditto.
17. Black marble from Galway, I.	.010 239 4	1.000 445 2	Mean of three ditto.
17*. Another rod of Galway black marble, containing more fossils, and softer than the former,	1.000 479 3	

28. *Remarks on the preceding Methods.*—These methods of experimenting are liable, it will be seen, to two classes of errors, one arising from inaccuracies in the measurement of the amount of expansion, the other from similar inaccuracies in the determination of the corresponding changes of temperature. In Ramsden's method, as we have remarked, the limits of the former class of errors may be determined, but in the others, though doubtless small, they cannot be ascertained with accuracy. In all these methods, however, there are only three or four temperatures which can be determined with rigid accuracy, namely, those of melting ice and of boiling water, oil, and mercury; and accordingly, though a few intermediate temperatures have been sought to be determined with extraordinary care, yet, owing to the difficulty of keeping all the parts of the bath at the same temperature, and maintaining the whole at a constant temperature for a length of time sufficient to insure the bar immersed in it having acquired the same, we are hardly justified in considering such observed temperatures, and the corresponding expansions, as more than approximations to the truth.

29. *M. Borda's comparative Method.*—Before concluding this part of our subject, we will describe a method by which the linear

expansion of bodies may be determined by comparison with some body of known expansion. This method was first employed by M. Borda to ascertain the expansion of the measuring rods made use of in the survey for determining the length of an arc of the meridian, and has been since applied to the determination of linear expansions generally by MM. Dulong and Petit. The method is this. A bar AB (Fig. 20) of the substance to be examined is placed on a similar bar CD , whose expansion is known, and is firmly secured to it at one end A , in a fixed line ef , while it is free to move on it at the other in a direction perpendicular to ef . At the end B it is bevelled off for a short distance to a sharp edge op , and after the two bars have been kept in melting ice for a sufficient length of time to bring them both to the temperature of 0°C. , a line $abcd$ is drawn parallel to ef . The bars are then placed in boiling water, and as soon as they have ceased to expand, the point c' on the lower bar is marked, to which the line abc on the upper bar, supposed more expansible, has advanced. Let the distance cc' at 0° be a , at the temperature T of boiling water it will be $= a(1 + \delta_T)$, δ_T being the coefficient of linear expansion of the bar CD for T° ; but this distance $a(1 + \delta_T)$ is the excess of the expansion of the length ea of the bar AB , over the same length of CD ; call this length at 0° l , and δ_T' the coefficient of linear expansion of the bar AB ; then we have $l(1 + \delta_T') - l(1 + \delta_T) = a(1 + \delta_T)$, or $\delta_T' = \delta_T + \frac{a}{l}(1 + \delta_T)$, which gives the expansion of the bar AB in terms of that of CD .*

30. *Method of determining the cubical Dilatation of Glass.*—

The linear expansion of solid bodies having been determined by any of the preceding methods, their cubical expansion may be determined from it by calculation, as will be shown hereafter; but the latter may also be ascertained by direct experiment, the cubical dilatation of mercury being first known. The method by which the expansion of mercury has been determined will be explained in the next section; we will, meantime, suppose it known, and show how, with its assistance, the cubical dilatation of other solid bodies may be ascertained.

* As it is only the ratio of a to l which occurs in this expression, those lengths may be measured at any common temperature.

And first, the cubical dilatation of glass is determined as follows. A cylindrical vessel of this material is constructed of the form represented in Fig. 21, terminating in a fine capillary tube. The vessel is filled with carefully purified mercury, which is then boiled for a considerable length of time, until all air and moisture have been completely expelled. It is next surrounded with melting ice, the end being kept immersed in mercury, until the whole mass has assumed the temperature of 0° . It is then removed from the ice, and placed, as represented in Fig. 22, in a vessel containing a small quantity of water, whose temperature is gradually raised to the boiling point, so that the glass cylinder is eventually enveloped in steam of the temperature T , at which water boils under the barometric pressure existing at the time of the experiment. During this process the mercury and glass are gradually expanding, and the portion of the former which is expelled is collected in a small capsule. When they have both attained the temperature of the surrounding steam, and have consequently ceased to expand, the vessel is removed, and the weight of the mercury remaining in it, as well as of the quantity expelled, is carefully ascertained. We have now all the data necessary for the determination of the cubical expansion of the glass vessel from 0° to T° . For let W be the sum of the weights of the mercury remaining in the vessel, and of that collected in the capsule, w the weight of the latter alone; then W is the weight of the mercury which filled the vessel at 0° , and if D be its density at that temperature, $\frac{W}{D}$ is the volume of the mercury, and consequently of the vessel, at 0° . The volume of the latter, therefore, at T , is $\frac{W}{D}(1 + \delta_T)$, if δ_T be the coefficient of cubical dilatation of glass for T° . It is clear also that $W - w$ is the weight of mercury which, at the temperature T , fills the expanded vessel; the volume of this mass at 0° , therefore, is $\frac{W - w}{D}$, and at T it is $\frac{W - w}{D}(1 + \Delta_T)$, Δ_T representing the coefficient of cubical expansion of mercury for T° . Equating the expressions of those quantities, namely, of the volume of the vessel at T , and of the volume of the mercury which fills it at that temperature, we have

$$\frac{W-w}{D}(1+\Delta_r) = \frac{W}{D}(1+\delta_r);$$

and therefore

$$\delta_r = \Delta_r - \frac{w}{W}(1+\Delta_r).$$

31. *Method of determining the cubical Dilatation of Metals.*—The expansion of glass being thus ascertained, the expansion of iron is determined as follows. Let a bar of iron, whose weight and density at 0° are known, be enclosed in a glass vessel shaped as in the last experiment, and the vessel then filled with mercury, and boiled as before. Let it be cooled down to 0° , and the mercury which is expelled when its temperature is subsequently raised to T° , the temperature of the vapour of boiling water, collected and weighed. Let the weight of that which remains be also ascertained. Let W be the sum of those weights, w the weight of mercury expelled, D the density of mercury at 0° , w' and d' the weight and density at 0° of the iron, Δ_r , δ_r , δ_r' , the coefficients of cubical expansion of mercury, glass, and iron for T° .

Then the mercury, whose weight is $W-w$, and accordingly its volume at $0^\circ = \frac{W-w}{D}$, and the mass of iron whose volume at the same temperature equals $\frac{w'}{d'}$ on being raised to T° , fill the vessel whose volume at 0° was $\left(\frac{W}{D} + \frac{w'}{d'}\right)$, and which at T° becomes $\left(\frac{W}{D} + \frac{w'}{d'}\right)(1+\delta_r)$; therefore we have

$$\frac{W-w}{D}(1+\Delta_r) + \frac{w'}{d'}(1+\delta_r') = \left(\frac{W}{D} + \frac{w'}{d'}\right)(1+\delta_r);$$

hence

$$\delta_r' = \delta_r + \frac{w}{w'} \cdot \frac{d'}{D}(1+\Delta_r) - \frac{W}{w'} \frac{d'}{D}(\Delta_r - \delta_r).$$

The cubical expansion of some of the metals which are chemically acted on by mercury may be obtained by the same method, if, previous to their immersion in the mercury, they are protected from its action by covering them with a thin film of their oxide. Or the cubical expansion of such metals may be

computed from the linear, and this latter may be determined by the comparative method described in (29), from the linear expansion of iron, previously computed from its cubical expansion.

The following Table contains the results of MM. Dulong and Petit's experiments on the cubical dilatation of glass and some metals.

TABLE of the cubical Dilatation of Glass, &c.*

Substance.	Mean Coefficient of Dilatation for 1° between 0° and 100°.		Mean Coefficient for 1° between 0° and 300°.	
	In vulgar Fractions.	In Decimals.	In vulgar Fractions.	In Decimals.
Glass, . .	$1 \div 38\,700$	0.000 025 839	$1 \div 32\,900$	0.000 030 395
Iron, . . .	$1 \div 28\,200$	0.000 035 461	$1 \div 22\,700$	0.000 044 052
Copper, .	$1 \div 19\,400$	0.000 051 546	$1 \div 17\,700$	0.000 056 497
Platina, .	$1 \div 37\,700$	0.000 026 525	$1 \div 36\,300$	0.000 027 548

MM. Dulong and Petit also give, as the value of the mean coefficient of glass for 1° between 0° and 200°, in vulgar fractions, $1 \div 36\,300$, or in decimals, 0.000 027 548.

M. I. Pierret† has since found, by the same method, that the mean coefficient of the cubical dilatation of glass for 1° between 0° and 100° varies in different specimens from 0.000 019 026 to 0.000 026 025.

32. *Laws of Dilatation of solid Bodies.*—Having thus briefly described the principal methods of determining by experiment the amount of dilatation of solid bodies, we will proceed next to state the general laws derived from them.

LAW I.—*All solid bodies, which do not undergo any physical change or loss of substance by the action of heat, increase in volume with increase of temperature, and on being restored to their initial temperature resume accurately their original volume.*

The case of clay balls, which contract on being exposed to a very high temperature, and do not return to their original volume,

* Annales de Chimie et de Physique, tome vii. p. 138.

† Id. tome xv. p. 335 (3^{me} Serie).

offers only an apparent exception to this rule. For it has been proved that the diminution of volume in this instance arises from an actual loss of substance, owing to the liberation of water, which is held in such intimate union by the alumina of the clay, that even a red heat is insufficient altogether to expel it.

We will see that this law holds also in the case of liquid and gaseous bodies, a few of the former, when near the temperature at which they undergo a change of state, being excepted.

A singular anomaly occurs in the case of Rose's fusible metal, —an alloy consisting of two parts bismuth, one lead, and one tin, which melts at 75° R. M. Erman* has found that the melted metal contracts, on cooling, according to the general analogy, until it solidifies at the temperature of 75° ; after this it still continues to contract until it attains to 55° , from which point to 35° , on the contrary, it expands, and after that contracts again, and continues to do so to the lowest temperatures. On tracing a curve whose ordinates shall represent the increments of volume, in parts of the original volume at 0° , and abscissæ the degrees on Reaumur's scale, the course of the curve from the origin to the abscissa, 35° , approaches nearly to a right line, showing that here the change of volume is nearly proportional to the temperature: at 35° there is a maximum ordinate: from 35° to 55° the curve descends towards the axis of the abscissæ, more rapidly at first, and more slowly as it approaches the latter point, when the ordinate has a minimum value: on passing 55° the ordinates increase, at first slowly, then more rapidly up to the abscissa corresponding to the seventy-fifth degree, the melting point of the metal: from 75° to 80° the rate of increase is still considerable, but after 80° it resumes a rate of progress which appears rigorously the same as that which was observed before the anomalous changes occurring between 35° and 75° ; so that in this metal the change of volume is *q. p.* proportional to the change of temperature, with the exception of the part of the scale between the points above-mentioned, where an anomalous oscillation occurs. It is to be remarked, that the volume at the melting point, 75° , is the same as that at the point of relative maximum, 35° , so that the

* Annales de Chimie et de Physique, tome xl. p. 197.

horizontal tangent to the curve at the latter point cuts the ascending branch of the curve, at the point whose abscissa is 75° . It is probable that this anomaly is connected with some change which takes place in the arrangement of the molecules of the metal consequent on the change from the liquid to the solid state, and that it is analogous to a similar phenomenon in the case of water, with this difference, that in the latter substance the anomaly begins to exhibit itself before the change takes place, but in the fusible metal not until afterwards.

33. LAW II.—*Homogeneous uncrystallized solid bodies expand uniformly in all their dimensions, so as to preserve similarity of figure.*

By means of this law we are enabled to establish a relation between the cubical, superficial, and linear expansions of a body, that is, the expansion which a body undergoes in volume, in the area of any surface, and in the length of any of its edges or sides.

Let V represent the volume of a body at any temperature, δ_t its coefficient of *cubical* expansion referred to that temperature for t° , so that $\delta_t V$ expresses its expansion for the given change of temperature. Similarly let S be the area of any surface at the original temperature, and δ'_t the coefficient of *superficial* expansion, L the length of any edge of the body, and δ''_t the coefficient of *linear* expansion, all these coefficients referring to the same original temperature and same change (t°), so that $V(1 + \delta_t)$, $S(1 + \delta'_t)$, $L(1 + \delta''_t)$, will express what V , S , and L become after expansion. Then as the volumes of similar bodies vary as the cubes of their homologous sides, we have $V(1 + \delta_t) : V :: L^3(1 + \delta''_t)^3 : L^3$, and $\therefore \delta_t = 3\delta''_t + 3\delta''_t{}^2 + \delta''_t{}^3$; whence if δ''_t be so small that we may neglect its square and higher powers, as is the case with most solid bodies, even for considerable changes of temperature, we have

$$\delta_t = 3\delta''_t.$$

In like manner, as similar surfaces vary as the squares of their homologous sides, we have $S(1 + \delta'_t) : S :: L^2(1 + \delta''_t)^2 : L^2$, and $\delta'_t = 2\delta''_t + \delta''_t{}^2$, or, neglecting $\delta''_t{}^2$,

$$\delta'_t = 2\delta''_t \text{ and consequently } \delta_t = \frac{3}{2} \delta'_t.$$

For any change of temperature, therefore, the corresponding coefficients of cubical, superficial, and linear expansion are in the ratio 3:2:1, if the last mentioned coefficient be so small that its square and higher powers may be neglected.

A hollow body, it may be remarked, expands in all its dimensions, exactly as a solid body of the same volume and material. For conceive a sphere composed of successive concentric shells, such a sphere must expand exactly as a solid one, and therefore every shell separately must expand as if the others were away.

34. LAW III.—*Some crystallized bodies do not expand uniformly in all directions under the action of heat.*

Change of temperature in such bodies is accompanied both by a change in their optical properties, and also in the dimensions of the solid angles formed by the faces of their crystals. M. Fresnel* appears to have been the first who discovered this property of crystals, to which he was led by observing that the double refracting power of sulphate of lime was sensibly diminished by a rise of temperature. He subsequently proved by a very simple experiment the unequal dilatation of this crystal in different directions. Having cut two very thin plates from a crystal of sulphate of lime, parallel to its prismatic axis, that is, to the line bisecting the acute angle formed by its optical axes, he placed one upon the other with an intermediate layer of cement, in such a position that the direction of the axis of one plate formed a right angle with that of the other. The temperature of the plates was then raised sufficiently to melt the cement, after which they were allowed to cool. When, in the process of cooling, the cement had become sufficiently hard to prevent one plate from sliding on the other, the unequal contraction of the plates caused them to warp or bend, in such a manner that each plate formed a concave surface in the direction in which it was most dilatable, that is, in the direction perpendicular to its axis, while it formed a convex surface in the direction at right angles to this.

The first direct experiments on the change of form of crystals, and hence on their unequal expansion in different directions by

* Bulletin de la Société Philomathique, December, 1823.

heat, were made by M. Mitscherlich.* Measuring the angles of crystals of carbonate of lime by the goniometer, he found that the obtuse angles of the edges of the rhombohedron diminished by $8'.5$ on a change of temperature from 0° to 100° C., and that the acute angles of the other edges increased. Hence it followed that the smaller axis of the rhombohedron was more elongated than the other diagonals, and that the form of the crystal consequently approached more nearly to a cube. From the alteration above mentioned, produced in the angles of crystals of carbonate of lime, it follows, that, supposing the dilatation in the direction perpendicular to its axis to be zero, its cubical dilatation should still exceed that of glass by about one-half. Experiment, however, proves, on the contrary, that it is less, from which we are forced to conclude, that while, on rise of temperature, this crystal *dilates*, according to the general analogy, in the direction of its axis, it actually *contracts* in the direction perpendicular to this. In confirmation of this conclusion, M. Mitscherlich records, that on measuring at different temperatures, by means of a spherometer, the thickness of a plate of calcareous spar, cut parallel to its axis, he obtained a result similar to the preceding. From this it is probable that sulphate of lime presents an analogous phenomenon, but in the inverse direction, namely, that the action of heat produces in its crystals a contraction in the direction of the axis, combined with a dilatation in the perpendicular directions.

M. Mitscherlich† gives the following as the results of his investigations into the action of heat on crystals, a detailed account of which he has published in the Transactions of the Academy of Berlin for 1825.‡

(1). Crystals belonging to the regular system, and which accordingly do not possess the property of double refraction, dilate uniformly in all directions, and exhibit no alteration in the value of their angles on rise of temperature.

(2). Crystals whose primitive form is a rhombohedron or hex-

* Annales de Chimie et de Physique, Janvier 4, 1834.

xxxii. p. 111.

† Poggendorff's Annalen, 1824, No. 5, and Ann. de Chimie et de Physique, tome

‡ And also in Poggendorff's Annalen, 1827, No. 5.

aedral prism, that is, those belonging to the hexagonal system,* are differently affected by heat in the direction of the principal axis, and in the direction of the three secondaries, but in the direction of the latter they are similarly affected. Thus we have seen that carbonate of lime expands in the direction of its principal axis, that is, the line uniting the obtuse solid angles of the rhombohedron, and contracts in the direction of the secondary axes perpendicular to the former.

(3). Crystals whose primitive form is a rectangular or rhomboidal octahedron,† or generally all whose double refraction depends on two axes, dilate unequally in all directions.

(4). The dilatation of crystals in different directions is always connected with the position of their axes of crystallization, and as these have a necessary relation to the optical axes, it follows that their dilatation has a fixed relation to the latter. In general the shorter axes dilate more than the longer, or dilate alone, while the latter contract. Hence Mitscherlich concluded that the tendency of heat is to increase the mutual distance of the molecules in the direction in which they are most condensed, and consequently to equalize those distances in different directions, and so bring the different axes to a state of equality.

35. LAW IV.—*Relation between the Expansion of Solids and the apparent Expansion of Mercury in Glass.*

Between the limits of 0° and 100° Cent., the expansion of all solid bodies is sensibly proportional to the change of temperature, as measured by the mercurial thermometer. In other words, between those limits their expansion for any change of temperature is proportional to the apparent expansion of mercury in glass for the same change. After passing the limit of 100° , however, the expansion of solid bodies is found to increase in a more rapid ratio than their temperature, the latter being still measured by the mercurial temperature. MM. Lavoisier and Laplace observed a striking exception to this latter rule in the case of tempered

* The third or rhombohedral system of Rose; optically, uniaxal. We may assume that this law applies also to the second or right square system of Rose, which, like the third, is optically uniaxal.

† Including the fourth or right prismatic,

and the fifth or oblique prismatic systems of Rose. Optically, biaxal. This law may be assumed to apply also to the sixth or doubly oblique system of Rose, which is, like the two former, optically biaxal.

steel, whose rate of expansion diminishes as the temperature increases.* This anomaly appears to be due to the fact that steel is partially deprived of its temper by the action of heat, and so brought gradually nearer to the condition of untempered steel, whose dilatability is less than that of tempered. We may take this opportunity of remarking, that in the process of tempering steel undergoes a permanent dilatation, varying in amount with the dimensions and shape of the mass, and the temperature at which it is suddenly cooled; and hence the violent torsions and alterations of form which pieces of steel frequently undergo in tempering.

36. *Expression for the Volume, &c., of a Solid in Terms of its Temperature and Coefficient of Expansion.*—It follows, from the law just stated, that for solid bodies, between the limits 0° and 100° , the coefficient of dilatation varies with the number of degrees expressing the corresponding change of temperature; for as the expansion may always be expressed in the form $\delta_t Q$, where δ_t is the coefficient of expansion corresponding to a given number t of degrees, and Q the volume, surface, or length, and as $\delta_t Q$ varies as t , it follows that δ_t varies as t , and hence if k expresses the value of the coefficient of expansion for 1° , kt will express its value for t° . In practice the amount of expansion determined is almost always that corresponding to the change of temperature from 0° to T° Cent., T° being the temperature of the vapour of water boiling under the atmospheric pressure existing at the time of the experiment, or, what is nearly the same thing, the temperature of the upper stratum of water boiling in a metallic vessel under that pressure. This temperature is, in many instances, assumed to coincide with the boiling-point of water under the normal pressure, that is, 100° C., or 212° F.; but where strict accuracy is desired, the value of T° should be determined for the given circumstances of the experiment by the method indicated in the second chapter of this Book.

The coefficient of expansion usually determined, therefore, is the fraction of the value at 0° C., by which a body expands in volume, surface, or length for T° , and the coefficient for 1° is

* Biot, *Traité de Physique*, tome i. p. 157.

$\frac{1}{T}$ th part of this. Hence if V_0 denote the volume at 0° , and V_t the volume at t° , we have $V_t = V_0(1 + kt)$, and similarly if $V_{t'}$ be the volume at t' , $V_{t'} = V_0(1 + kt')$, whence in general

$$\frac{V_t}{V_{t'}} = \frac{1 + kt}{1 + kt'}$$

by means of which expression we can compare directly the volumes at t and t' .

We have, in the same way,

$$\frac{S_t}{S_{t'}} = \frac{1 + kt}{1 + kt'}, \text{ and } \frac{L_t}{L_{t'}} = \frac{1 + k''t}{1 + k''t'};$$

k , k' , k'' , the coefficients of cubical, superficial, and linear expansion for 1° C. being, as was shown before $q : p$ in the ratio $3 : 2 : 1$.

The numerical values of the coefficients k , k' , k'' , which are generally given in physical treatises, refer, as we have mentioned, to the volume at 0° C. As there is no reason, however, for fixing upon this volume as the standard, except the facility of determining the corresponding temperature, the coefficients may be referred to the volume at any other temperature, and their corresponding numerical values obtained as follows. In the equation

$$V_t = V_{t'} \frac{1 + kt}{1 + kt'}$$

put $t = t' + \tau$, then

$$V_t = V_{t'} \left(1 + \frac{k}{1 + kt'} \tau \right) = V_{t'} (1 + \kappa \tau);$$

$$\text{or } V_\tau = V (1 + \kappa \tau);$$

if, therefore, we count our degrees from a temperature t° , and denote them, when counted from this point, by the symbol τ , and take the volume at t' as the standard, then the coefficient, as referred to this standard, bears to the coefficient k , referred to the volume at 0° as standard, the relation expressed by the equation

$\kappa = \frac{k}{1 + kt'}$.* Similar relations hold, of course, for the coefficients

* If k be expressed by a vulgar fraction whose numerator is unity, and denominator

n , κ will be expressed by a similar fraction whose denominator is $n + t'$.

of superficial and linear expansion. It must be borne in mind that these methods of expressing the volume at one temperature in terms of the volume at another, only apply between the limits within which the fourth law is true.

37. *Remarks on the Values of the Coefficient of Dilatation of various Solids.*—The coefficient of dilatation of glass varies considerably with its composition. MM. Lavoisier and Laplace found it to be less in proportion as the glass contained more lead.* M. Regnault, who has been led to examine this subject with particular care, has found not only that the dilatation of *different* kinds of glass varies within very wide limits, but that the value of the coefficient varies in different pieces of the *same* kind, according as they are in the form of tubes, or have been blown into bulbs of different dimensions, and that even the dilatation of the same apparatus, between the same limits of temperature, is not always identical. "The differences," says M. Regnault, "observable in the dilatation of the same piece of glass, when in the form of a tube, and when blown into balls of different dimensions, do not appear to follow any simple law. Thus, ordinary white and green glass dilate less in the form of a ball than in that of a tube. The contrary is true in the case of Swedish, infusible French glass, and crystal. The same glass, formed into a ball, appears to dilate more as its diameter is greater, or perhaps as the thickness of its sides is less. In every case we see how liable we are to fall into serious errors, in experiments requiring extreme accuracy, from calculating the dilatation of an instrument of glass, by means of the coefficient obtained even from direct experiments on a tube or ball of the same material, much more from the linear dilatation of a rod of the same kind of glass, as has been done by many eminent physicists."†

The coefficients of cubical dilatation of glass for 1° between 0° and 100° vary, as we have seen (p. 40), according to M. I. Pierre's experiments, from .000019026 to .000026025, so that the difference between the extreme values exceeds one-fourth of the

* Biot, *Traité de Physique*, tome i. p. 157.

† *Ann. de Chim. et de Phys.* 3^{me} Ser. tome iv. p. 67.

least. M. Regnault has also found that different kinds of glass follow different laws in their dilatation, those whose coefficients of expansion are smallest increasing less rapidly in the rate of their dilatation.*

The coefficient of iron also differs very much in different specimens, thus confirming the result derived from other sources, that the iron of commerce is by no means an identical metal. The same remark applies to brass; in fact, it is to be observed, generally, that unless we are certain that the strictest physical identity in composition and molecular arrangement exists between two bodies, we are not justified in assuming that their coefficients of expansion possess the same numerical value.

SECT. II.—DILATATION OF LIQUIDS.

38. *Definitions; Relation between Coefficients of apparent and real Expansion.*—As liquids can be experimented on only in solid envelopes, and as on a rise of temperature the latter must increase in volume as well as the former, it is obvious that the expansion of the liquid in the envelope, or, as it is called, its apparent expansion, will be modified by the expansion of the envelope itself. Before proceeding to describe the methods of determining by experiment the expansion of liquids, we will point out the relation existing between the real expansion of the liquid, that is, its actual increase of volume due to any rise of temperature,—its apparent expansion, or the increase of its volume over that of the envelope,—and the expansion of the latter. For this purpose let us suppose the liquid contained in a graduated tube with bulb attached, similar to a thermometer tube, and let V_0 represent the volume occupied by the liquid at the temperature 0° ; let V'_t represent its volume at t° , as estimated by the number of divisions it fills in the tube supposed inexpandible, and V_t the volume which it really occupies at that temperature, the expansibility of the envelope being taken into account. Further, let α_t denote the coefficient of *apparent* expansion of the liquid for t° , δ_t its coefficient of *real* expansion, and k_t the coeffi-

* Ann. de Chim. et de Phys., 3^me Ser., tome v. p. 102.

cient of expansion of the envelope for the same change of temperature. Then since $V_t = V_o (1 + \delta_t)$, and $V_t = V'_t (1 + k_t)$, we have

$$V_o (1 + \delta_t) = V'_t (1 + k_t);$$

but we have also $V'_t = V_o (1 + a_t)$; therefore

$$V_o (1 + \delta_t) = V_o (1 + a_t) (1 + k_t),$$

or

$$\delta_t = a_t + k_t + a_t k_t.$$

The last term of the right hand member of this equation is so small that it may, in most cases, be neglected, so that we may, in general, assume

$$\delta_t = a_t + k_t,$$

that is, *the coefficient of the real expansion of a liquid for a given change of temperature equals q. p. the coefficient of its apparent expansion + the coefficient of expansion of the solid envelope for the same change.*

39. *MM. Dulong and Petit's Method.*—The first method of ascertaining the expansion of a liquid to which we will direct the attention of the student, is founded on a principle suggested by the Hon. Mr. Boyle, for the determination of the relative density of different fluids, and applied by MM. Dulong and Petit to the investigation of the expansion of mercury. Although not so sensitive in the appreciation of small changes of volume as other methods which will be described subsequently, this method yet possesses the great advantage of enabling the experimenter to determine directly the true expansion of any liquid whose physical properties admit of its application, without a knowledge of the expansion of the envelope in which it is contained.

The principle on which this method is based is, that if a U-shaped tube contains in its vertical arms fluids of different densities, when those fluids are *in equilibrio*, the weight of a cylindrical column of the fluid in one arm, standing on any base, is equal to the weight of a cylindrical column of the fluid in the other arm, standing on an equal base in the same horizontal plane; and the volumes of those columns will be as their heights, since their bases are equal. Hence we can

obtain immediately the ratio of the volumes of equal weights of the same liquid at different temperatures, and consequently at different densities, or, what is the same thing, the ratio of the volumes of the same mass at two different temperatures, from which we can calculate, as usual, the coefficient of expansion for the corresponding change of temperature. For, let the fluids in the two arms be at 0° and t° , and conceive two cylindrical columns standing on equal bases; let the heights and volumes of these columns be $H_0, H_t; V_0, V_t$; we have $V_t : V_0 :: H_t : H_0$; but $V_t = V_0 (1 + \delta_t) \therefore V_0 (1 + \delta_t) : V_0 :: H_t : H_0$ and $\delta_t = \frac{H_t - H_0}{H_0}$.

The following is a brief description of the apparatus employed by MM. Dulong and Petit for the application of this principle; a complete account of it is contained in their original memoir, which appeared in the seventh volume of the *Annales de Chimie*.

The tube containing the mercury, which, as we have mentioned, was the liquid to which this method was applied by MM. Dulong and Petit, was placed, as in Fig. 23, on a strong T-shaped bar of iron, furnished with two levels, and placed on a board provided with levelling screws, so that the upper surface of the iron bar, and consequently the branch BC of the tube, could be rendered perfectly horizontal. The branch BC and the lower portions of the vertical arms were of very small diameter, for the purpose of diminishing the mass of mercury employed in the experiment, and also preventing the ready communication of the fluids, at different temperatures, in the two arms. Two pillars supported the vertical arms; of these, one carried an index, whose height above the level of the bar was accurately determined, and, being maintained at the temperature of 0° , remained constant during the whole of the experiment. This pillar and the bar AB were surrounded by a tin cylinder containing melting ice, and having a small aperture at the top, through which, on removing a portion of the ice, at any instant, the surface of the mercury in the tube could be seen. The other arm was surrounded by a cylinder of copper, filled with a fixed oil, which can be raised to the tempe-

perature of 300° C. without boiling, and kept at a constant level by means of a waste pipe. Plunged in the oil, parallel to the arm DC, were two thermometers, one an air thermometer, constructed in a manner which will be explained in the next section, the other a mercurial *weight thermometer*. This latter instrument is similar in its construction to that described in (30), and its application to the measure of temperatures is easily explained. From what was said in the passage referred to, it appears that if W be the weight of mercury filling such an instrument at 0° , w the weight expelled by the expansion due to the temperature t° , we have

$A_t = \frac{w}{W-w}$; but A_t , the coefficient of relative expansion of mercury in glass for t° , $= Kt^{\circ}$, if K be the coefficient for 1° ; hence

$t = \frac{1}{K} \frac{w}{W-w}$. Similarly, if w' be the weight expelled at T° , the temperature of water boiling under a known pressure, we have

$T = \frac{1}{K} \frac{w'}{W-w'}$; hence $t = T \frac{w}{w'} \cdot \frac{W-w'}{W-w}$, from which equation,

knowing w' in a given instrument, we know t° , the temperature corresponding to w .

The level of the mercury in the arm DC was always kept one-half a millimetre (about one-fiftieth of an inch) above the level of the surrounding oil; this was effected by adding or removing a small quantity from the arm AB. A brick furnace surrounding the copper cylinder was used for the purpose of heating the oil; and before making the experiment all the apertures of the furnace were closed, by which means the oil could be maintained at a constant temperature for a considerable period.

Such were the arrangements for determining the temperature of the mercury in the two arms; it remains now to describe the method of ascertaining the height at which it stood in them. This was effected by means of an instrument, since called a *kathetometer*, and extensively used in physical investigations for the purpose of measuring small differences of vertical heights. This instrument (Figs. 24, 25, 26) consists of a massive vertical pillar a , standing on a tripod, which rests on levelling screws. Attached to the pillar, and moveable round it as an axis, is a

finely graduated rule bb' , which carries a horizontal telescope d , of short focus.* The telescope is also furnished with a spirit level e , and a screw k , for altering slightly its inclination to the vertical. Preparatory to using this instrument, the pillar, or rather the axis round which the rule and telescope turn, must be rendered perfectly vertical, and the telescope itself truly horizontal; and this will be known to be the case when the telescope preserves its horizontality, as shown by its spirit level, during a complete revolution round the axis. The kathetometer being thus adjusted, the telescope is directed towards the index x , and the cross wires in its focus brought to coincidence with the extremity of the index; it is then directed successively to the levels of the mercury in AB and DC , and the quantities by which the telescope is depressed along the graduated rule measure the differences between the altitude of the index and those of the mercurial columns; and the height of the former above the base bar being known, the heights of the latter are determined.

Great care was requisite, in the use of this apparatus, to insure uniformity of temperature through the whole of the liquid in the copper cylinder; and the observations made with the kathetometer required to be conducted with the utmost care, in order to obtain accuracy in the measurement of the expansions, as the greatest difference of level in the two tubes was very small, being only $1 \div 55.5$ th part of the shortest between 0° and 100° .

By this method MM. Dulong and Petit determined the coefficient of expansion of mercury, from 0° to 100° , to be $= 1 \div 55.5$, or the circulating decimal 0.018.

40. *Method of Densities.*—The *second* method of ascertaining the expansion of a liquid from 0° to t° consists in determining the weights of known volumes of the liquid at 0° and t° , from which we can calculate the volumes of equal weights, or of the same mass of liquid at those two temperatures. This may be effected in several ways.

(1). The liquid may be contained in an envelope similar to that

* This telescope is attached to a frame f , which is capable of being moved with gentle friction along the rule bb' , and when it has been thus brought nearly to its position, the

frame is clamped by means of the screw g , and the more accurate adjustment of the telescope is effected by means of the micrometer screw h .

in Fig. 21, whose expansion has been previously determined from the expansion of mercury by the method described in 30. The liquid, after having been long and carefully boiled, to free it from air, is placed in melting ice, the orifice of the capillary tube being kept immersed in a quantity of the same liquid, until the whole mass has been brought to the temperature of 0° . It is then removed from the ice, and the temperature gradually raised to t° , the liquid which escapes, in consequence of the expansion, being collected in a little capsule. Let w be the weight of liquid expelled, $W - w$ the weight of that remaining in the vessel at t° , and let V_0 be the volume of the vessel at 0° , k its coefficient of expansion for 1° . Then we have from the experiment:

W = the weight of the volume V_0 of the liquid at 0° , and

$W - w$ = the weight of the same volume V_0 at t° , if we neglect the expansion of the envelope.

$W - w$ = the weight of the volume $V_0(1 + kt)$ at t° , if we take the expansion into account.

In the former case, as V_0 is the volume of liquid at t° , corresponding to a weight, $W - w$, the volume at the same temperature, corresponding to the weight, W , will be $V_0 \frac{W}{W - w}$; the *apparent* expansion of the mass of liquid corresponding to the weight W therefore from 0° to t° is

$$V_0 \frac{W}{W - w} - V_0 = V_0 \left(\frac{w}{W - w} \right);$$

and consequently the coefficient of *apparent* expansion

$$a_t = \frac{w}{W - w}.$$

In the latter case the volume at t° corresponding to the weight $W - w$, is $V_0(1 + kt)$; therefore the volume at the same temperature of the weight, W , will be $V_0(1 + kt) \frac{W}{W - w}$; hence the *real* expansion of the mass whose weight is W is

$$V_0(1 + kt) \frac{W}{W - w} - V_0 = V_0 \left(\frac{Wkt + w}{W - w} \right);$$

and consequently the coefficient of *real* expansion

$$\delta_t = \frac{Wkt + w}{W - w}.$$

(2). The liquid may be contained in a vessel similar to that represented in Fig. 27, and the weight of the quantity filling it up to a certain height, *mn*, at two temperatures, 0° and t° , ascertained. Then, the coefficient of expansion of the vessel being known, the coefficients of apparent and real expansion of the liquid may be determined as in the last method.*

(3). Or lastly, a ball of some material whose expansion is known, may be weighed in air and in the liquid, at the temperatures 0° and t° , care being taken to allow the ball, on both occasions, to acquire the temperature of the liquid. If *w* be the difference between the weights in air and in the liquid at 0° , and *w'* at t° , we obtain, by a process of reasoning similar to that in the preceding methods,

$$a_t = \frac{w}{w'} - 1,$$

$$\delta_t = \frac{w}{w'} (1 + kt) - 1.$$

This, which is called the areometric method, was employed by M. Hälström in the determination of the expansion of water; it is inapplicable, however, as well as some other of the preceding methods, to the case of such liquids as are altered by oxygen or atmospheric moisture, as well as to those which are highly volatile.

41. *Method of graduated Tubes.*—The *third* method of determining the expansion of liquids consists in constructing with them thermometers similar to the ordinary ones, and comparing their indications with those of a standard mercurial thermometer. M. I. Pierre has employed this method in an extensive series of experiments, which appear to have been conducted with the greatest care, and of which he has published a detailed account in the fifteenth, nineteenth, and twentieth volumes of the *Annales de Chimie et de Physique* (3^{me} Serie).

* *Annales de Chemie*, 3^{me} Ser., vol. ix. p. 338.

The apparatus employed by M. I. Pierre consisted of a cylindrical vessel, AB , Fig. 32, of galvanized metal, twenty-five centimetres in diameter, and twenty-six centimetres high.

On its lid, AC , which was furnished with a tundish, E , was soldered a rim, hh' , of the same metal, within which was fixed, by means of a collar of leather, a cylindrical envelope of glass, twenty-seven centimetres high and seventy-seven millimetres in diameter. In the centre of the lid and of the rim was an opening with a short flange, in which were fixed, by means of a cork, the standard thermometer, r , and the thermometer r' , containing the liquid to be examined.

The stems and reservoirs of these two thermometers were as nearly as possible of the same dimensions, and the reservoirs were always kept at the same height, so as to occupy the same stratum of the liquid in the vessel AB , designed to raise their temperature.

Two other thermometers were completely plunged in the water contained in the glass envelope, to the points to which the liquids filling them respectively rose. The first, t , was a mercurial thermometer, and had a very long reservoir, whose diameter differed little from that of the stems of the larger thermometer; t' was a thermometer containing the same liquid as r' . These two thermometers were intended, the first, t , to give the temperature of the water in the envelope, and of the portions of the stems plunged in it, and t' to estimate the correction to be applied to the apparent volume of the liquid in r' .

To maintain the temperature of the water in the upper vessel constant, which was rendered difficult by its contact with the lower vessel, AB , especially when the temperature of the latter was at all elevated, a stream of water was conducted from a cistern, R , through a lead pipe, into the lower part of the envelope, and a syphon, $SS'S''$, furnished with a cock, constantly drew away an equal quantity from the upper part of the vessel. The quantity of water necessary to be supplied from the cistern varied with the temperature of AB , and was regulated by the cock, r .

A double agitator, $aa'bb'' cc'dd''$, served to agitate simultaneously the water in the two vessels, and to establish a uniform temperature in the liquid masses occupying them respec-

tively. The upper part of this agitator is represented in plan in Fig. 33.

For temperatures above that of the surrounding medium, the vessel AB was placed over a large furnace, whose draught could be so regulated as to maintain a constant temperature in the apparatus, during each observation, for a period of fifteen or twenty minutes. The agitator being kept constantly in motion, the instant was noticed when the liquids in τ and τ' attained their maximum heights, and those heights were observed by means of a small horizontal telescope. The instant of the maximum of τ' differed scarcely at all from that of τ . The indications of the thermometers t and t' were then successively noted.

When it was required to produce a temperature in AB lower than that of the surrounding medium, a stream of cold water was introduced through the tun-dish E, and an equal quantity drawn off by the cock B. When the temperature became stationary, the cock B was closed, the stream of cold water stopped, and the agitator kept in motion until the instant of the minimum in τ and τ' .

For temperatures below 0° a somewhat simpler form of vessel was used, filled with a mixture of alcohol and water, and the reduction of temperature was effected by surrounding the apparatus with a mixture of pounded ice or snow and crystallized chloride of calcium. By adding suitable quantities of snow or chloride, a stationary temperature of -35°C. , or even -36°C. , was obtained for at least fifteen minutes, when the temperature of the surrounding medium did not exceed $+1^\circ$ or $+2^\circ$.

The thermometers employed in these experiments were constructed with every precaution, and the expansion of the crystal glass of which they were composed ascertained separately for each. The chemical and physical properties of the liquids examined were also accurately determined by M. I. Pierre himself.

The calculation of the observations was made as follows:

Let X be the true temperature of the lower vessel;
 x the temperature as observed on the thermometer τ ;
 θ that of the upper vessel given by t ;

k' the coefficient of apparent dilatation of mercury in the glass forming the envelope of τ : then the formula

$$X = x + nk'(X - \theta).$$

will give the value of X .*

If now we denote

- By V_o the volume at 0° of the liquid contained in τ' ;
- By V_o' the volume at 0° of the part of the thermometer τ' in the lower vessel;
- By V_o'' the volume at 0° of the part in the upper vessel;
- By V_x the total volume which the liquid would occupy at X° ;
- By δ_x the coefficient of true expansion of the liquid for X° ;
- By δ_θ the same coefficient for θ° ; and finally,
- By k the coefficient of cubic dilatation of the envelope of τ' :

then we will have

$$V_x = V_o(1 + \delta_x).$$

$V_o'(1 + kX)$ will represent the volume at X° of the liquid in the lower vessel; and

$V_o''(1 + k\theta)$ the volume at θ° of that in the upper.

And first it is requisite to determine what this latter volume would become at X° .

Let U_o be the volume occupied by the liquid at 0° in the thermometer τ' ;

U_o' the volume apparently occupied by the same at θ° ; and

k'' the coefficient of expansion of the envelope of τ' :

then the volume really occupied by the liquid in τ' at θ° is

$U_o'(1 + k''\theta)$; therefore we have

$$U_o'(1 + k''\theta) : U_o :: V_o''(1 + k\theta) : \frac{U_o V_o''(1 + k\theta)}{U_o'(1 + k''\theta)},$$

* This equation is only approximately true. It would be strictly true if k' represented the coefficient of dilatation referred to the volume at θ ; that is, if for k' we

substitute $\frac{k'}{1 + k'\theta}$ (36), in which expression k' denotes the coefficient of apparent expansion referred to the volume at 0° , as usual.

the volume at 0° of the liquid which at θ° occupies $V_o''(1+k\theta)$; therefore the volume of this quantity at X° would be

$$\frac{U_o V_o''(1+k\theta)}{U_o'(1+k'\theta)} (1+\delta_x);$$

consequently we have

$$V_o(1+\delta_x) = V_o'(1+kX) + \frac{U_o V_o''(1+k\theta)}{U_o'(1+k'\theta)} (1+\delta_x);$$

and therefore

$$1+\delta_x = \frac{U_o' V_o'(1+kX)(1+k'\theta)}{U_o' V_o(1+k'\theta) - U_o V_o''(1+k\theta)}.$$

The volumes V_o , V_o' , V_o'' , U_o , U_o' were all ascertained by gauging with mercury.

Laws of Dilatation of Liquids..

42. LAW I.—*All liquids expand on increase and contract on diminution of temperature.*

A singular anomaly occurs in the case of water and some saline solutions, which possess a point of maximum density, on either side of which they expand, whether on rise or fall of temperature. The following apparatus exhibits this property in a very striking manner. *ab* (Fig. 28) represents a glass cylinder, surrounded at *cd* by a brass envelope. Above and below this envelope the cylinder is pierced, so as to allow the introduction of two horizontal thermometers, *e*, *f*, fitted in pieces of cork. On filling the cylinder with water at the ordinary temperature, a little above the level of the upper thermometer, and introducing a frigorific mixture into the space between the cylinder and envelope, after a short time the lower thermometer begins to fall, and continues to do so until it has reached the temperature of about 4° Cent. above zero, at which point it remains stationary. After this the upper thermometer, which had fallen slowly to 6° or 7° C., begins to fall more rapidly, and continues falling until it reaches to 0° , and the water surrounding its bulb is congealed.

These phenomena may be thus explained. The water in the space *cd*, being cooled by the effect of the freezing mixture, is contracted in volume, and being thus rendered specifically heavier, it falls to the bottom of the vessel, while its place is supplied by the lighter fluid from below, which, in its turn, is cooled, and, being increased in density, again sinks down; and this double current of the colder and heavier water descending, and of the warmer and lighter ascending, continues until the whole mass below *c* has attained its maximum density. Meanwhile the smaller mass of water above *c* has been slowly cooled by contact with the glass cylinder, which, although imperfectly, conducts the effect of the frigorific mixture, until it has attained a density approaching nearly to the maximum. After this, the reduction of temperature continuing in the portion *cd*, which has attained its maximum density, this mass expands, and shortly becoming specifically lighter than that above *c*, it rises into and mixing with it rapidly reduces its temperature to the freezing point. The stationary temperature, therefore, of about 4° C., which, as we have mentioned, is maintained by the lower thermometer, corresponds to the maximum density of water; and any change of temperature, either above or below this point, produces a decrease of density, and consequently an expansion of volume.

This property of water is attended with important consequences in the economy of nature. It preserves the deep fresh-water lakes and rivers in high latitudes from being completely frozen, since, after having been cooled down to the temperature of 4° by the effect of currents, as described above, they can only be reduced below that by direct conduction, which, as we shall see subsequently, is effected with the utmost difficulty through water or ice.

In a scientific point of view, the determination of the temperature corresponding to the maximum density of water is a matter of considerable interest and importance, as the unit of volume of water, at its maximum of density, has been proposed as the unit of weight to be generally adopted in scientific researches, and has been actually adopted as such in France for all purposes.

The precise point of maximum density has accordingly been

investigated by several physicists; its exact determination, however, is a problem of extreme difficulty, owing to the very slow alteration which the density undergoes for several degrees on either side of the temperature corresponding to the maximum. We subjoin a table of the results arrived at by several experimenters, and of the methods which they adopted.

TABULAR VIEW of the Results of the principal Experiments made with the View of determining the Temperature corresponding to the maximum Density of Water.

Name of Observer.	Method of Experiment.	Temperature.	Remarks.
Sir Chas. Blagden and Mr. Gilpin ^a , Lefevre Gineau ^b ,	Second method of densities. Third method of densities.	3°.890 C. 4°.400	From direct observation. Ditto.
Hope ^c ,	By means of apparatus described in p. 59.	3°.605	The mean of two sets of observations, one made by cooling water from the ordinary temperature, the other by heating it from 0°.
Tralles,	Ditto.	4°.350	
Rumford ^d ,	Ditto.	4°.440	
Hälström ^e ,	Third method of densities.	4°.108	From calculation by means of the formula representing the densities corresponding to different temperatures, as ascertained by experiment.
Ditto ^f ,	Ditto.	4°.031	
Muncke ^g ,	Method of graduated tubes.	3°.780	

^a Philosophical Transactions, 1792, p. 428.

^b Journal de Physique, tome xlix. (ann. 1799) p. 170.

^c Edinburgh Transactions, vol. v. (1805) p. 379.

^d Nicholson's Journal, vol. xi. (1805)

p. 228.

^e Memoirs of Academy of Sweden, 1823. Ann. de Chim., Janvier, 1825.

^f Memoirs of Academy of Sweden, 1833. Annalen di Poggendorf, 1835, No. 2.

^g Memoirs of Royal Academy of St. Petersburg, vol. i. 1830.

Name of Observer.	Method of Experiment.	Temperature.	Remarks.
Stamfer ^b ,	Third method of densities.	3°.750	The mean of several direct observations.
Ditto ⁱ ,	Ditto.	3°.762	From calculation by means of formula.
Hälström ^j ,	Ditto.	3°.900	From a combination of the results Nos. 6, 7, 8, 9, 10.
Ditto ^k ,	By means of apparatus in p. 59.	4°.004	Mean of two sets of experiments as in Nos. 3, 4, 5.
Despretz ^l ,	Method of graduated tubes.	4°.007	From a graphic representation of the curve of densities, laid down from direct observation.
Ditto ^m ,	By a method similar in principle to that in p. 59.	3°.982	From graphic representation.

Sea water and water holding in solution various salts in different degrees of saturation, have also been found by M. Despretz to possess a point of maximum condensation, in some instances above, in others below that of ordinary congelation. We annex the table of his results as given by M. Pouilletⁿ.

It may be remarked that M. I. Pierre has shown that a maximum density does not exist in any of the very extensive series of liquids examined by him, except water or watery solutions.

^b Annalen di Poggendorf, 1831, No. 1.

ⁱ Ibid.

^j Memoirs of Academy of Sweden, 1833.

Annalen di Poggendorf, 1835, No. 2.

^k Annalen di Poggendorf, 1827, No. 4.

^l Comptes Rendus, 1837, part i. Annales de Chim., &c., January, 1839.

^m Ibid.

ⁿ Elements de Physique, 4^{me} Edit., tome i. p. 284.

TABLE of the Temperatures corresponding to the maximum Density of various aqueous Solutions.

Substances.	Densities.	Weight of Substance in 997.45 of Water.	Maximum.	Point of Congelation.	Temperature during Congelation.
Sea Water, .	1.027 at 20°	- 3°.67	- 2°.55	- 1.88
Chloride of Sodium, . .	1.009 „ 6	12.346	+ 1.19	- 1.21	0.71
Ditto,	1.018 „ 6.26	24.692	- 1.69	- 2.24	1.41
Ditto,	1.027 „ 6.60	37.039	- 4.75	- 2.77	2.12
Ditto,	74.078	- 16.00	- 4.30
Chloride of Calcium, .	1.005	6.173	+ 3.24	- 0.38	- 0.22
Ditto,	1.010	12.346	+ 2.05	- 0.53	- 0.53
Ditto,	1.020	24.692	+ 0.06	- 1.12	- 1.03
Ditto,	1.031	37.039	- 2.43	- 3.92	- 1.61
Ditto,	1.060	74.078	- 10.43	- 5.28	- 3.56
Sulphate of Potash, . .	1.005	6.173	+ 2.92	- 0.15	- 0.15
Ditto,	1.010	12.346	+ 1.91	- 0.27	- 0.27
Ditto,	1.020	24.692	- 0.10	- 0.56	- 0.56
Ditto,	1.030	37.039	- 2.28	- 2.09	- 0.77
Ditto,	1.058	74.078	- 8.37	- 4.08	- 1.50
Sulphate of Soda, . . .	1.006	6.173	+ 2.52	- 0.27	- 0.17
Ditto,	1.012	12.346	+ 1.15	- 1.14	- 0.33
Ditto,	1.023	24.692	- 1.51	- 0.83	- 0.69
Ditto,	1.034	37.039	- 4.33	- 2.39	- 1.10
Ditto,	1.066	74.078	- 12.26	- 2.17	- 1.13
Carbonate of Potash, . .	1.033	37.039	- 3.95	- 3.21	- 1.17
Ditto,	1.075	74.078	- 12.41	- 2.25	- 2.25
Carbonate of Soda, . . .	1.039	37.039	- 7.01	- 2.85	- 1.37
Ditto,	1.075	74.078	- 17.30	- 2.20	- 2.02
Sulphate of Copper,	57.996	- 0.62	- 1.32	- 0.37
Pure Potash, .	1.032	37.039	- 5.64	- 2.10	- 2.03
Ditto,	1.062	74.078	- 15.95	- 4.33	- 4.33
Alcohol, . . .	0.988	74.078	+ 2.30	- 2.83	- 2.83
Sulphuric acid,	1.008	12.346	+ 0.60	- 0.47	- 0.47
Ditto,	1.016	24.692	- 1.92	- 1.09	- 0.90
Ditto,	1.024	37.039	- 5.02	- 1.34	- 1.34

43. LAW II.—*The amount of expansion of liquids is not proportional, as in the case of solids, to their change of temperature; they do not, therefore, expand uniformly with mercury in glass, nor indeed do any two liquids expand uniformly with one another.*

The two following tables demonstrate this in a very striking manner. The first, due to M. Deluc, was constructed as follows: A number of thermometer tubes, with their stems accurately graduated, were filled to a convenient height with the fluids enumerated, and then sealed with the precautions ordinarily used in the construction of thermometers. The points at which the liquid stood in each at the temperatures of melting ice and boiling water were marked, and the intervening space divided into eighty equal divisions, as in Reaumur's scale. The thermometers were then all plunged into the same bath, whose temperature was gradually raised by intervals of 5° R., and the number of divisions occupied by the fluid in each, at the temperature given in the first column, is registered in the succeeding columns.

TABLE of the comparative Indications of various Thermometers.

Mercury.	Olive Oil.	Essential Oil of Chamomile.	Essential Oil of Thyme.	Water saturated with Muriate of Soda.	Alcohol highly rectified.	One Part of Alcohol and one of Water.	One Part of Alcohol and three of Water.	Water.
80	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
75	74.6	74.7	74.3	74.1	73.8	73.2	71.6	71.0
70	69.4	69.5	68.8	68.4	67.8	66.2	62.9	62.0
65	64.4	64.3	63.5	62.6	61.9	60.6	55.2	53.5
60	59.3	59.1	58.3	57.1	56.2	54.8	47.7	45.8
55	54.2	53.9	53.3	51.7	50.7	49.1	40.6	38.5
50	49.2	48.8	48.3	46.6	45.3	43.6	34.4	32.0
45	44.0	43.6	43.4	41.2	40.2	38.4	28.4	26.1
40	39.2	38.6	38.4	36.3	35.1	33.3	23.0	20.5
35	34.2	33.6	33.5	31.3	30.3	28.4	18.0	15.9
30	29.3	28.7	28.6	26.5	25.6	23.9	13.5	11.2
25	24.3	23.8	23.8	21.9	21.0	19.4	9.4	7.3
20	19.3	18.9	19.0	17.3	16.5	15.3	6.1	4.1
15	14.4	14.1	14.2	12.8	12.2	11.1	3.4	1.6
10	9.5	9.3	9.4	8.4	7.9	7.1	1.5	0.2
5	4.7	4.6	4.7	4.2	3.9	3.4	0.1	- 0.4
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
- 5				- 4.1	- 3.9			
- 10				- 8.0	- 7.7			

The next table is due to M. Gay-Lussac; it gives the amount of contraction of four liquids,—water, alcohol, sulphuret of carbon, and sulphuric ether,—for every 5° Cent., counted in each from the point at which they boil in air. M. Gay-Lussac fixed upon this as the point of comparison, because the integrant molecules of all liquids appear to have the same cohesive power at their boiling points; and he hoped to be able to obtain some general expression for the amount of their expansions in terms of their temperature, starting from this point. In this he did not succeed, but his investigations have led to one very striking result. On inspecting the third and fourth columns of the following table, it will be seen that alcohol and sulphuret of carbon contract equally; and M. Gay-Lussac has proved that equal volumes of these liquids produce also equal volumes of vapours.

*TABLE of the Contraction of the undermentioned Liquids for successive Intervals of 5° , counted from their Boiling Points, the Volumes at those Points being equal to 1000.**

Temperatures.	Water.	Alcohol.	Sulphuret of Carbon.	Ether.
0	0.00	0.00	0.00	0.00
5	3.34	5.55	6.14	8.15
10	6.61	11.43	12.01	16.17
15	10.50	17.51	17.98	24.16
20	13.15	24.34	23.80	31.83
25	16.06	29.15	29.65	39.14
30	18.85	34.74	35.06	46.42
35	21.52	40.28	40.48	52.06
40	24.10	45.68	45.77	58.77
45	26.50	50.85	51.08	65.48
50	28.56	56.02	56.28	72.01
55	30.60	61.01	61.14	78.38
60	32.42	65.96	66.21	
65	34.02	70.74		
70	35.47	75.48		
75	36.70	80.11		

“This result,” says M. Gay-Lussac, “that alcohol and sulphuret of carbon dilate equally, and produce the same volumes

* Ann. de Chim. et de Phys., tome ii. p. 134.

of vapours, is certainly very remarkable; it would seem to warrant the presumption that there is an intimate relation between the dilatation of a liquid and the expansion which it undergoes when reduced to the state of vapour. This ratio ought to be independent of the density and volatility of the liquids, or at least ought not to depend on these properties only, since in alcohol and sulphuret of carbon they differ so widely.*

44. *Formula representing absolute Dilatation of Liquids.*—As the expansion of a liquid by heat is not proportional to its change of temperature, we cannot express its corresponding coefficient by the simple formula,

$$\delta_t = kt,$$

as in the case of solid bodies (36). M. Biot† accordingly proposed the following expression,

$$\delta_t = at + bt^2 + ct^3,$$

where a , b , c are coefficients constant for all temperatures in the case of the same liquid, but varying with different liquids. This formula generally represents the expansion of a liquid with considerable accuracy; but M. I. Pierre‡ has shown that in some cases we must adopt different coefficients in the upper and in the lower parts of the thermometric scale. Thus in the case of amylic alcohol, or fusel oil, the coefficients for temperatures from -15° to $+80^\circ$ C. differ from those suited to the range from 80° to 131° . M. I. Pierre states also,§ that a formula of this kind fails to represent the expansion of water between the limits $-13^\circ.14$ and 100° , and that he has not succeeded in discovering any simple expression which will do so.

If we consider the volume of a liquid at 0° as equal to unity, its increment for $t^\circ = \delta_t = at + bt^2 + ct^3$, and its *mean* coefficient of dilatation at the temperature t° , for 1° , is expressed by $\frac{\delta_t}{t} = a + bt + ct^2$. Its *true* coefficient, however, at the same temperature, is expressed by the ratio of the increment of volume to the increment of temperature, that is, by $\frac{d(1 + \delta_t)}{dt} = a + 2bt + 3ct^2$. In re-

* Ann. de Chim. et de Phys., tome ii. p. 136.

† Traité de Physique, tome i. p. 210.

‡ Annales de Chimie et de Physique, tome xix. p. 210 (3^{me} Serie).

§ Ann. de Chim. et Phys., xv. p. 351.

ference to these coefficients, M. I. Pierre has arrived at the following conclusions:*

(1). That in all the liquids examined by him the *mean* and *true* coefficients increase with the temperature.

(2). That the true coefficient always surpasses the mean at temperatures above 0° , and that the converse holds true below 0° .

(3). That the difference between these two quantities is sometimes very considerable, amounting to 38 per cent. of the latter at $131^{\circ}.8$, in the case of amylic alcohol, and to 27 per cent. at 59° , for terchloride of silicon.

(4). That the *true* coefficient increases sometimes with great rapidity; thus, for an interval of less than 132° , its increment equals 80 per cent. of its original value, in the case of amylic alcohol; and for a change of 59° it equals 53 per cent. in the case of terchloride of silicon.

45. *Graphic Representation of Curve of Dilatation of Liquids.*—These results admit of being represented graphically as follows:

Let a line AB be drawn representing the apparent dilatation of mercury in glass from 0° to 100° , and let it be divided into 100 parts, corresponding to the degrees of the mercurial thermometer. At different points, $p, p', p'', \&c.$, let ordinates be raised, representing, on the same scale as the base line, the absolute dilatation of a liquid corresponding to the temperatures expressed by the abscissæ, $Ap, Ap', Ap'', \&c.$, and let the points $A, o, o', o'', \&c.$, be connected. The line passing through these points, if they be sufficiently numerous, will form a curve convex to the axis of the abscissæ, which may be called the curve of dilatation, and whose equation is of the form

$$y = ax + bx^2 + cx^3.$$

If at the extremities of the abscissæ, Ab, Ab' , corresponding to the temperatures $t, t+dt$, we erect ordinates, ab, ab' , and if we further connect the points A, a , and draw at a a tangent am , to the curve of dilatation, cutting the axis of abscissæ in n , the *mean* coefficient of dilatation for 1° at the temperature t will be

* Ann. de Chim. et de Phys., tome xx. p. 351 (3^{me} Serie).

represented by the ratio $\frac{ab}{Ab}$, and the *true* coefficient by $\frac{ab}{nb}$, Ab and nb being expressed by the number of degrees to which they correspond. And Aa being produced to cut the ordinate $a'b'$ in c , and ad being drawn parallel to AB , the increment of absolute dilatation at the temperature t , for the increase of temperature dt , will be represented by cd , if calculated from the *mean* coefficient, and by $a'd$ if calculated from the *true*, the change dt being supposed, in the latter case, to be so small that the tangent am coincides with the curve from a to a' . We have seen above that $a'c$, in some cases, is equal to $.38 cd$. It appears also, from M. I. Pierre's results, that the true increment of dilatation for the same small change of temperature increases rapidly with the temperature: thus for amyl alcohol its value at 132° is to its value at 0° in the ratio of 180 : 100.

As in this construction the ordinates represent the dilatation of the liquid under examination on the same scale as the base line represents the apparent dilatation of mercury from 0° to 100° , it follows that the ratio of any ordinate to its abscissa expresses the ratio of the dilatation of the liquid to the apparent dilatation of mercury, for the change of temperature indicated by the abscissa.

46. *Relation between the Temperatures corresponding to absolute and apparent Maxima of Density.*—If a liquid admits of a minimum dilatation, or, in other words, of a maximum density above its point of congelation, the corresponding temperature is given by the expression,

$$\frac{d(1 + \delta_t)}{dt}, \text{ or } a + 2bt + 3ct^2 = 0;$$

while if we seek the temperature corresponding to the apparent maximum in a given envelope, we will obtain it by differentiating the expression for the apparent volume with respect to the temperature, and equating it to zero.

This expression, as appears from (38), is,

$$1 + \Delta_t = 1 + (a - k)t + bt^2 + ct^3,$$

where Δ_t is the coefficient of apparent expansion, and k the co-

efficient of expansion of the envelope for 1° . The temperature corresponding to the apparent maximum, therefore, is given by the expression,

$$\frac{d(1 + \Delta_t)}{dt}, \text{ or } (a - k) + 2bt + 3ct^2 = 0.$$

If we call (T) the temperature corresponding to the absolute maximum of density, and T that corresponding to the apparent, and neglect the term multiplied by c , which is, in general, exceedingly small, we have, approximately,

$$(T) = -\frac{a}{2b},$$

$$T = -\frac{a}{2b} + \frac{k}{2b},$$

and consequently,

$$T = (T) + \frac{k}{2b},$$

which exhibits the relation between the temperatures corresponding to the absolute maximum and the apparent maximum in an envelope of known expansion.

M. Biot* gives the following tabular comparison of the results of this formula applied to pure water in various envelopes, with some experiments made by Mr. Dalton.†

Material of Envelope.	Value of k for 1° R.	Temperature of apparent Maximum in Degrees Reaumur.	
		Calculated.	Observed.
Flint glass, . .	0.000 030 03	$4^\circ.236$	$4^\circ.222$
Iron,	0.000 045 78	$5^\circ.072$	$4^\circ.667$
Copper,	0.000 063 09	$5^\circ.960$	$6^\circ.000$
Brass,	0.000 070 02	$6^\circ.319$	$6^\circ.222$
Pewter,	0.000 072 66	$6^\circ.456$	$6^\circ.664$
Lead,	0.000 106 89	$8^\circ.246$	$7^\circ.778$

47. *Absolute Dilatation of Mercury, &c.*—The mean coefficients of the dilatation of mercury from 0° to 100° , 200° , and 300° , are given by MM. Dulong and Petit as follows:‡

* *Traité de Physique*, tome i. p. 240.

† *Nicholson's Journal*, vol. x. p. 93, 1805.

‡ *Ann. de Chim., &c.*, tome vii. p. 124.

Temperatures determined by Air Thermometer.	Mean absolute Dilatation of Mercury for 1°.		Temperature indicated by the Dilatation of Mercury supposed uniform.
	In vulgar Fractions.	In Decimals.	
0°	0	0.	0°.00
100	1 ÷ 5550	0.000 180 180	100 .00
200	1 ÷ 5425	0.000 184 331	204 .61
300	1 ÷ 5300	0.000 188 679	314 .15

In consequence of the uncertainty attached to these results, owing to various causes, M. Regnault has repeated the experiments on the dilatation of mercury, with a form of apparatus constructed on the same principle as that employed by MM. Dulong and Petit, but considerably improved in its details. He has found* that the expansion of mercury between the limits 0° and 300° C. is represented with remarkable fidelity by the formula

$$\delta_T = bT + cT^2,$$

in which

$$\text{Log } b = \bar{4}.252\ 869\ 0,$$

$$\text{Log } c = \bar{8}.401\ 944\ 1,$$

T being the temperature as given by an air thermometer, and δ_T the coefficient of expansion from 0° to T° .

In the following Table, calculated by means of this formula, the first column contains the temperatures (T); the second, the value of the coefficient δ_T ; the third and fourth, the *mean* (δ) and *real* (Δ) coefficients of dilatation for 1° C., calculated by means of the formulæ given in (44); the fifth, the temperature (θ) which would be indicated by a thermometer graduated on the supposition of the uniform dilatation of mercury. These temperatures are given by the formula

$$\theta = 100 \frac{\delta_T}{\delta_{100}};$$

and finally, in the sixth column are given the differences ($\theta - T$) between the indications of such a thermometer and the standard air thermometer.

* Memoires de l'Institut, tome xxi. p. 326.

TABLE of absolute Dilatation of Mercury from 0° to 350° C.

Tem- pera- ture. <i>T.</i>	Dilatation of Unit of Volume from 0° to T° . δ_T .	Mean Coefficient of Dilatation for 1° between 0° and T° . δ .	Real Coefficient of Dilatation for 1° between 0° and T° . Δ .	Tempera- ture deduced from Dila- tation of Mercury. θ .	Difference of θ and T . $\theta - T$.
0	0.000 000	0.000 000 00	0.000 179 05	0	0
10	0.001 792	0.000 179 25	0.000 179 50	9.872	- 0.128
20	0.003 590	0.000 179 51	0.000 180 01	19.776	- 0.224
30	0.005 393	0.000 179 76	0.000 180 51	29.709	- 0.291
40	0.007 201	0.000 180 02	0.000 181 02	39.668	- 0.332
50	0.009 013	0.000 180 27	0.000 181 52	49.650	- 0.350
60	0.010 831	0.000 180 52	0.000 182 03	59.665	- 0.335
70	0.012 655	0.000 180 78	0.000 182 53	69.713	- 0.287
80	0.014 482	0.000 181 02	0.000 183 04	79.777	- 0.223
90	0.016 315	0.000 181 28	0.000 183 54	89.875	- 0.125
100	0.018 153	0.000 181 53	0.000 183 05	100.000	0.000
110	0.019 996	0.000 181 78	0.000 184 55	110.153	+ 0.153
120	0.021 844	0.000 182 03	0.000 185 05	120.333	+ 0.333
130	0.023 697	0.000 182 28	0.000 185 56	130.540	+ 0.540
140	0.025 555	0.000 182 54	0.000 186 06	140.776	+ 0.776
150	0.027 419	0.000 182 79	0.000 186 57	151.044	+ 1.044
160	0.029 287	0.000 183 04	0.000 187 07	161.334	+ 1.334
170	0.031 160	0.000 183 29	0.000 187 58	171.652	+ 1.652
180	0.033 039	0.000 183 55	0.000 188 08	182.003	+ 2.003
190	0.034 922	0.000 183 80	0.000 188 59	192.376	+ 2.376
200	0.036 811	0.000 184 05	0.000 189 09	202.782	+ 2.782
210	0.038 704	0.000 184 30	0.000 189 59	213.210	+ 3.210
220	0.040 603	0.000 184 56	0.000 190 10	223.671	+ 3.671
230	0.042 506	0.000 184 81	0.000 190 61	234.154	+ 4.154
240	0.044 415	0.000 185 06	0.000 191 11	244.670	+ 4.670
250	0.046 329	0.000 185 31	0.000 191 61	255.214	+ 5.214
260	0.048 247	0.000 185 57	0.000 192 12	265.780	+ 5.780
270	0.050 171	0.000 185 82	0.000 192 62	276.379	+ 6.379
280	0.052 100	0.000 186 07	0.000 193 13	287.005	+ 7.005
290	0.054 034	0.000 186 32	0.000 193 63	297.659	+ 7.659
300	0.055 973	0.000 186 58	0.000 194 13	308.340	+ 8.340
310	0.057 917	0.000 186 83	0.000 194 64	319.048	+ 9.048
320	0.059 866	0.000 187 08	0.000 195 15	329.786	+ 9.786
330	0.061 820	0.000 187 33	0.000 195 65	340.550	+ 10.550
340	0.063 778	0.000 187 58	0.000 196 16	351.336	+ 11.336
350	0.065 743	0.000 187 84	0.000 196 66	362.160	+ 12.160

We subjoin tables of the values of the coefficients in the expression $1 + \delta_t = 1 + at + bt^2 + ct^3$, for various liquids, compiled from M. I. Pierre's memoirs; and also tables of the density and volume of water at different temperatures, according to MM. Hälstrom and Despretz.

TABLE of the Values of the Coefficients in the Expression $1 + \delta_1 = 1 + at + bt^2 + ct^3$ for various Liquids.

Name of Liquid.	Atomic Constitution.	Density at 0° C., re- ferred to Water at 4°.	Temperature of boiling Point, and corresponding Pressure.	Value of Coefficient a.	Value of Coefficient b.	Value of Coefficient c.
1. Oxide of ethyle (<i>sulphuric ether</i>),	C ₄ H ₆ O.	0.73574	33°.5	0.001 513 244 795 062	0.000 002 359 182 881	0.000 000 040 051 240
2. Ethylic alcohol (<i>alcohol</i>), . . .	C ₂ H ₅ O, HO.	0.81509	78.3	0.001 048 630 106 063	0.000 001 750 990 620	0.000 000 001 345 183
3. Methylalcohol (<i>pyroxylic spirit</i>),	C ₂ H ₅ O, HO.	0.82074	66.3	0.001 185 569 707 407	0.000 001 564 932 615	0.000 000 009 111 344
4. Amylic alcohol (<i>fusel oil</i>), . . .	C ₁₀ H ₁₈ O, HO.	0.82705	131.8	{ 0.000 890 010 672 000 0.000 898 883 635 000	0.000 000 657 293 250	0.000 000 011 845 833 ^a
5. Chloride of ethyle,	C ₂ H ₅ Cl.	0.92138	11.0	0.001 574 578 423 400	0.000 002 813 658 100	0.000 000 010 096 172 ^b
6. Bromide of ethyle,	C ₂ H ₅ Br.	1.47339	40.7	0.001 337 627 678 064	0.000 001 501 348 513	0.000 000 016 900 026
7. Iodide of ethyle,	C ₂ H ₅ I.	1.97546	70.0	0.001 142 251 234 512	0.000 001 963 811 340	0.000 000 006 406 415
8. Bromide of methyle,	C ₂ H ₅ Br.	1.66443	13.0	0.001 415 206 168 177	0.000 003 315 281 301	0.000 000 010 051 210
9. Iodide of methyle,	C ₂ H ₅ I.	2.19922	43.8	0.001 199 590 835 123	0.000 002 163 318 353	0.000 000 010 051 210
10. Formiate of oxide of ethyle, . . .	C ₂ H ₅ O, C ₂ H ₃ O ₂ .	0.93565	52.9	0.001 325 204 736 811	0.000 002 862 484 865	0.000 000 006 618 006
11. Acetate of oxide of ethyle, . . .	C ₄ H ₈ O, C ₄ H ₅ O ₂ .	0.90691	74.1	0.001 258 495 886 000	0.000 002 956 880 447	0.000 000 001 492 150
12. Butyrate of oxide of ethyle, . . .	C ₄ H ₈ O, C ₆ H ₇ O ₂ .	0.90193	119.0	{ 0.001 202 792 100 000 0.000 632 742 605 000	0.000 000 072 233 787	0.000 000 022 634 601 ^c
13. Acetate of oxide of methyle, . . .	C ₂ H ₅ O, C ₄ H ₅ O ₂ .	0.86684	59.5	0.001 295 934 262 601	0.000 002 909 820 130	0.000 000 004 256 971
14. Butyrate of oxide of methyle, . . .	C ₂ H ₅ O, C ₆ H ₇ O ₂ .	1.02928	102.1	0.001 239 895 926 000	0.000 000 626 024 909	0.000 000 013 065 664
15. Terchloride of phosphorus, . . .	PCl ₃ .	1.61616	78.3	0.001 128 618 932 000	0.000 000 872 880 045	0.000 017 923 565 000
16. Terbromide of phosphorus, . . .	PBr ₃ .	2.92489	175.3	{ 0.000 847 204 934 000 0.000 824 268 122 000	0.000 000 436 718 628	0.000 000 002 327 555 ^e
17. Terchloride of arsenic,	AsCl ₃ .	2.20495	133.8	0.000 979 072 746 000	0.000 000 966 948 200	0.000 000 000 055 001 ^f
18. Bichloride of tin,	SnCl ₂ .	2.26712	115.4	0.001 132 800 769 000	0.000 000 011 710 706	0.000 000 001 777 204
19. Bichloride of titanium,	TiCl ₂ .	1.76088	136.0	0.000 942 569 004 000	0.000 001 345 791 937	0.000 000 000 888 044
20. Terchloride of silicon,	SiCl ₄ .	1.52371	59.0	0.001 294 119 069 000	0.000 002 184 143 631	0.000 000 040 864 220
21. Terbromide of silicon,	SiBr ₄ .	2.81280	153.4	0.000 952 572 440 000	0.000 000 756 742 000	0.000 000 000 292 074
22. Bichloride of ethyle,	C ₂ H ₄ Cl ₂ .	1.28034	84.9	0.001 118 932 379 000	0.000 001 446 861 382	0.000 000 010 341 738
23. Bibromide of ethyle,	C ₂ H ₄ Br ₂ .	2.16292 ^h	132.6	{ 0.000 952 096 190 000 0.001 016 765 988 000	0.000 001 316 506 858	0.000 000 001 062 687 ⁱ
24. Bromine,	Br.	3.18718	63.0	0.001 038 186 255 000	0.000 001 711 380 853	0.000 000 005 447 118
25. Sulphuret of carbon,	CS ₂ .	1.29312	47.9	0.001 139 803 833 952	0.000 001 370 651 328	0.000 000 019 122 546

^a From -15° to 80°. ^b From 80° to 131°.8. ^c From 13° to 90°. ^d From 90° to 118°.
^e From 0° to 100°. ^f From 100° to 175°.3. ^g For this liquid t is counted from 20°.09, its point of congelation. ^h At 20°.79.
ⁱ From 20°.09 to 100°.16. ^j From 100°.16 to 132°.6.

TABLE of the Density and Volume of Water, from -4° C., to 100° C., according to M. Despretz. (The Density and Volume at 4° taken as Units.)

Temperature.	Volume.	Density.	Temperature.	Volume.	Density.
-9	1.001 631 1	0.998 371	46	1.010 20	0.989 903
-8	1.001 373 4	0.998 628	47	1.010 67	0.989 442
-7	1.001 135 4	0.998 865	48	1.011 09	0.989 032
-6	1.000 918 4	0.999 082	49	1.011 57	0.988 562
-5	1.000 698 7	0.999 302	50	1.012 05	0.988 093
-4	1.000 561 9	0.999 437	51	1.012 48	0.987 674
-3	1.000 422 2	0.999 577	52	1.012 97	0.987 196
-2	1.000 307 7	0.999 692	53	1.013 45	0.986 728
-1	1.000 213 8	0.999 786	54	1.013 95	0.986 243
0	1.000 126 9	0.999 873	55	1.014 45	0.985 756
1	1.000 073 0	0.999 927	56	1.014 95	0.985 270
2	1.000 033 1	0.999 966	57	1.015 47	0.984 766
3	1.000 008 3	0.999 999	58	1.015 97	0.984 281
4	1.000 000 0	1.000 000	59	1.016 47	0.983 798
5	1.000 008 2	0.999 999	60	1.016 98	0.983 303
6	1.000 030 9	0.999 969	61	1.017 52	0.982 782
7	1.000 070 8	0.999 929	62	1.018 09	0.982 231
8	1.000 121 6	0.999 878	63	1.018 62	0.981 720
9	1.000 187 9	0.999 812	64	1.019 13	0.981 229
10	1.000 268 4	0.999 731	65	1.019 67	0.980 709
11	1.000 359 8	0.999 640	66	1.020 25	0.980 152
12	1.000 472 4	0.999 527	67	1.020 85	0.979 576
13	1.000 586 2	0.999 414	68	1.021 44	0.979 010
14	1.000 714 6	0.999 285	69	1.022 00	0.978 473
15	1.000 875 1	0.999 125	70	1.022 55	0.977 947
16	1.001 021 5	0.998 979	71	1.023 15	0.977 373
17	1.001 206 7	0.998 794	72	1.023 75	0.976 800
18	1.001 39	0.998 612	73	1.024 40	0.976 181
19	1.001 58	0.998 422	74	1.024 99	0.975 619
20	1.001 79	0.998 213	75	1.025 62	0.975 018
21	1.002 00	0.998 004	76	1.026 31	0.974 364
22	1.002 22	0.997 784	77	1.026 94	0.973 766
23	1.002 44	0.997 566	78	1.027 61	0.973 132
24	1.002 71	0.997 297	79	1.028 23	0.972 545
25	1.002 93	0.997 078	80	1.028 85	0.971 959
26	1.003 21	0.996 800	81	1.029 54	0.971 307
27	1.003 45	0.996 562	82	1.030 22	0.970 666
28	1.003 74	0.996 274	83	1.030 90	0.970 027
29	1.004 03	0.995 986	84	1.031 56	0.969 405
30	1.004 33	0.995 688	85	1.032 25	0.968 757
31	1.004 63	0.995 391	86	1.032 93	0.968 120
32	1.004 94	0.995 084	87	1.033 61	0.967 482
33	1.005 25	0.994 777	88	1.034 30	0.966 837
34	1.005 55	0.994 480	89	1.035 00	0.966 183
35	1.005 93	0.994 104	90	1.035 66	0.965 567
36	1.006 24	0.993 799	91	1.036 39	0.964 887
37	1.006 61	0.993 433	92	1.037 10	0.964 227
38	1.006 99	0.993 058	93	1.037 82	0.963 558
39	1.007 34	0.992 713	94	1.038 52	0.962 908
40	1.007 73	0.992 329	95	1.039 25	0.962 232
41	1.008 12	0.991 945	96	1.039 99	0.961 547
42	1.008 53	0.991 542	97	1.040 77	0.960 827
43	1.008 94	0.991 139	98	1.041 53	0.960 125
44	1.009 38	0.990 707	99	1.042 28	0.959 434
45	1.009 85	0.990 246	100	1.043 15	0.958 634

TABLE of the *Density and Volume of Water from 0° C. to 30° C., according to M. Hälström.*

Temperature.	Density and Volume at 0° taken as Units.		Density and Volume at 4°.1, taken as Units.	
	Density.	Volume.	Density.	Volume.
0°	1.0	1.0	0.999 891 8	1.000 108 2
1	1.000 046 6	0.999 953 6	0.999 938 2	1.000 061 7
2	1.000 079 9	0.999 920 2	0.999 971 7	1.000 028 1
3	1.000 100 4	0.999 899 6	0.999 992 0	1.000 007 8
4	1.000 108 17	0.999 891 8	0.999 999 5	1.000 000 2
4.1	1.000 108 24	0.999 891 77	1.0	1.0
5	1.000 103 2	0.999 896 8	0.999 995 0	1.000 005 0
6	1.000 085 6	0.999 914 4	0.999 977 2	1.000 022 6
7	1.000 055 5	0.999 944 5	0.999 947 2	1.000 052 7
8	1.000 012 9	0.999 987 2	0.999 904 4	1.000 095 4
9	0.999 957 9	1.000 042 1	0.999 849 7	1.000 150 1
10	0.999 890 6	1.000 109 4	0.999 782 5	1.000 220 0
11	0.999 811 2	1.000 188 8	0.999 703 0	1.000 297 0
12	0.999 719 6	1.000 280 4	0.999 611 7	1.000 388 8
13	0.999 616 0	1.000 384 1	0.999 508 0	1.000 492 4
14	0.999 500 5	1.000 499 7	0.999 392 2	1.000 608 1
15	0.999 373 1	1.000 627 3	0.999 264 7	1.000 735 7
16	0.999 234 0	1.000 766 6	0.999 126 0	1.000 874 7
17	0.999 083 2	1.000 917 6	0.998 975 2	1.001 025 9
18	0.998 920 7	1.001 080 5	0.998 812 5	1.001 188 8
19	0.998 746 8	1.001 254 8	0.998 638 7	1.001 363 1
20	0.998 561 5	1.001 440 6	0.998 453 4	1.001 549 0
21	0.998 324 8	1.001 637 9	0.998 257 0	1.001 756 0
22	0.998 156 9	1.001 846 5	0.998 048 9	1.001 954 9
23	0.997 937 9	1.002 066 4	0.997 830 0	1.002 174 6
24	0.997 707 7	1.002 297 6	0.997 600 0	1.002 405 8
25	0.997 466 6	1.002 539 8	0.997 358 7	1.002 648 3
26	0.997 214 6	1.002 293 2	0.997 107 0	1.002 901 6
27	0.996 951 8	1.003 057 5	0.996 843 9	1.003 166 2
28	0.996 678 3	1.003 332 8	0.996 570 4	1.003 441 4
29	0.996 394 1	1.003 618 9	0.996 286 4	1.003 727 4
30	0.996 099 3	1.003 916 0	0.995 991 7	1.004 024 5

47. *Comparative Contraction of various Liquids, starting from their respective boiling Points.*—We will conclude this section by stating the results of M. I. Pierre's investigations as to the comparative contractions of various liquids, starting from their respective boiling points.

(1). In the first place* he has verified the law of M. Gay-Lussac (p. 65), as to the equality existing between the amounts of contraction, for the same changes of temperature, of equal volumes of sulphuret of carbon and ethylic alcohol, taken at their boiling points.

(2). He has shown† that amylic, ethylic, and methylic alcohol follow sensibly the same law of contraction, that is, that equal volumes of those liquids, at their respective boiling points, will preserve their equality at all temperatures equidistant from those points.

(3). That the same law holds true for the bromides of ethyle and methyle.

(4). And also for their iodides.

(5). That it is also true for the acetate of the oxide of ethyle, and the acetate of the oxide of methyle.

(6). As also for the butyrates of those oxides.

From which he concludes, that "the agreement observed in the progress of the contraction of the liquids contained in each of those groups leads us to believe, that in general the homologous compounds derived from ethylic and methylic alcohol, and probably those also from amylic alcohol, follow the same law of contraction, starting from the respective boiling points of those liquids, and comparing them at temperatures equidistant from those points."

On inquiring further, whether this law extended to all binary groups formed in an analogous manner by the union of one common element, simple or compound, with isomorphous elements, simple or compound, M. I. Pierre examined, first, some binary groups formed by the combination of a common *simple* element, and secondly, some formed by the combination of a *compound* element, with simple isomorphous bodies. Of the first class were the groups, PhCl_3 , PhBr_3 ;‡ PhCl_3 , AsCl_3 ; SnCl_2 , TiCl_2 ;

* Annales de Chimie, &c. (3^{me} Serie), tome xv. p. 401.

† Ib., tome xix. p. 220.

‡ M. I. Pierre remarks that chlorine and

bromine are usually considered isomorphous, as, although chlorine has not yet been observed in a crystalline form, their corresponding combinations are of that character.

SiCl_3 ,* SiBr_3 ; and of the second, $\text{C}_4\text{H}_5\text{Cl}$, $\text{C}_4\text{H}_5\text{Br}$; $\text{C}_4\text{H}_5\text{Cl}$, $\text{C}_4\text{H}_5\text{I}$; $\text{C}_4\text{H}_5\text{Br}$, $\text{C}_4\text{H}_5\text{I}$; $\text{C}_2\text{H}_3\text{Br}$, $\text{C}_2\text{H}_3\text{I}$; $\text{C}_4\text{H}_4\text{Cl}_2$, $\text{C}_4\text{H}_4\text{Br}_2$.

The results of M. I. Pierre's investigations were:

(1). That in general† two liquids formed by the combination of a common principle with isomorphous elements follow different laws of contraction, starting from their respective boiling points.

(2). That the difference of contraction increases, and always in the same direction for each group, according as the distance of the temperature of comparison from the boiling point increases.

(3). That this difference is sometimes very considerable. Thus in the group consisting of the terchloride and terbromide of silicon, the difference of contractions of those liquids amounts to one-half of the contraction of one of them.

DILATATION OF GASES.

48. *Notice of early Experiments.*—The earliest recorded experiments on the dilatation of gases were made in the case of atmospheric air by M. Amontons, and are described in the *Memoires de l'Academie Royale des Sciences* for the years 1699 and 1702. He observed‡ that the increase of the elastic force of air produced by the heat of boiling water was equal to the third part of the weight with which it was charged, if the experiment was made in spring or autumn, when the atmosphere has a mean temperature; and hence he concluded, from Mariotte's law, that the increase of volume of air due to a rise of temperature from the mean atmospheric temperature to that of boiling water equalled about one-third of its original volume, and this whatever its original density might have been.

The subject of the dilatation of atmospheric air was subsequently investigated by M. Miguet,§ M. De la Hire,|| M. Stancari¶

* M. Pierre assigns for the bodies of this group the formula SiCl , SiBr . This, however, is not in accordance with the generally received view of their constitution.

† *Annales de Chimie* (3^{me} Serie), tome

xx. p. 51.

‡ *Memoires de l'Academie Royale*, An. 1702, p. 4.

§ *Ib.*, An. 1708, p. 11.

|| *Ibid.*

¶ *Ibid.*

of Bologna, M. Deluc,* Colonel Roy,† and M. Saussure.‡ Dr. Priestley§ first extended the inquiry to other permanent gases, and was followed by MM. Monge, Berthollet, and Vandermonde,|| MM. De Morveau and Duvernois,¶ and M. Charles, who, although he did not publish the results of his experiments, had arrived at the conclusion that oxygen, azote, hydrogen, carbonic acid, and atmospheric air, all dilated equally between 0° and 80° R., while for the gases soluble in water he found different values of dilatation for each different gas.

49. *M. Gay-Lussac's Experiments.*—An interesting account of the preceding methods is given by M. Gay-Lussac in the introduction to his memoir on the dilatation of gases and vapours, in the forty-third volume of the *Annales de Chimie*.** In this memoir M. Gay-Lussac has shown that the chief cause of error affecting all those methods, and which led, in some instances, to the most incongruous results, was the presence of moisture in the apparatus made use of, which, furnishing fresh quantities of vapour as the temperature increased, caused the expansion of the gas to appear much greater than it really was. Avoiding this source of error as far as possible, M. Gay-Lussac, by means of an apparatus described in the memoir above cited, arrived at the result that all gases and vapours expanded by equal quantities for equal increments of temperature; and that, for a change from 0° to 100° C., this expansion amounted to 37.5 parts, the volume at 0° being represented by 100. As M. Gay-Lussac has not mentioned in his memoir having made any correction for the expansion of the glass vessel in which the gas was contained, it has been suggested†† that the value of the expansion, according to his experiments, was really 37.8 for 100 parts at 0° , or 378 for 1000. M. Laplace,‡‡ however, mentions that M. Gay-Lussac, having repeated the experiments, at his

* *Recherches sur les Modif. de l'At.*, p. 36.
part iv. c. iii.

† *Philos. Trans.*, 1777, p. 704.

‡ *Essai sur l'Hygrometrie*, p. 108.

§ *Exper. and Obser.*, vol. ii. p. 450.

¶ *Memoires de l'Academie*, An. 1786,

¶ *Annales de Chimie*, tome i. p. 264.

** *Ibid.*, tome xliii. p. 137 (1802).

†† Gilbert, *Annalen der Physik*.

‡‡ Laplace, *Mecanique Celeste*, tome iv. p. 270 (1805).

request, with a different form of apparatus, on making all necessary corrections, among which that for the expansion of the glass envelope is expressly included, arrived at the same value, 375, for the expansion of all gases from 0° to 100° , the volume at the former being 1000.

The apparatus employed by M. Gay-Lussac in this latter series of experiments, appears to have been that described by M. Biot in his *Traité de Physique*,* and is represented in Figs. 30, 31. A thermometer tube was carefully graduated into portions of equal volume, and the volume of the ball at its extremity ascertained in terms of the divisions of the tube. All moisture was then expelled by filling it with mercury, which was boiled for a considerable time. The extremity of the tube was then luted into a tube of larger dimensions, containing the gas to be examined, and also a quantity of chloride of calcium or other desiccating substance. The gas was admitted into the ball by introducing a fine platina or iron wire into the tube, which, not being moistened by the mercury, left a free space all round it, through which the gas ascended and displaced the mercury, of which, finally, there was only left in the tube a small quantity to serve as an index. The tube, being thus filled with dry gas, was first placed in melting ice, and the point marked at which the index of mercury stood, and also the height of the barometer at the time noted. It was next placed in the vessel represented in Fig. 31, containing water whose temperature was gradually raised to the boiling point, when the position of the index and the barometric height were again observed. Let n express the number of units of volume occupied by the gas at 0° , and H the height of the barometer, n' its volume at the temperature T of boiling water, and H' the pressure to which it was then subjected. The volume actually occupied at the temperature T , and under the pressure H , was $n'(1 + kT)$, k being the coefficient of cubical expansion of glass for 1° ; and this volume at the pressure H' would have been $n'(1 + kT) \frac{H'}{H}$; accordingly, the expansion of the volume n was represented by $n'(1 + kT) \frac{H'}{H} - n$, and the ex-

* Tome i. p. 182 (1816).

pansion of the unit of volume, or the coefficient, α_r , of expansion for T° , by

$$\alpha_r = \frac{n'}{n} (1 + kT) \frac{H'}{H} - 1.$$

50. *Dr. Dalton's Experiments.*—A few months previous to the publication of M. Gay-Lussac's first memoir, there appeared in the fifth volume of the Transactions of the Manchester Society, published in 1802, a paper by Dr. John Dalton, which had been read before the Society in the preceding year.

In this paper Dr. Dalton states as the result of his experiments, that all gases and vapours expand equally for equal changes of temperature, and that 1000 parts of any one of them, at 55° Fahr., become 1325 at 212° F.; giving a change of volume of 325 parts for a difference of 157° F., and consequently of 372 for a difference of 180° F. or 100° C.

The accordance of this result with M. Gay-Lussac's is at first sight very striking; it is not, however, in reality so close as it appears to be, since the unit of volume is taken in the two cases at different temperatures: at 0° by M. Gay-Lussac, at 55° F. or $12^\circ.78$ C. by Dr. Dalton. To obtain the coefficient of expansion for 100° , referred to the volume at 0° , from Dr. Dalton's data, we may make use of the formula,

$$\frac{V}{V'} = \frac{1 + \alpha t}{1 + \alpha' t'},$$

since gases, like solids, expand uniformly with the mercurial thermometer. Putting in this expression for V and t , 1000, and $12^\circ.78$, and for V' and t' , 1325 and 100° , we find α for $1^\circ = .00392$, and for $100^\circ = .392$, a result notably different from M. Gay-Lussac's.

51. *Recent Experiments by Rudberg.*—For upwards of thirty years the law of the equal dilatation of all gases was admitted by the physicists of Europe to be the law of nature; and the value of the expansion assigned by M. Gay-Lussac, and confirmed, as was supposed, by Dr. Dalton's independent researches, was universally held to be correct. M. Rudberg, a Swedish physicist, was the first to point out that the expansion of dry air had a value sensibly different from that given by M. Gay-Lussac,

and that it probably did not exceed .365 of the volume at 0° ; and his views have since been confirmed, with numerical results slightly different, by M. Pouillet, M. Magnus, and M. Regnault. The two last-named experimenters have also shown that the rate of expansion differs very considerably in different gases.

In these more recent investigations, as in the earlier, the methods of experimenting have been of two different kinds: in the first, as in M. Gay-Lussac's, the expansion is deduced from the actual increase of volume which a gas receives on change of temperature, the pressure to which it is subjected remaining constant; in the second, as in M. Amonton's, the volume remaining the same, or nearly so, the increase of its elastic force due to a change of temperature is ascertained, and thence, by Mariotte's law, its expansion under a constant pressure is calculated.

As instances of the application of those methods, we extract the following description of two forms of the apparatus employed by M. Regnault from his Memoirs on the Dilatation of Gases, contained in the fourth and fifth volumes of the *Annales de Chimie et de Physique*.

52. *Apparatus employed by M. Regnault for determining directly the Dilatation of Air under constant Pressure.*—The apparatus of the first kind, as employed by M. Regnault for determining the dilatation of atmospheric air, was constructed and used as follows:

A cylindrical reservoir, AB (Fig. 34), terminating in a thermometer tube, was passed through the cover of a tin vessel, v, containing some water. When this water was boiled, the vapour had to pass round the annular space, L, L', in order to escape through the pipe M, thus preventing the temperature of the inner wall surrounding the reservoir, AB, from being reduced below that at which the water boiled under the existing atmospheric pressure. After a short time, when the temperature of the reservoir was raised to that of the vapour of the water, the extremity, c, of the tube was connected, by means of caoutchouc collars, with the desiccating apparatus, GG. This consisted of two tubes, each 1 metre long and 20^{mm} in diameter, filled with pumice pounded and saturated with concentrated sulphuric acid. These tubes were connected with a small air-pump, P. When the re-

servoir, AB, was connected with this apparatus, the air was drawn from it by means of the pump, and again slowly admitted through the tubes, GG, by opening a cock communicating with the atmosphere. This was done five-and-twenty or thirty times, until the air in the reservoir and the tubes attached was perfectly dry. The cocks communicating with the external air were then left open, and the apparatus allowed to remain in this state for half an hour or an hour. The desiccating apparatus was then removed; this was done by first opening the joint, *d*, in order that if, owing to any obstruction in the tubes, GG, the pressure in AB was less than that of the external air, the pressure might be equalized by the compression of the dry air between *a* and *b* into the reservoir. The joint, *d*, was then loosed, and the reservoir left for some minutes in free communication with the atmosphere, after which the end of the tube, CD, was sealed by means of a blow-pipe, and the barometric pressure at the time noted. The difference of pressures, if any, between the external air and the steam in the vessel, *v*, as indicated by the manometer, FN, was also observed. The reservoir, AB, was thus filled with dry air under a pressure equal to the atmospheric, and at the temperature T° of vapour generated under the pressure existing in the vessel, *v*. The next step was to ascertain the contraction which this air would undergo on being cooled down to 0° , and the volumes of the reservoir at 0° and T° , or rather the ratio of those quantities.

For this purpose the reservoir was removed from the chamber, *v*, and placed in the apparatus represented in perspective in Fig. 35, its tube passing downwards through a hole in the circular plate, EF', and the reservoir itself, supported by three pieces projecting from the plate, and further secured by the steadying screw, *v*. One of the feet, *r'*, of this apparatus carried an arm capable of being fixed at different heights by means of the screw, *r*. On this arm moved a little piece, *nk*, whose lower part, *k*, formed a little iron spoon, which carried a small piece of soft wax. Another foot, *r*, carried the arm, *st*, through the end of which was passed a screw terminating above and below in slightly rounded points. The reservoir was fixed in the apparatus in such a manner that the curved part of the tube, CD, was directed exactly

towards the foot, P' , and the height was marked at which the arm, m , should be fixed, so that the little spoon, k , should be precisely at the height and in the direction of cd .

This being arranged, the apparatus was placed over a vessel of mercury, in such a manner that the end of the tube was five or six centimetres below the level of the mercury. A fine file-scratch had previously been made on the tube, cd , at the place where it was intended to be broken. The point was then detached by means of a forceps, and the mercury rose to a certain height in the reservoir. This latter was next surrounded with snow or pounded ice, and the apparatus was left in this state for an hour or an hour and a half, until it was brought to the temperature of 0° . The little spoon was then advanced along the arm, m , until the end of the tube was closed by the wax, and the height of the barometer at the time noted. The arm, st , was moved along the foot, P , until the lower end of the screw at its extremity reached the level of the mercury; the ice was then removed, and the elevated column of mercury allowed to assume the temperature of the surrounding air.

It now remained to measure the height of this column, in order to ascertain the pressure to which the air in AB had been subjected at the temperature of 0° . For this purpose the vessel of mercury was removed, by previously taking away the support, s , and the difference of levels of the upper surface of the mercurial column, and the lower point of the screw on st , was measured by a kathetometer, which read, by means of its vernier, the fiftieth part of a millimetre.

The reservoir, AB , with the mercury contained in it, was then removed and weighed. It was next filled completely with mercury which had been well boiled to expel all air and moisture, and finally surrounded with ice, its point remaining plunged in a vessel full of mercury. At the expiration of an hour and a half or two hours, when the mercury remained perfectly stationary at the orifice of the point, the ice was removed, and the mercury which was expelled by its subsequent dilatation collected in a capsule. The reservoir was then suspended in the vessel, v (Fig. 34), which had served to dilate the air; the height of the barometer was noted when the water contained in it boiled; and the mercury,

which continued to be expelled from AB until it had acquired the temperature of the vapour, still collected in the capsule. The weight of this quantity was then ascertained, and also the weight of that remaining in the reservoir. All the data were then known requisite to calculate, first, the expansion of the envelope; secondly, that of the air contained in it.

For let

H^* express the height of the barometer when the point of the tube, CD, was sealed;

T the temperature at which the water then boiled in the vessel, v ;

H' the barometric height when the point was closed with wax;

h the height of the mercurial column which rose into the reservoir;

W the weight of the same column;

W' the weight of mercury filling the reservoir at 0° ;

w the weight expelled by dilatation from 0° to T_1 , the temperature of water boiling under the pressure H' ;

δ the coefficient of cubical dilatation of glass for 1° ; and finally,

a the coefficient of dilatation for dry air:

then to determine δ we have, as in (30),

$$(W' - w) \left(1 + \frac{T_1}{5550} \right) = W'(1 + \delta T_1);$$

and therefore

$$\delta T_1 = \frac{T_1}{5550} - \frac{w}{W} \left(1 + \frac{T_1}{5550} \right).$$

Next to determine a . If D be the density of mercury at 0° ,

$\frac{W' - W}{D}$ is the volume of the air at 0° and under the pressure

$H' - h$, which, at the temperature T and pressure H , filled the expanded reservoir; therefore we have

$$\frac{W' - W}{D} \cdot \frac{H' - h}{H} (1 + aT) = \frac{W'}{D} (1 + \delta T);$$

* The barometric heights are supposed to be reduced to their equivalents at 0° .

and therefore

$$1 + \alpha T = \frac{W'}{W' - W} \cdot \frac{H}{H' - h} \cdot (1 + \delta T).$$

The mean of fourteen experiments made with this form of apparatus gave $\alpha = 0.0036623$.

53. *Apparatus employed by M. Regnault for determining the Dilatation of Air, from the Change of its elastic Force due to Change of Temperature.*—In the second form of apparatus (Figs. 36, 37) the air was contained in a ball, A, whose capacity equalled 800 or 1000 cubic centimetres. This ball was connected, by means of a fine thermometer tube, with the larger tubes, FH, IJ, represented in profile in Fig. 36, and in full view in Fig. 37. These two tubes were cemented into the iron socket, IH, by means of which they had free communication, and which was provided with a cock, K. The thermometer tube, DE, consisted of two pieces let into the opposite arms of a small copper socket, mno, of three arms, into the under one of which was cemented a short tube, op, drawn out at its extremity into a capillary termination. The temperature of the ball was varied by placing it in a vessel, MN, which could either be filled with the vapour of boiling water, or surrounded with melting ice.

In order to dry the ball, the tube, DE, was removed from the copper socket, the extremity, D, of the latter closed with a cover of caoutchouc, and the temperature of AB raised to that of boiling water by means of the furnace under MN. The tube, op, was then put in communication with the desiccating apparatus of Fig. 34, and the ball, AB, dried in the manner described in the preceding paragraph. In the same way the tubes, FH, IJ, were also carefully dried, and immediately filled with warm mercury up to the end, ED, which was closed with caoutchouc to prevent the entrance of moisture from the atmosphere. The tube, DE, was then introduced into the socket at D, which it fitted closely, op still continuing in connexion with the desiccating apparatus. Mercury was now allowed to flow out of the cock, K, until the level in the tube, FH, sank to a fixed mark, a, and in consequence of the connexion with the desiccating apparatus the space da was filled with dry air. Both tubes, also, FH and IJ, being in free

communication with the atmosphere, the mercury stood at the same height in both. The tube, *op*, was now detached from the drying apparatus, and its extremity, *p*, sealed with a blow-pipe, the atmospheric pressure at the time being noted. The furnace, *o*, which had heated the water in *MN*, was then removed, and the air in *A* cooled, by first pouring cold water over it, and finally surrounding it with melting ice. The elastic force of the air being thus diminished, the mercury in *FH* had a tendency to rise, but was kept constantly at the level *a*, by allowing a suitable quantity of mercury to flow out through the cock, *x*. When the ball, *A*, had attained the temperature of 0° , the height of the barometer, and the difference of level, *ab*, were observed, and thus all the data were obtained requisite to determine the expansion of dry air.

In making the calculations it is necessary to take into account the small volume of air in the space, *dDEF*, which remains through the whole of the experiment at the temperature *t* of the surrounding medium. We will suppose the ratio of its volume, *v*, to that of the ball, *V*, known by the method of gauging with mercury.

Let *H* be the height of the barometer when the tube, *op*, was sealed;

T the temperature of water boiling under that pressure;

H' the barometric height at the conclusion of the experiment;

k the difference of levels, *ab*;

v the volume of the space, *dDEF*;

V the volume of the ball; and

t° the temperature of the space *dDEF* throughout the experiment, δ and α having the same significations as before.

Now when the ball was surrounded with ice, the air under the pressure $H' - k$ occupied the volume *V* at the temperature 0° + the volume *v* at the temperature t° . At the temperature 0° the sum of these volumes would have been $V + \frac{v}{1 + \alpha t}$, and under the pressure *H*,

$$\left(V + \frac{v}{1 + \alpha t} \right) \frac{H' - k}{H};$$

but the *same* mass of air, when the ball was surrounded by the

vapour of boiling water, occupied, under the pressure H , a volume $V(1 + \delta T)$ at the temperature T° , + a volume v at t° ; therefore at 0° , and under the pressure H , this mass would have occupied the volume

$$\frac{V(1 + \delta T)}{1 + \alpha T} + \frac{v}{1 + \alpha t}.$$

Equating these expressions for the volume of the same mass at 0° and the pressure H , and solving for $1 + \alpha T$, we get

$$1 + \alpha T = \frac{H(1 + \delta T)}{H' - h' - \frac{v}{V} \frac{1}{1 + \alpha T} (H - H' + h')}.$$

The value of α determined by this method was $\alpha = 0.003665$.

54. *Modification of this Form of Apparatus, by Means of which it may be applied to the direct Measurement of the Dilatation of Air under constant Pressure.*—A slight modification of this form of apparatus fitted it for the direct measurement of the expansion of dry air under a constant and given pressure. This arrangement is represented in Figs. 38, 39. The tubes, FH, IB, were contained in a rectangular vessel, two of whose opposite sides were formed of plates of glass. This vessel was filled with water, which, in order to insure uniformity of temperature through its different strata, was frequently stirred by means of the agitator, $ff'gg'$. The tubes were fitted into an iron socket, which, instead of being furnished with a single stopcock, was constructed as is represented in section in Fig. 39. The cock, R, merely served to put the tube, IJ, in communication with the exterior, but the cock, R', served either to connect FH with IJ, as in (a), or simply to open a communication between FH and the exterior, as in (b).

To use this apparatus, the ball, A, being connected as before with FH, was surrounded with melting ice, the tube, op , being in communication with the desiccating apparatus. Mercury was now poured into the tube, BI, until it rose to the level, a , and the cock, R', being in the position (a), the mercury necessarily stood at the same height in both tubes. The tube, op , was then sealed, the barometric height noted, and also the temperature of the water surrounding the tubes.

The ice having been removed, the water in the vessel, MN,

was brought to the boiling point. To keep the mercury in the two tubes at nearly the same level, it was necessary to allow some to flow out through the cock, *R*. A portion of air thus passed from the ball into the tube, *FH*, and the two columns standing nearly at the same level, β , their difference of height was measured exactly by means of a kathetometer, and at the same time the height of the barometer and temperature of the water round the tubes noted.

To derive from this experiment the expansion of the air, it was necessary to know the volume of the ball, *A*, the volume, *v*, from *E* to *a*, occupied by the air in the tube, *FH*, when the ball was surrounded by ice, and the volume *v'*, from *E* to *b*, when the ball was at the temperature of boiling water. The first volume was easily determined by filling the ball with mercury at 0° , in the manner formerly described. The volumes *v*, *v'* were determined as follows:

The point, *p*, was broken so as to put the interior of the tube, *EF*, in communication with the atmosphere. Mercury was then poured into *BI* until it rose to γ . The cock, *R'*, was put in the position (*b*), and the quantity of mercury was collected and weighed, which flowed out while the level was falling to *a*; this served to give the volume *v*; the quantity was then collected which escaped on the level sinking further to β , and this, added to the former, determined *v'*. For let *w*, *w'* be the weights of mercury corresponding to the two volumes, *t* the temperature of the water; then $w \left(1 + \frac{t}{5550}\right)$, $w' \left(1 + \frac{t}{5550}\right)$, are the weights of mercury at 0° , which would fill those spaces; and accordingly we have

$$v = \frac{w}{D} \left(1 + \frac{t}{5550}\right), \quad v' = \frac{w'}{D} \left(1 + \frac{t}{5550}\right).$$

To these volumes should be added the capacity of the capillary tubes outside the vessel in which the water was boiled. In M. Regnault's apparatus this capacity never exceeded the $1 \div 2000$ th part of the volume of the ball.

To determine the relation between the data of the experiment and the coefficient *a*,

Let H be the barometric height when the ball was surrounded with melting ice;

h the small difference of level at the same time in FH and IB ;

H' the height where the ball was surrounded by the vapour of boiling water;

h' the difference at the same time of levels in the tubes; ,

t° the temperature of the water surrounding FH and BI during the first part of the experiment;

t'° during the latter.

Now when the ball was at the temperature T° of boiling water, the air, under the pressure $H' + h'$, occupied a volume $V(1 + \delta T)$, at the temperature $T^\circ + a$ a volume v' at the temperature t' ; therefore at 0° , and under the pressure $H + h$, it would have occupied a volume

$$\left(\frac{V(1 + \delta T)}{1 + \alpha T} + \frac{v'}{1 + \alpha t'} \right) \frac{H' + h'}{H + h};$$

but in the first part of the experiment the *same* mass of air, under the pressure $H + h$, occupied a volume V at $0^\circ + a$ a volume v at t° ; therefore at 0° , and under the pressure $H + h$, it would have occupied the volume

$$V + \frac{v}{1 + \alpha t}.$$

Equating these expressions for the volume of the same mass of air at the same temperature and pressure, we obtain the result,

$$\left(V \frac{1 + \delta T}{1 + \alpha T} + v' \frac{1}{1 + \alpha t'} \right) (H' + h') = \left(V + v \frac{1}{1 + \alpha t} \right) (H + h).$$

55. *Application of preceding Methods to the Case of other permanent Gases.*—We have hitherto supposed the apparatus employed in the determination of the expansion of air. If any of the other permanent gases are operated on, after the balloon has been perfectly dried in the manner already explained, it is exhausted, and put in communication with the gasometer by means of a second tube in the lower part of the pump P . The gas is then admitted, and withdrawn through the desiccating apparatus, four or five times, until we are sure of its perfect freedom from moisture. The remainder of the operation is then conducted in the same manner as in the case of atmospheric air.

If we wish to operate on air under a constant pressure greater or less than the atmospheric, it is only necessary to force it into the ball, or draw it from it, until its pressure reaches the desired limit, as indicated by the height, h , of the mercurial column which it balances in IB, and throughout the experiment to maintain, as before, this height, h , as nearly constant as possible.

Laws of Dilatation of Gases.

56. *Laws of Dilatation under the atmospheric Pressure.*—In the year 1801 Mr. Dalton* announced as the result of his investigations, that all elastic fluids under the same pressure expand equally by heat, and that for any given expansion of mercury, the corresponding expansion of air is proportionally something less, the higher the temperature.

By a coincidence not unusual in the history of the physical sciences, M. Gay-Lussac arrived, about the same time, at similar conclusions, although his experiments were not published until a few months after Mr. Dalton's. M. Gay-Lussac states his results as follows:

(1). "All gases, whatever be their density, and the quantity of water which they hold in solution, and all vapours, undergo the same dilatation by the same change of temperature."

(2). "For the permanent gases, the augmentation of volume which each of them receives from 0° to 100° is $100 \div 266.66$ of the initial volume at 0° ."

According to Mr. Dalton, a gas at any temperature increases in volume for a change of temperature of 1° by a constant fraction of its volume *at that temperature*. But according to M. Gay-Lussac, its increment at any temperature for 1° is a constant fraction of its volume *at some given temperature*. According to the first view, if α be the coefficient of expansion for 1° , the unit of volume at 0° becomes, at t° , $(1 + \alpha)^t$; according to the latter, it becomes $1 + \alpha t$. The latter view is now generally considered to be correct, and the coefficients of expansion of gases are always referred to the volume at 0° .

* Memoirs of the Manchester Philosophical Society, vol. v. part ii. p. 600.

As gases expand uniformly with the mercurial thermometer, at least between 0° and 100° , the formulæ (36) applicable to solids apply also to them.

Those laws of Dalton's and Gay-Lussac's have, however, been found, as we have seen, to be only approximations to the truth. Gases have not all the same coefficient of dilatation, nor is this coefficient constant for the same gas under different pressures. They are, however, true, in M. Regnault's opinion,* *at the limit*, that is, they approach more nearly to the truth according as we apply them to gases in a greater state of dilatation, and are strictly applicable to a perfect gaseous state.

We subjoin a tabular view of the results of recent investigations on the expansion of gases under the ordinary atmospheric pressure.

TABLE of the Coefficients of Expansion of Gases from 0° to 100° , under the Pressure of 760^{mm}.

Gases.	According to M. Rudberg.†	According to M. Pouillet.‡	According to M. Regnault.§	According to M. Magnus.
Hydrogen,	0.366 130	0.365 659
Air,	0.364 570	0.368 000	0.367 060	0.366 508
Carbonic oxide,	0.366 880
Carbonic acid,	0.370 990	0.369 087
Protoxide of azote,	0.371 950
Cyanogen,	0.387 670
Sulphurous acid,	0.390 280	0.385 618

The values¶ given in the fourth column were obtained by M. Regnault by the *direct method of dilatation* (54).

He remarks that the coefficients of carbonic acid gas and protoxide of azote, obtained by this method, were greater than those determined by the *method of elastic forces*. The difference

* Annales de Chimie et de Physique (3^{me} Serie), tome v. p. 83.

† Taylor's Scientific Memoirs, vol. ii. p. 516.

‡ Elemens de Physique (4^{me} Edit.), tome i. p. 259.

§ Annales de Chimie et de Physique, tome v. p. 75.

|| Transactions of the Royal Academy of Berlin, 1841, p. 80.

¶ It may be remarked, that adopting the value .36666 for the coefficient of the dilatation of air, this coefficient may be expressed by the equivalent vulgar fraction $11 \div 30$, which will be found very convenient in calculations.

between the results of these methods is still greater in the case of the more compressible gases, as cyanogen and sulphurous acid. Thus for sulphurous acid he obtained, by the first method, the value 0.390 280; by the latter, 0.384 530; and for cyanogen, the values, 0.387 670, and 0.382 900. The difference between these two methods shows that, in the case of highly compressible gases, Mariotte's law ceases to be strictly applicable.

57. *Laws of Dilatation under Pressures greater and less than the Atmospheric.*—On comparing the expansion of gases at different pressures, M. Regnault found that the coefficient increases with the pressure. The following Table exhibits his results in the case of air, as derived from the *method of elastic forces* (53).

TABLE showing the Expansion of Air under different Pressures.

Pressure at 0°.	Pressure at 100°.	Density of Air at 0°, taking as Unity the Density of Air at 0° under the Pressure of 760 ^{mm} .	1 + 100 α .
109 ^{mm} .72	149 ^{mm} .31	0.1444	1.36482
174 .36	237 .17	0.2294	1.36513
266 .06	395 .07	0.3501	1.36542
374 .67	510 .35	0.4930	1.36587
375 .23	510 .97	0.4937	1.36572
760 .00	1.0000	1.36650
1678 .40	2286 .09	2.2084	1.36760
1692 .53	2306 .23	2.2270	1.36800
2144 .18	2924 .04	2.8213	1.36894
3655 .56	4992 .09	4.8100	1.37091

Experiments conducted by the same method lead to the following results in the case of carbonic acid gas.

TABLE showing the Expansion of Carbonic Acid Gas under different Pressures.

Pressure at 0°.	Pressure at 100°.	Density of Gas at 0°.	1 + 100 α .
758 ^{mm} .47	1034 ^{mm} .54	1.0000	1.36856
901 .09	1230 .37	1.1879	1.36943
1742 .73	2387 .72	2.2976	1.37523
3589 .07	4759 .03	4.7318	1.38598

On investigating the relation between the expansion of gases and their pressure, by the *direct method of dilatation* (54) M. Regnault arrived at the following results:

Gases.	Value of $1 + 100 \alpha$ under the Pressure of 760 ^{mm} .	Value of $1 + 100 \alpha$ under the Pressure of 2530 ^{mm} .
Hydrogen,	1.36613	1.36616
Air,	1.36706	1.36944
Carbonic acid,	1.37099	1.38455

And for sulphurous acid:

Gas.	Value of $1 + 100 \alpha$ under the Pressure of 760 ^{mm} .	Value of $1 + 100 \alpha$ under the Pressure of 982 ^{mm} .
Sulphurous acid, . . .	1.39028	1.39804

From these tables it appears that the difference between the coefficients of expansion of gases increases with the pressure to which they are subjected, and hence the conjecture which, as we have stated above, M. Regnault has advanced, that Dalton and Gay-Lussac's laws, as well as the law of volumes, &c., are true of gases at the limit, that is, when they are taken in their greatest state of dilatation.

The remarkable increase in the expansion of sulphurous acid for so small a change of pressure as that from 760^{mm} to 980^{mm} renders it probable, as the same author has observed, that vapours have coefficients of dilatation very different from that of air at temperatures not far removed from their point of condensation. The importance of this remark will be seen when we come to treat of the density of vapours.

SECT. IV.—ON THE COMPARABILITY OF THERMOMETERS AND THE MEASURE OF TEMPERATURES.

58. *Mercurial Thermometers.*—It has been shown (10) that thermometers constructed of any material, for which the increment of volume for any given change of temperature bears to the origi-

nal volume a constant ratio, and which are graduated according to the method there explained, are perfectly uniform and comparable in their indications. It was there also assumed that mercury in glass envelopes fulfils the former condition; but careful experiments have shown this assumption not to be strictly true, and that mercurial thermometers, when exposed to high temperatures, and great changes of temperature, especially if made of different kinds of glass, not only fail in being perfectly comparable, but that even the same thermometer is not strictly uniform in its indications.

The following table, given by M. Regnault,* establishes the former assertion. The temperature was ascertained by the method of weights. The thermometer, No. 1, was formed of a piece of ordinary thermometer tube, with a ball blown at the end; No. 2, of a small globe of the same description of glass, united to a capillary tube; and No. 3 of a ball blown on a tube of crystal glass. The three balls had sensibly the same diameter, and the tubes the same bore and length

1.	2.	3.	Difference between 1. and 3.
0°	0°	0°	0°
100	100	100	0
190 .51	190 .84	191 .66	+ 1 .15
246 .68	247 .02	249 .36	+ 2 .68
251 .87	252 .06	254 .57	+ 2 .70
279 .08	279 .31	282 .50	+ 3 .42
310 .69	311 .14	315 .28	+ 4 .59
333 .72	333 .76	340 .07	+ 6 .35

A want of uniformity in the indications of the same thermometer, arising from a progressive displacement of the 0°, has been already referred to, but in addition to this some researches by M. Regnault and M. I. Pierre show that this point, as well as the interval between it and 100°, are liable to irregular changes, amounting, in some instances, to a displacement of 1° C., in the

* Ann. de Chim. et de Phys., tome v. p. 427, 3^m Ser.

zero, and to an alteration equivalent to $0^{\circ}.6$ in the length of the column between 0° and 100° .

59. *Air Thermometers.*—The differences in the indications of mercurial thermometers formed of different kinds of glass arise, doubtless, from the fact that different glasses have not only different coefficients of expansion, but also vary in the law of their dilatation at high temperatures; and as the amount of absolute dilatation of mercury is small, this variation in the expansion of the envelope produces irregularities of considerable magnitude in the apparent dilatation of mercury. As the real expansion of air is much greater, its apparent expansion in glass is not affected to the same extent by these variations in the rate of expansion of the latter; and accordingly in an air thermometer the rate of expansion of the glass may be considered as sensibly uniform. When corrected, therefore, for the expansion of its envelope, such an instrument forms the most perfect thermometer with which we are acquainted in the present state of science.

Any of the various forms of apparatus devised for the purpose of ascertaining the expansion of air may be employed, when this expansion is known, as an air thermometer. The volume occupied by the air in the instrument at 0° being known, its *true* increment at any temperature,—derived from its *apparent* increment and the expansion of its envelope, and corrected for variations of pressure,—divided by its volume at 0° , is the index of the temperature; and this ratio, being divided by .00366, gives the number of degrees expressing the temperature on the centigrade scale.

60. *Comparative Indications of Mercurial and Air Thermometers.*—The following table gives a view of the comparative indications of an air thermometer corrected for the expansion of its envelope, and of a mercurial thermometer constructed with the peculiar description of glass employed by M. Regnault in his experiments. It will be seen the accord between the two instruments is perfect up to 200° C.

Air Thermometer.	Mercurial Thermometer.	Difference.
0°	0°	0
50	50 .2	+ 0 .2
100	100	0
150	150	0
200	200	0
250	250 .3	+ 0 .3
300	301 .2	+ 1 .2
325	326 .9	+ 1 .9
350	353 .3	+ 3 .3

“These results apply to the comparative progress of an air thermometer, corrected for the expansion of the glass envelope, and a mercurial thermometer constructed with glass tubes of French manufacture, identical, in short, with those employed in my experiments. The correction might be very different if the mercurial thermometer were constructed with glass of a different kind.”*

61. *Changes of Temperature as measured by Air Thermometers, formerly supposed proportional to the Quantities of Heat producing those Changes.*—When Dalton and Gay-Lussac's laws, however, of the equal dilatibility of all gases, were first announced, physicists were of opinion that the air thermometer was not only best fitted to serve for the indication of temperatures, but that it was also capable of measuring the forces to which changes of temperature are due. For, they argued, as the power of heat to produce dilatation of volume is the same in all gases, however they may differ in their physical properties, we must conclude not only that the mutual attraction of the molecules of bodies is destroyed when they assume the gaseous state, but also that the specific action of those particles on heat, which might modify the repulsive power of the latter, cease to have any influence. That the force of heat is accordingly exhibited without any deduction or diminution in the dilatation of gases, and that this effect, therefore, may be assumed as the measure of the power which produces it.†

* Regnault. *Annales de Chimie et de Physique*, tome v. (3^{me} Serie) p. 100.

† Laplace, *Mecanique Celeste*, tome iv. (Preface).

62. *Determination, on this Hypothesis, of the absolute Zero.*—

Hence they were led to form certain speculations relative to the absolute zero, that is, the point on the thermometric scale at which the amount of heat in a body would be reduced to nothing. This point they investigated thus. As the expansion of a gas is the measure of the force or quantity of heat producing it, we may assume as the unit of heat the quantity producing an expansion of the unit of volume; now taking the expansion due to a change of temperature of 1° C. as the unit of volume, a mass of gas at 0° contains, according to Gay-Lussac, $266\frac{2}{3}$ units of volume, and therefore $266\frac{2}{3}$ of heat. Subtracting, therefore, this number of units of heat from it, which, as we have seen, would reduce its temperature to $-266\frac{2}{3}$ C., the quantity of heat which it contained would be totally exhausted, and therefore the absolute zero corresponds to the temperature of $-266\frac{2}{3}$ C.

This determination of the absolute zero appeared to be confirmed by some speculations of MM. Clement and Desormes on the subject of specific heats, to which we will subsequently refer. For the present we may state that the coincidence between the two results can only be attributed to accident, as they are both based on hypotheses which have been since proved to be unfounded. The investigations of MM. Rudberg and Regnault, which show the inaccuracy of Dalton and Gay-Lussac's laws, overthrow all these views with respect to the air thermometer, and reduce it to the same class with instruments constructed of other dilatable substances, from which it differs only in its superior sensibility, and in the *degree* in which it possesses other important thermometric properties. It is still, therefore, entitled to hold its place as the standard instrument for the *measure of temperatures*, but can no longer be regarded as a *true measure of the forces* producing changes in the latter.

63. *Concluding Remarks on the Measure of Temperatures.*—

In fine, the subject of the measurement of temperature stands thus. Any substance possessing the property formerly referred to (10), serves perfectly, when properly graduated, to estimate and indi-

cate temperatures, and if we assume as the measure of changes of temperature the effect of these changes in producing dilatation in the substance selected, then, consistently with this definition, one change of temperature is two, three, or four times another, if the expansion of the thermometric substance due to the former is so many multiples of that due to the latter. On comparing different substances, however, we find that although all, except liquids, agree very well in the relation between their expansions and changes of temperature from 0° to 100° C., yet at higher temperatures a notable difference is perceivable, so that the ratio of two changes of temperature, measured by the corresponding expansions of one substance, differs sensibly from the ratio of the same quantities as estimated by the dilatations of another. The question then arises, as to what substance we shall fix upon for our standard; is there any one possessing any property marking it out as *peculiarly* suitable for this purpose, or do all differ merely in the *degree* in which they possess various properties, which render them, under different circumstances, more or less suitable, so that our standard must be, to a certain extent, arbitrarily chosen? If we were acquainted with any body, or class of bodies, whose changes of volume were proportional to the forces producing them, such bodies would have a decided peculiarity marking them out as suitable for thermometric purposes, inasmuch as changes of temperature indicated by these would be proportional to the forces or quantities of heat to which those changes were due. Such bodies gases were supposed to be, on certain hypotheses derived from Dalton's and Gay-Lussac's laws; but these laws failing, the hypotheses based upon them fall to the ground, and accordingly gases are reduced to the same category with other thermometric substances, and our choice of a standard thermometer must be determined by other considerations which have been already adverted to. These, we have seen, lead us to adopt the air thermometer as the standard; the sensibility of its indications, the extent of its range, limited only by the nature of its envelope, and the identity attainable in the thermometric substance, all combine to render it superior for this purpose to any other with which we are acquainted. It must not be forgotten, however, that changes of temperature are estimated and measured in this

thermometer, as in all others, by changes in volume of the thermometric substances; that both these changes are co-ordinate effects of a common cause, but that of the relation of each to such cause we are yet ignorant; and that accordingly we are still unprovided with an instrument whose indications of changes of temperature are proportional to the forces by which such changes are produced.

SECT. V.—APPLICATIONS OF THE LAWS OF DILATATION.

64. *Measures of Length and Weight.*—We have seen that the absolute dimensions of all bodies are altered by changes of temperature, and accordingly if these dimensions are expressed by the number of the corresponding units, supposed to be of invariable magnitude, which they contain, the value of the dimension in question, whether length, surface, or volume, will vary with the temperature of the body. If, however, we are acquainted with its rate of expansion, and know the value of any of its dimensions at a given temperature, in terms of the invariable unit, we can easily determine its value at all other temperatures. In practice, however, it is impossible to determine any of the dimensions of a body in reference to such unit directly, as it can only be ascertained by the application of a measure or rule which is itself liable to contraction and expansion, and which, at some given temperature, bears a known ratio to the corresponding unit. In order, therefore, to ascertain the value of the dimensions of a body, corresponding to various temperatures, in terms of the invariable unit, it is necessary to apply to the results of actual observation various corrections and reductions, which it is our purpose to point out in the present section.

In like manner, although the absolute weight of the same mass of matter, that is, its weight *in vacuo*, is not altered by changes of temperature, yet its weight in air or any other fluid, and the absolute weight of a given volume of it, as well as the ratio which the weight of such volume bears to the weight of the same volume of any other substance selected as a standard of reference, are all affected by such changes.

Before proceeding to point out the relations existing between

the absolute and relative dimensions and weights of bodies, and their temperature, it will not be irrelevant to our present subject to make some remarks on the units to which these quantities are usually referred.

65. *Units of Length and Weight anciently adopted.*—Amongst most nations we find that the earliest standards of length were taken from the human body; of this the *δακτυλος, πους, σπιθαμη, πηχυς*, and *οργυια* of the Greeks, the *digitus, pollex, palmes minor* and *major, pes, gradus, passus*, and *ulna* of the Latins, and the foot, yard, and fathom of the English, are obvious instances. Besides these, however, we frequently find a further ultimate unit, of dimensions so small as to be considered to require no farther subdivision, and of a nature so constant as to serve to correct the great diversity of measures derived from the sources formerly referred to. Of this nature was the length or breadth of a grain of barley-corn, which we find adopted as the ultimate unit by the Hindoos, Hebrews, and several nations of modern Europe. Thus, in England three barley-corns taken from the middle of the ear, and placed end to end, formed the standard of an inch, twelve inches constituted a foot, and three feet a yard, which was made the unit for all higher denominations.

We find, in like manner, the unit of weight frequently derived from the same source. “Thus, among the Hindoos, the weights are determined by reference to the *barley-corn* and *gunja*, the seed of the *Abrus precatorius*, which is considered equivalent to two of the former. The Greeks also made two *σιτάρια*, or grains of barley, equivalent to the *χαλκος*, their most minute piece of copper money, four of these equal to the *κερατιον*, or carob seed, and eight to the *θερμος*, or lupine. The Romans made their weights, however, terminate in the *siliqua*, or *κερατιον*, deriving them directly from the Greeks, and therefore not proceeding lower than such weights as were in actual use. Amongst the Italians, and all other European nations, the grain of barley and the carat, which is equivalent to four of them, have been assumed as the basis of all existing weights.”*


66. *Attempts to fix Standards in England; Committee of 1758.*—In England one of the provisions of the Magna Charta

* Encyclopædia Metropolitana, Art. Arithmetic.

ordained that one common measure and one common weight should exist all through the kingdom. The weight which, in accordance with this provision, it was the constant object of the Legislature to establish, was that which has long been denominated the *pound troy*, and which contains twelve ounces, each ounce being equivalent to twenty pennyweights, and each pennyweight to twenty-four grains. Besides this pound, however, there has also been in use, from a very early period, another one-fourth greater, called the *libra mercatoria*, and nearly equivalent to our pound avoirdupois of sixteen ounces, which was employed almost universally in mercantile transactions, the use of the legal and statutable pound, or pound troy, becoming gradually limited to the precious metals, and, with a different subdivision, to medical purposes.

Notwithstanding the various and repeated efforts of the Legislature to secure uniformity of weights and measures throughout the kingdom, it appears that, owing either to the inconsistency of their regulations, the frequency of their changes, or the imperfections of the standards (made by rude artists, and tried by methods equally rude), the desired uniformity was never attained, and it was accordingly found necessary to appoint a Parliamentary Committee in 1758, to inquire into the original standards of weights and measures, and to examine those preserved in the Exchequer, Guildhall, and elsewhere. The committee presented their Report, which was drawn up by Lord Carysfort, on the 28th of May of the same year, and recommended that, in consequence of the diversities and inaccuracies of the old standards, a new yard and pound troy should be made by Mr. Bird, from the mean of those preserved in the Exchequer, or rather from accurate copies which had been made of them, by Mr. Graham,* for

* It appears from the paper in the Philosophical Transactions (vol. xlii. p. 541), which gives an account of Mr. Graham's comparison of the old standards with one which had been made for the Royal Society, that he examined those contained in the Exchequer, the Guildhall, the Tower, and in the office of the Clockmakers' Company. Of these the most ancient was an eight-sided brass rod, deposited in the Ex-

chequer, and representing a yard, very coarsely made, and rudely divided into three feet, and one of the feet into twelve inches, stamped with an old English  and a crown, and supposed to have been the standard lodged there in the reign of Henry VII. In the same office were two other squared rods of brass, used as the standard yard and ell, about half an inch in breadth and thickness, and fitted into hollow

the Royal Society, and that these should be adopted as the standards for the kingdom. A second Report was made in the following year, but the bills founded upon them, and proposed in 1765, never received the sanction of Parliament.

67. *Determination of Standards in France.*—Shortly before the Revolution, the subject of establishing a uniformity of weights and measures, by reference to an invariable standard determined on scientific principles, engaged the attention of the French Government; and on the motion of M. Talleyrand in 1790, the Constituent Assembly appointed a commission consisting of MM. Borda, Lagrange, Laplace, Monge, and Condorcet, to report on the measures proper to be taken to effect this object. Their Report, delivered in the following year, after noticing the proposals* which had been made to adopt as the unit of measure the length of a pendulum vibrating seconds at the equator, or at 45° , and stating their objection to the adoption of such standard, as involving the heterogeneous element of time, recommended that the 10 000 000th part of a quadrantal arc of a meridian on the earth's surface should be adopted as the primary standard of length, weight, and capacity, under the denomination of a *mètre*; that to determine this length an arc of the meridian, extending from Dunkirk to Barcelona, should be measured; and that subsequently the weight of a *decimètre*, cubed, of distilled water at the temperature of 0° C., should be determined as the unit of measures of weight, under the designation of a *kilogramme*; and, moreover, that the subdivisions of all measures should proceed according to the decimal scale.

beds or matrices, cut on two sides of a substantial brass bar, about forty-nine inches long. These were stamped with a crowned E, and appeared to have been placed there in the reign of Queen Elizabeth. It was from these that the standards in the other offices seem to have been taken,—those at the Guildhall and Clockmakers' Hall in the reign of Charles II., and that at the Tower in the reign of George I.

The standard troy and avoirdupois weights in the Exchequer, which Mr. Graham examined, were also placed there in the reign

of Queen Elizabeth, and were the originals from which those belonging to the Founders' Company and the Mint were derived.

* “The first proposal to adopt the length of the pendulum as a standard of measure, and to establish the third part of the seconds' pendulum (supposed to be everywhere of equal length) as a *pes honorarius*, . . . is found in Huygens' *Horologium Oscillatorium*, 1673, prop. 25.”—*Humboldt's Cosmos*, vol. i. notes, p. xliii. (Sabine's translation).

The recommendations of this committee having been carried into effect, with this exception, that the decimètre cubed of distilled water at its *greatest density*, and not at the temperature of 0° , as first proposed, was adopted as the standard of weight, “the original *mètre** and *kilogramme* (*les étalons prototypes*) were presented on the 4th Messidor, 1798, with a pompous address, to the two councils of the legislative body. In speaking of the *mètre* it is said: ‘Cette unité, tirée de plus grand, et des plus invariables des corps que le l’homme puisse mesurer, a l’avantage de ne pas différer considérablement de la demitoise et des plusieurs autres mesures usitées dans les différens pays: elle ne choque point l’opinion commune. Elle offre un aspect qui n’est pas sans intérêt. Il y a quelque plaisir pour un père de famille à pouvoir se dire: Le champ qui fait subsister mes enfans est une telle portion du globe. Je suis dans cette proportion conpropriétaire du monde.’” To provide against the loss of the unit resulting from the possible destruction of the standard by “un tremblement de terre,” or “un affreux coup de foudre,” it is added that M. Borda had determined with great accuracy the length of the second’s pendulum at Paris, and the repetition of the experiments at any future period would furnish the means of recovering the original relation of its length to that of the *mètre*, and, consequently, of determining the length of the metre itself.

Notwithstanding all the facilities, however, which the circumstances of France offered for the introduction into actual life of a system of weights and measures founded on scientific principles, the opposition to its adoption, from prejudice or habit, was so great, that in 1812 the Legislature was obliged to sanction a partial return to the old system, and to this day the *duodecimal* subdivision, and several of the names of the old measures, are applied to the new.

68. *Determination of Standards in England; Committee of 1818.*—In the year 1818 the attention of Parliament was again directed in England to the subject of uniformity of weights and measures; and a commission was appointed, consisting of Sir Joseph Banks, P. R. S., Sir George Clerk, Mr. Davies Gilbert,

* Encyclopædia Metropolitana, Art. Arithmetic.

Dr. W. H. Wollaston, Dr. Thomas Young, and Captain Kater, for the purpose of forming new standards of weights and measures, or of determining the relations of those already in use to some invariable standard existing in nature. These commissioners presented three Reports, bearing date, respectively, the 24th June, 1819, 13th July, 1820, and 31st March, 1821. In the first of these Reports they give it as their opinion, that with respect to the actual magnitude of the standards of length there is no sufficient reason for altering those which at present are generally employed, and that there is no practical advantage in having a quantity commensurable to any original quantity existing, or which may be imagined to exist, in nature, except as affording some little encouragement to its common adoption by neighbouring nations. The great inconvenience, however, resulting from a change which would be so extensively felt, they considered to outweigh the possible advantage which might accrue from its adoption, and which would only affect those engaged in foreign commercial affairs and scientific pursuits. They recommended, moreover, the continuance of the existing subdivisions, both of measures of length and weight, as being practically more convenient than the decimal scale, inasmuch as they admit of a greater number of divisions without the occurrence of fractional parts. As the unit of length they further recommended the adoption of General Roy's standard yard, employed for the measurement of the base on Hounslow Heath; but they subsequently, in their Second Report, recommended, in preference, Mr. Bird's parliamentary standard, executed in 1760. They further determined that the length of a pendulum vibrating seconds, in the latitude of London, at the level of the sea, and *in vacuo*, was equal to 39.13929 inches measured on this standard; and that the length of a platina mètre at 32° F. equalled 39.37079 inches, the English standard in both cases being at the temperature of 62° F.

With respect to measures of weight, they recommended that one-half of the double pound troy, made by Mr. Bird, should be considered as the imperial standard pound troy; that this pound should contain 5760 grains, and that 7000 such grains should make a pound avoirdupois. They further determined, for the purpose of furnishing the means of verifying these weights, that

a cubic inch of distilled water *in vacuo* at 62° , as opposed to weights also *in vacuo*, weighed 252.722 grains, and in air at the same temperature, and under the mean barometric pressure, 252.456 grains.

As the unit of capacity they recommended the adoption of the gallon containing ten pounds avoirdupois of distilled water, at the temperature of 62° F., and weighed in air when the barometer is at 30° ; and they determined the volume of this quantity of water to be 277.276 cubic inches.

A Bill embodying these recommendations was drawn up by Sir George Clerk, and passed in 1824.

The standard of length, then, in England, is Mr. Bird's standard brass yard at 62° F., and in France Borda's platina mètre at 0° C.; and in case these measures should be lost or destroyed, the means of restoring them is provided by the known relation existing between them and the length of pendulums vibrating seconds in the capitals of the respective countries.

In the same way, the standard weight in England is Mr. Bird's double pound troy, and in France M. Borda's platina kilogramme; and the means of reproducing those standards is derivable from the knowledge of the volume of distilled water at a known temperature whose weight is equivalent to them respectively.

We now proceed to point out the relations between the absolute and relative measures and weights of bodies and their temperature; and first of their weights.

69. *Relation between absolute Weight and Weight in Air.*—The absolute weight of a body, or its weight *in vacuo*, is, as we have remarked, independent of its temperature; this, however, is not the case with its weight in air. For from the principles of hydrostatics it appears that the weight of a body in air is less than its weight *in vacuo*, by the weight of a volume of air, at the existing temperature and pressure, equal to the volume of the body. To obtain the relation between those quantities, let us suppose that the body whose weight is sought, and its counterpoise, are both at the temperature of the surrounding air, which we will also suppose free from moisture. We will further suppose them placed in perfectly similar scales. Then let W be the weight of the body *in vacuo*, v its volume at 0° , k its coefficient of expansion

for 1° , and therefore $v(1 + kt)$, its volume at t° , the temperature existing at the time of the experiment; further let m be the weight of the unit of volume of dry air at 0° , and under the barometric pressure H , its weight at t° and h , the temperature and pressure during the experiment will be $m \frac{h}{H} \frac{1}{1 + at}$; and accordingly the weight of a volume $v(1 + kt)$, under the same circumstances, will be

$$mv \frac{h}{H} \frac{1 + kt}{1 + at};$$

and consequently the weight w of the body in air will be

$$w = W - mv \frac{h}{H} \frac{1 + kt}{1 + at}.$$

In the same way, if W' be the absolute weight of the counterpoise, v' its volume, and k' its coefficient of expansion, its weight w' in air will be

$$w' = W' - mv' \frac{h}{H} \frac{1 + k't}{1 + at};$$

but these two quantities are equal to one another, therefore we have

$$W = W' + \{v(1 + kt) - v'(1 + k't)\} m \frac{h}{H} \frac{1}{1 + at}.$$

The weight of a *constant volume* of any body also varies with its temperature, as it is inversely proportional to the volume occupied by a constant weight at that temperature. Consequently, if W , W' represent the weights of a given volume, at the temperatures t , t' , of a body whose coefficient of expansion is k , we have

$$\frac{W}{W'} = \frac{1 + kt'}{1 + kt}.$$

70. *Relation between tabular Density and Temperature.*—In like manner the ratio which the weight of a given volume of any body bears to the weight of the same volume of some substance selected as a standard, varies with the temperature of both. This ratio is called the specific gravity or tabular density of the body,

as referred to that standard, and we will proceed to exhibit the relation which exists between it and the temperature of the body itself, and of the standard to which it is referred.

For this purpose let W be the absolute weight of the body, and V its volume at the temperature t° , then $\frac{W}{V}$ will represent the weight of the unit of volume at the temperature t° . Let Ω represent the weight of the unit of volume of the standard at any temperature T ; then the density of the body corresponding to the temperature t of itself, and T of the standard, and which we will represent by D , is given by the expression,

$$D = \frac{W}{V} \cdot \frac{1}{\Omega}.$$

To ascertain the density D , corresponding to any other temperatures t' and T' of the body and the standard, it is to be observed that the volume of the body, which at t° was equal to V , at t' is equal to

$$V \frac{1 + \delta_{t'}}{1 + \delta_t},$$

if δ_t , $\delta_{t'}$ express the coefficients of expansion from 0° to t and t' respectively; and similarly the weight of the unit of the standard, which at T was Ω , at T' becomes

$$\Omega \frac{1 + \Delta_{T'}}{1 + \Delta_T};$$

Δ_T , $\Delta_{T'}$ expressing the coefficients of its expansion from 0° to T and T' . Therefore we have

$$D = \frac{W}{V} \frac{1 + \delta_t}{1 + \delta_{t'}} \cdot \frac{1}{\Omega} \cdot \frac{1 + \Delta_{T'}}{1 + \Delta_T},$$

or

$$D = D \frac{1 + \delta_t}{1 + \delta_{t'}} \cdot \frac{1 + \Delta_{T'}}{1 + \Delta_T}.$$

In the case of *gases*, the weight of matter contained in a given volume depends not only on the temperature but also on the pressure. Hence the pressures, both of the gas whose density is sought, and of that to which it is referred as a standard, must be taken into account.

Let D be the density of the gas at the temperature t and pressure h , as referred to a standard gas at the temperature T and pressure H , we have, as before,

$$D = \frac{W}{V} \frac{1}{\Omega};$$

if D' represent the density at t' and h' , referred to the standard at T and H' , V and Ω become

$$V \frac{1 + \alpha t'}{1 + \alpha t} \cdot \frac{h}{h'}, \text{ and } \Omega \frac{1 + \alpha' T}{1 + \alpha' T'} \cdot \frac{H'}{H},$$

α and α' being the coefficients of expansion of the gas and of the standard; consequently,

$$D' = D \frac{H}{H'} \cdot \frac{h'}{h} \cdot \frac{1 + \alpha t}{1 + \alpha t'} \cdot \frac{1 + \alpha' T'}{1 + \alpha' T}.$$

If we suppose the standard unchanged, those expressions become, in the case of *solids and liquids*,

$$D' = D \frac{1 + \delta_t}{1 + \delta_v},$$

and in the case of *gases*,

$$D' = D \frac{h'}{h} \cdot \frac{1 + \alpha t}{1 + \alpha t'},$$

which are obviously true, as they are merely the algebraic statements (69) of the fact, that when the standard is unaltered the tabular density of a body varies directly as the weight of a given volume of the material of which it is composed.

As examples of the application of these expressions, let it be required to determine, first, the density of mercury at 15°C. , in reference to water at $4^\circ.1$, being given its density 13.598, in reference to water at 0° .

Here $D = 13.598$, $\Delta_T = 0$, $\Delta_{T'} = .0001$ (by Hälström's table),

$\delta_t = 0$, $\delta_v = \frac{15}{5550}$; therefore

$$D' = 13.562.$$

Secondly, let it be required to determine the density of cyanogen at 50°C. and under the pressure of twenty-eight inches of

mercury, its density being 1.8064 at 0° and thirty inches, the density in both cases being referred to air at 0° and thirty inches.

Here $D = 1.8064$, $h' = 28$, $h = 30$, $\alpha = .00387$, $t = 0^\circ$, $t' = 50^\circ$; therefore

$$D' = 1.412.$$

71. *Correction of barometric Heights for Temperature.*—In all calculations in which the weight and pressure of the atmosphere are regarded as proportional to the corresponding height of the barometer, it is supposed that the column of mercury in the latter has a constant temperature, as otherwise its weight will not be simply proportional to its height, as it is assumed to be in such calculations. In all accurate observations, therefore, it is necessary to reduce the height of the barometric column to that which a column of equal weight would have at some constant temperature. To do this, suppose h to be the height of the column at the time of observation, when the temperature is t° , and let H be the height of a column of equal weight, at the standard temperature T° ; then, as the volumes of the mercurial columns in the same tube are proportional to their heights, we have

$$\frac{H}{h} = \frac{1 + \delta T}{1 + \delta t},$$

where δ , the coefficient of the expansion of mercury for 1°C. , equals $1 \div 5550$. If $T^\circ = 0^\circ$ we have simply

$$H = h \frac{5550}{5550 + t}.$$

72. *Correction of Measures of Length for Temperature.*—In determining the linear dimensions of bodies by the application of a rule of known length, any error arising from alterations in the latter from changes of temperature is, in ordinary cases, quite inappreciable. The errors, in fact, resulting from other sources, chiefly from the inaccurate adjustment of the rule to the body to be measured, far exceed any arising from expansion. When the distance to be measured, however, is considerable, as in the case of the base line of a trigonometrical survey, and extreme care is taken to avoid other sources of error, then alterations in the length of the rule by changes of temperature produce sensible

errors, which may be obviated either by some mechanical contrivance for counteracting the expansion,* or by determining the true length of the rule when it is applied to the measurement of a given portion of the line. This length is known immediately from the formula,

$$l_t = l_t \frac{1 + kt'}{1 + kt},$$

if we know the corresponding temperature, t' , and the length of the rule, l_t , at a known temperature, t . The difficulty in applying this formula consists in the determination of the true temperature of the rule at the time of observation, for if we assume its temperature to be the same as that of the atmosphere, as shown by a delicate thermometer, then if the atmospheric temperature undergoes sudden changes, the thermometer, on account of its smaller mass, will be quicker in experiencing the corresponding changes than the rule, and accordingly will not accurately indicate the temperature of the latter. This difficulty may be obviated by the following method, due to M. Borda, in which the rule is made its own thermometer.

Suppose, as in Fig. 20, two bars of metals of different expansibility, secured immoveably in a certain line, ef , at one extremity, and free to move at the other. Let the standard length from the fixed line be laid down on one of those bars, AB , when both have the same known temperature, which we will suppose to be 0° C. Let this length be ea , and draw $abcd$ parallel to the fixed line. Let the temperature of the bars be raised to T° , the temperature of boiling water, and suppose the upper bar to have gained the distance ce' on the lower, and let this distance be marked on the upper bar, and divided into any number, m , of equal parts. Then it is easy, at any instant, to determine the true length of the distance ea . For suppose at any time that the bar AB has gained n divisions on the bar CD , both having the temperature t° ; then, if those divisions were of invariable magnitude, the number gained, in consequence of the rise of temperature t° , would be to that gained by T° as those temperatures; therefore we should

* For a description of an ingenious contrivance of this kind by Colonel Colby see Williams's Geodesy, p. 65.

have $\frac{n}{m} = \frac{t}{T}$; but the actual length of the n divisions at t° is to that of the m divisions at T° , in the ratio $n(1 + kt) : m(1 + kT)$, k being the coefficient of expansion for 1° of the bar AB , on which we have supposed the divisions traced; therefore we have

$$\frac{n(1 + kt)}{m(1 + kT)} = \frac{t}{T},$$

which gives

$$t = T \frac{n}{m + (m - n)kT};$$

and therefore if L represent the length ea at 0° , or the standard length, its length at t° , when the upper rod has gained n divisions, will be $L(1 + kt)$, or

$$L \left(1 + kT \frac{n}{m + (m - n)kT} \right).$$

If kT , the coefficient of expansion of the bar AB for T° , be represented by the vulgar fraction $\frac{1}{q}$, the preceding expression may be put under the form

$$L \frac{(1 + q)}{(1 + q) - \frac{n}{m}},$$

which expresses very simply the true length of the rule corresponding to an advance of n divisions.

73. *Effect of Change of Temperature on Measures of Time.*—Measures of time are also affected, as well as measures of length, by changes of temperature; for the immediate object of the instruments in general use as measures of time is to register the number of vibrations made from a given epoch by some oscillating body, either a pendulum, as in the case of clocks, or a balance wheel, as in that of watches; and it is on the supposition that the duration of each vibration is a constant quantity, that the number of such vibrations is assumed to measure the portion of time which has elapsed from that epoch; but we learn from the principles of mechanics that the duration of the vibration of an oscillating body depends on the distance of its particles from

the axis of suspension or centre of motion, and this distance being affected by change of temperature, it follows that the duration of the body's vibrations is affected by the same cause.

It has been shown by mathematicians that if a material point is connected, by means of an inflexible rod devoid of weight, with a horizontal axis of suspension, and oscillates *in vacuo* in a vertical plane, the duration of each vibration is independent of the magnitude of the arc it describes, provided that this arc be very small, and depends only on the force of gravity, and on the distance of the moving point from the axis of suspension. To this abstraction they have given the name of the *simple pendulum*. It is, however, not only impossible to construct a pendulum exactly fulfilling the above conditions, but it is not even possible in practice to approach them nearly, for as all pendulums execute their vibrations in air, a considerable mass must be given to them to prevent their motion from being materially disturbed by the resistance of that medium, and to secure inflexibility the parts uniting the moving mass with the axis of suspension must necessarily be possessed of considerable strength and weight. They have an analogy, however, to the simple pendulum in this respect, that they generally consist of a mass of matter called the bob, of considerable weight as compared with other parts of the apparatus, and connected by a simple or compound rod with the axis of suspension. And the heavier the bob is in proportion to the rest of the apparatus, and the more condensed the matter in it is, the more striking is this analogy, and the more nearly are the vibrations of this *compound pendulum*, as it is called, in opposition to the afore-mentioned abstraction, isochronous with those of a *simple pendulum*, whose length is equal to the distance of the centre of gravity of the bob from the axis of suspension.

In the general case it has been shown that the vibrations of a compound pendulum are isochronous with those of a simple pendulum, whose length is equal to the quotient of the moment of inertia of the former, in reference to its axis of suspension, divided by its statical moment with respect to the same axis. Accordingly, if m , m' denote the masses of the suspending frame and of the bob, $m A^2$, $m' B^2$ the moments of inertia of the frame and bob round their respective centres of gravity, C , D the distances

of those centres from the axis of suspension, and L the length of the isochronous simple pendulum, we have

$$L = \frac{mA^2 + m'B^2 + mC^2 + m'D^2}{mC + m'D}.$$

74. *Principle of Correction.*—The quantities mA^2 , $m'B^2$, &c., are all functions of the weight and dimensions of the several parts of which the pendulum is composed, and of the distances of their centres of gravity from those of the frame and bob, and from the axis of suspension. Now as all these dimensions and distances are affected by change of temperature, it follows that the duration of the vibrations of the pendulum will be altered, unless there is such a relation established between the various parts that the changes of volume they experience shall counteract one another, and the preceding value of L be thus retained unchanged. This condition will be fulfilled if we give to the different parts of the instrument values consistent with the equation which is obtained by equating to cypher the variation of the preceding expression for L , arising from change of temperature; and as this variation is always exceedingly small, we may obtain it by means of the ordinary rules of the differential calculus.

Without pursuing this investigation, however, it is easy to see that we can, at least to a certain extent, fulfil the above condition, if, by any mechanical contrivance, we can cause the distance of the centre of gravity of the frame or of the bob from the axis of suspension, to remain unaltered, or rather slightly to diminish, while the other dimensions and distances are increased by rise of temperature. And we will accordingly now proceed to explain how this object has been sought to be effected, and first will describe some of the methods devised for counteracting the increase of the distance of the centre of gravity of the bob from the axis by increase of temperature.

75. *Graham's Compensation Pendulum.*—Mr. Graham, an English clockmaker, appears to have been the first who constructed a pendulum in which this object was attained. The pendulum which he proposed consisted of a rod of glass, to the extremity of which was attached a cylindrical vessel containing a quantity of mercury. As the rod expanded by heat, the centre of gravity

of the mercury was lowered, but it was at the same time raised by the expansion of the mercury itself, and if the lengths of the rod and of the mercury were duly proportioned to their coefficients of expansion, the distance of the centre of gravity from the axis of suspension might not only be retained unaltered, but even diminished by rise of temperature. Then let l be this distance at any temperature t° , h the height of the cylinder of mercury at the same temperature, k and Δ the coefficients of expansion of the rod and of mercury in glass for 1° , referred to the temperature t° , then, for any rise of temperature τ , the centre of gravity would be lowered by the expansion of the rod by the quantity $k\tau l$; but it would be raised by the expansion of the mercury, by the quantity $\Delta \frac{h}{2} \tau$, and accordingly, if those two quantities were equal, that is, if

$$kl = \Delta \frac{h}{2},$$

the distance of the centre of gravity of the bob from the axis would remain unaltered, and if the right hand member of the equation were the greater, this distance would be diminished.

A pendulum of this description is represented in Fig. 40. The correction to bring it to time at the mean temperature was made by means of the weight d , which was capable of motion along the rod; and the correction for imperfect compensation, by increasing or diminishing the quantity of mercury in the jar. This arrangement, however, was considered objectionable, as it altered the range of the index t on the arc of motion; and accordingly the construction of the mercurial pendulum in Fig. 41, in which the compensation is regulated by a screw which raises and lowers the cylinder, is that now generally adopted.

76. *Ellicot's Pendulum*.—In the pendulum represented in Fig. 42, and which is constructed after the plan suggested by Mr. Ellicot, this object is differently effected. The rod AB consists of two bars, to one of which are attached two short levers, turning on the centres o , o' , and acting at one extremity on the ends of the screws a , a' , attached to the bob; the other bar, which is permanently screwed to the former at b , acts on the other extremity c of the same levers; and accordingly when, by the effect of in-

creased temperature, the first bar is elongated, and the bob depressed, the second bar, expanding, acts on the end c of the levers, and raises the bob by a quantity depending on the difference of expansion of the bars, and on the ratio of the arms of the levers. The amount of correction may be very accurately regulated by means of the screws a, a' , the advancing or withdrawing of which alters the ratio of the effective portion of the arms of the levers. Let l represent the distance of the centre of the bob from the axis of suspension at the temperature t° , k the coefficient of expansion of the main bar for 1° referred to that temperature, l' the length of the compensating bar, and k' its coefficient, and $m \div n$ the ratio expressing the effect of the levers; then, in order that the length l should remain unaltered, we should have

$$lk = l'(k' - k)\frac{m}{n}.$$

77. *Harrison's Gridiron Pendulum*.—The next form of compensating pendulum to which we will direct the attention of the student is the invention of Mr. Harrison, and from its form is usually called the *gridiron pendulum*. In this pendulum the bob is suspended from a system of rods, some of which, by their connexion, can only expand downwards, and others upwards, and the lengths of the rods being so determined with reference to their expansibility, that those expansions in opposite directions shall mutually counteract each other, the centre of gravity of the bob may be made to retain its distance from the axis unaltered. The arrangement of the rods is shown in Fig. 43: those shaded with dark vertical lines represent rods of iron, those with the fainter horizontal shading rods of brass or zinc, or some metal more expansible than iron. By the construction exhibited in the figure it will be seen that the iron rods all *depending* from the cross traverses can only expand downwards, while the others *resting on* those traverses must expand upwards. It will also readily appear, from examination of the figure, that if we denote by $\Sigma\lambda$ the sum of the lengths of all the iron rods, and by $\Sigma\lambda'$ the sum of the brass, and by l the distance of the centre of the bob from the axis of suspension, we have

$$\Sigma\lambda - \Sigma\lambda' = l;$$

but in order to maintain l invariable it is necessary that we should have

$$k\Sigma\lambda - k'\Sigma\lambda' = 0,$$

k and k' being the coefficients of expansion of iron and brass, for 1° referred to the temperature at which the lengths are estimated; accordingly we have

$$\frac{k}{k' - k} l = \Sigma\lambda',$$

by means of which equation we can determine the length to give the more expansible metal, in order to preserve the distance l unaltered.

From the above equation it appears that a compensating pendulum may be constructed on this plan with any two metals, for which k' and k have different values; if the difference between these quantities, however, is very small, the sum $\Sigma\lambda'$ of the lengths of the compensating bars must be very great, and as the length of each of them must be less than l , their number in such case must be very considerable. In such case, also, the mass of the frame is very large in proportion to that of the bob, and the pendulum deviates widely from the form of the simple pendulum.

The correction for bringing the pendulum to time at the mean temperature is made by means of the screw d , while that for imperfect compensation is effected by having one of the cross traverses, generally the last, capable of being shifted on the rods to which it is attached, by which means the sum $\Sigma\lambda'$ of the lengths of the compensating bars is altered, and consequently the amount of the variation in the distance l adjusted.

78. *Compensation Pendulum with compound cross Bar.*—In the last form of compensating pendulum which we will notice, the compensation is effected by counteracting the alteration in the distance of the centre of gravity of the frame, or rather of the common centre of the frame and bob, from the axis of suspension. It depends on this principle, that if two metallic bars of different expansibility are firmly secured to each other, any change of temperature will cause such a system to warp or bend, the more

expansible bar forming the outside of the curve if the temperature is raised, and the less expansible if it is lowered. Accordingly, if such a compound bar, having two balls, a, a' , moveable on screws attached to its extremities, is secured at right angles to a pendulum rod, as in Fig. 44, with the more expansible metal placed underneath, the effect of increase of temperature will be to raise the centre of gravity of the two balls, a, a' , and thus the descent of the centre of the bob, owing to the same cause, may be counteracted. The amount of the correction may be regulated by moving the balls a, a' to or from the rod.

79. *Compensation Balance in Watches.*—In watches the regulation of the movement is effected by a balance-wheel, to which a vibratory motion is communicated by a spiral spring, alternately closing and opening. If the length of the spring and dimensions of the wheel be altered by change of temperature, the periods of the vibrations will also be altered. A compensation of this alteration may be effected by forming the periphery of the wheel, as in Fig. 45, of two compound bars similar to that last described, loaded with small screws. The effect of a change of temperature will be to move those screws nearer to the centre or farther from it; in the first case a smaller force will be necessary to effect the vibratory movement, in the latter a greater. And the position of the screws may be so arranged as to counteract the effect of change of temperature in altering the force of the spiral spring and the dimensions of the wheel.

80. *Pyrometers. Wedgwood's Pyrometer.*—Instruments destined for the measure of high temperatures are called pyrometers; these are constructed either of refractory solids, or of air contained in a refractory envelope. Wedgwood proposed a pyrometer formed of clay cylinders, which, contrary to the general analogy, contract on being exposed to high temperatures. Their diameter was measured by a gauge formed of two converging rules, before and after they had been exposed to the temperature to be estimated, and their change of volume was supposed to bear a fixed relation to the change of temperature to which they had been submitted. It has been ascertained, however, that their contraction, which is due to loss of moisture, is influenced by the length of time to which

they are exposed to a given temperature; and that exposure for a long time to a lower temperature produces as great a contraction as for a shorter time to a higher.

81. *Guyton de Morveau's Pyrometer*.—In the year 1803 M. Guyton de Morveau* published a description of a pyrometer composed of a bar of platina 45 millimetres long, 5^{mm} broad, and 2^{mm} thick, which was placed in a groove sunk in a slab of well-baked porcelain. The bar pressed at one end against the extremity of the groove, and at the other against the shorter arm of a platina lever 2.5 millimetres in length, whose longer arm, 50 millimetres long, traversed a graduated arc, also of platina, which, by means of a vernier, was capable of being read off to the one-tenth of a degree. The expansion of the platina bar, calculated from the arc described by the end of the longer arm of the lever, served to measure the temperature to which the apparatus was exposed. As it appears, however, that platina softens at high temperatures, it is probable that the motion of the index arm round its centre and along its scale would be impeded by this cause, and that the indications of this instrument, accordingly, at high temperatures, could not be safely relied upon.

82. *Daniell's Pyrometer*.—In this respect the apparatus employed by Mr. Daniell (see p. 31) for the measurement of expansions, and which may also be employed as a pyrometer, possesses a considerable advantage, as its scale is not exposed to any change of temperature. When used as a pyrometer this instrument is furnished with a platina or iron rod, whose expansions, determined by means of the scale, indicate the corresponding changes of temperature. If it is desired to compare the indications of this instrument with those of a mercurial or air thermometer, and to make its degrees identical with those of the latter instrument, a near approximation is obtained by supposing the rate of expansion of platina or iron the same as that of air or mercury in glass envelopes. Then, knowing the expansion of those metals for a given temperature, as measured on the mercurial or air thermometer, a simple proportion gives the temperature, calculated according

* *Annales de Chimie et de Physique*, tome xlvi. p. 276. *Memoires de l'Institut*, tome ix. (1808), tome xii. (1811).

to the same scale, corresponding to an observed expansion of the metal in the register. Thus having ascertained that an expansion amounting to .0152 corresponds to a change of temperature of 600° F., we obtain the temperature corresponding to the expansion .0508 by the proportion

$$.0152 : .0508 :: 600^\circ : 2005^\circ.$$

In this way the following table has been obtained by Mr. Daniell.

TABLE of the fusing Points of the Metals enumerated (Temperature at Time of Observation being 65° F.)

Metals.	Bar in Register.	Expansion for 600° F.	Expansion observed.	Temperature.
Copper fused,	Platina,	.0152	.0508	2005° + 65 = 2070°
Gold do.	Do.	.0159	.0537	2026 + 65 = 2091
Do. do.	Iron,	.0229	.0787	2061 + 65 = 2126
Silver do.	Platina,	.0116	.0363	1877 + 65 = 1942
Do. do.	Iron,	.0203	.0645	1906 + 65 = 1971
Iron do.	Platina,	.0116	.0546	2824 + 65 = 2889
Zinc do.	Iron,	.0203	.0239	708 + 65 = 773
Do. inflamed,	Do.	.0245	.0358	876 + 65 = 941

83. *Air Pyrometer*.—The expansion of air may be employed to measure changes of temperature, through a very considerable range, by means of the apparatus represented in Figs. 36, 37, 38, 39, and described in (53), (54). The ball A may be made of glass for ordinary temperatures, but for very high temperatures, when the instrument is used as a pyrometer, a ball of platina is required. The coefficient of expansion of air being known, the equations in pages 86, 88, give the temperature corresponding to an observed expansion.*

84. *Breguet's metallic Thermometer*.—A very sensitive metallic thermometer has been constructed by M. Breguet for ascertaining changes of atmospheric temperature. It consists (Fig. 46) of a cylindrical spiral formed of three metals, silver, gold, and pla-

* See also Pouillet, *Elements de Physique*, vol. i. p. 253 (4^{me} Edition).

tina, superposed, and rolled out until their joint thickness does not exceed one-sixtieth part of a millimetre, or .000656 inch. Its mass being so small, the least change of atmospheric temperature immediately affects it, and, owing to the different expansibility of the silver and platina it coils and uncoils, carrying an index needle round an arc graduated by comparison with a delicate mercurial thermometer. The whole apparatus is enclosed within a glass case, to preserve it from external causes of disturbance.

A different form of this instrument is represented in Fig. 47, where the apparatus is enclosed in a case similar to that of a common watch. The thermometric bar, *ab*, acts on one end of a lever, whose other end carries a rack which gives motion to the pinion to which the index, *cd*, is attached. A spiral spring, *h*, serves to bring back the index and lever when the bar returns to its original position.

85. *Force of Contraction and Expansion of Metals.*—The force which a metallic bar exerts at any instant in the direction of its length, when contracting on change of temperature, is equal to the force which would be necessary to extend it, at its temperature at that instant, by a quantity equal to the amount of contraction. Thus a bar cooling from 81° F. to 80° F. exerts a force equal to that which would be required to extend it at the temperature of 80° F. by a quantity equal to its contraction from 81° to 80° . The only experiments on the laws of tension of metallic bars with which we are acquainted are those made in the case of malleable iron, and at the ordinary temperatures, by Mr. Barlow.* According to this author, a bar of malleable iron is extended about $1 \div 10000$ th part of its length by a weight of one ton per square inch of section, and the elastic or restitutive force is weakened or destroyed by a weight of ten tons per inch, or an extension of $1 \div 1000$ th part of the length. Supposing these values to hold good, as they probably do, within the limits of atmospheric changes of temperature, it is easy to calculate the force exerted by a bar when contracting on any change of temperature within those limits. For as wrought iron expands $1 \div 10000$ th

* Barlow on the Strength of Materials, p. 315.

of its length at the mean temperature, for a change equal $q. p.$ to $15^{\circ}25$ F., it follows that a bar whose temperature is originally $15^{\circ}25$ F. above the mean, secured at its extremities to points maintained at an invariable distance, tends to draw its supports together by a force equal to one ton for each square inch of its section, on cooling down to the mean temperature; for a change of $30^{\circ}5$ F. its force of traction would equal two tons per square inch; and so on, within the limits above referred to. Calling, therefore, t° the higher temperature, t° the mean, and s the section of the bar in square inches, the force of traction τ may be expressed by

$$\tau = s \frac{t^{\circ} - t}{15 \cdot 25}.$$

Admitting, therefore, 76° F. to be the extreme range of temperature in this climate, it follows that a bar permanently fixed at the highest limit to supports maintained at an invariable distance, would be subjected, at the lower limit, to a strain of five tons per square inch of section, or one-half of that which it could bear without the destruction of its elastic force. If the above law held as far as 212° F., a bar fixed at that temperature would lose its elastic force altogether on cooling down to 60° F.

A familiar instance of the application of the contractile force of wrought iron when cooling is exhibited in the mode of securing the tires on wheels. The tires being made red hot, and thus considerably expanded, are dropped over the periphery of the wheel, and then cooled. If the wheel be of cast metal, the tire, when cold, embraces it with such force as to render its removal a matter of extreme difficulty. Care must, of course, be taken not to make the tire too small, as it might in that case be exposed, when cold, to a permanent strain, which would greatly weaken its cohesion, and render it liable to fracture. If the wheel be of wood, the contraction of the tire not only secures itself upon the rim, but also presses home the joints of the spokes in the felloes and nave, and holds all secure. If τ , as before, expresses the contractile force in tons, the normal force on any small portion da of the circumference will be $\tau \frac{da}{r}$, r being the radius

of the wheel; and therefore the force over the whole periphery will be equal to $2\pi\tau$, π being the ratio of the circumference of a circle to its diameter; and the felloes in the interval between the spokes being supposed inflexible, the pressure p in the direction of the spokes forming any diameter may amount to

$$p = \frac{2\pi}{n} s \frac{t' - t}{15^{\circ}.25},$$

n being the number of spokes. t' in this expression represents the temperature at which the tire just fits the rim, and s , as before, the area of the section of the tire.

An ingenious application of this force was also made in the case of a gallery in the Conservatoire des Arts et Metiers in Paris, whose walls were forced outwards by some horizontal pressure. To draw them together, M. Molard, formerly Director of the Museum in that establishment, had iron bars passed across the building, and through large plates of metal bearing on a considerable surface of the external walls. The ends of these bars were formed into screws, and provided with nuts, which were first screwed close home against the plates. Each alternate bar was then elongated by means of the heat of oil lamps suspended from it, and when expanded the nuts were again screwed home. The lamps being removed, the bars contracted, and in doing so drew the walls together. The other set of bars was then expanded in the same manner, their nuts screwed home, and the wall drawn in through an additional space by their contraction. And this series of operations was repeated until the walls were completely restored to the vertical, in which position the bars then served permanently to secure them.

The force with which a metallic bar expands is in like manner equal to the force which would compress it through a space equal to that by which it is elongated. It appears from Mr. Barlow's experiments, that in bars of malleable iron, when deflected, the depth of tension bears to the depth of compression a ratio varying from 5:1 to 3:1. From this it follows, that the force requisite to compress this material through a small space bears to the force required to extend it through the same, a ratio varying from $5^2:1$ to $3^2:1$. The force arising from the expansion

of metals by heat appears, therefore, to be considerably greater than that arising from their contraction, but it is, for many reasons, less manageable, and less capable of practical application.

86. *Allowance to be made for Contraction and Expansion of Rails and Pipes.*—The force of the expansion and contraction of metals being so considerable, allowance must be made for it in all works in which large masses are united together, as otherwise distortions of shape, or even destruction of part of the work, might result. On railroads, accordingly, an interval should be left between the rails when they are being laid, the amount of which should depend on their temperature at the time, on the change of temperature to which they will probably be exposed, and on the length of the rail. Thus if L be the known length of the rail at a given temperature t , L' its length at the temperature t' at which it is laid, and L'' its length at the temperature t'' , the highest to which it will be exposed, then an interval equal to $L'' - L'$ must be left between the rails when being laid; but we have (36)

$$L'' = L \frac{1 + kt''}{1 + kt}, \text{ and } L' = L \frac{1 + kt'}{1 + kt},$$

if k be the coefficient of dilatation of wrought iron for 1° ; therefore,

$$L'' - L' = L \frac{k(t'' - t')}{1 + kt};$$

and if k be expressed by the vulgar fraction $\frac{1}{n}$, the interval i is given by the formula

$$i = L \frac{t'' - t'}{n + t}.$$

In long systems of gas or water pipes provision must also be made for the effects of contraction and expansion. "When iron water pipes were first adopted, it was the practice to make a flanch at each end, for the purpose of connecting them together by screw bolts and nuts; however, a little experience demonstrated the inconvenience of this mode of constructing them, for the expansion and contraction by the variations of temperature occasioned their joinings either to become loose and leaky, or the breaking

of the pipes. The various injurious effects and disadvantages proceeding from this cause led to an alteration in their form, which proved an effectual preventive of any defects from the action of heat and cold upon the metal. This important improvement consisted in forming one end of each pipe larger than the other, with a kind of socket several inches in length, into which the narrow end of the corresponding pipe could be inserted, and the space remaining between them closely filled up with hemp wadding and lead, so as to render the joining perfectly water-tight. By adopting this method of connecting the pipes, the changes of temperature have not the effect of breaking them, though occasionally their joinings become imperfect, and require a little repairing to prevent the escape of water."*

87. *Effects of Expansion and Contraction on arched Bridges.*—

In *arched* iron bridges, which must bear against their abutments, no room, of course, can be left for expansion in the direction of the chord of the arch; the effect of increase of temperature in such structures, accordingly, is exhibited in a rise in the crown, which, however, in ordinary cases, is so small in amount, as not to exceed the limits which the elasticity of iron admits of without danger of fracture. Thus, in one of the arches of Southwark bridge, for which the length of the chord of extrados is 246 feet, and its versed sine 23 feet 1 inch, and accordingly the length of the arch, which is a segment of a circle, 3020.8 inches, the rise in the crown of the arch for a change of temperature of 1° F. was observed by Mr. Rennie† to be about one-fortieth of an inch; for 50° F., accordingly, it would amount to about 1.25 inch. In stone bridges of several arches, the effect of the contraction produced by cold is perceived in the opening of the joints of their parapets over the springing of the arches, and a depression over the crowns, while the expansion due to heat is exhibited, on the contrary, by an opening of the joints over the crowns. Instances of these effects were noticed by Mr. Rennie, in the case of the bridge over the Thames at Staines, in the year 1834, and described in the memoir just referred to.

* *Hydraulia*, an Account of the Water-Works of London, &c., by William Matthews, p. 70.

† *Transactions of Institution of Civil Engineers*, vol. iii. p. 201.

88. *Allowance to be made for Expansion and Contraction in the case of tubular and Lattice Bridges.*—In tubular and lattice iron bridges, however, in which such alteration of form is inadmissible, a space must be left at either end to admit of the expansion and contraction due to the change of temperature to which they will be exposed, and some provision also must be made, especially if their weight is very considerable, to facilitate, by some form of friction rollers,* the sliding motion of the free extremity, as otherwise the support at that end might be shaken or destroyed. The expression

$$i = L \frac{t'' - t'}{n + t},$$

in which L is the length of the bridge at any temperature t , and t'' , t' the highest and lowest temperatures to which it will be exposed, gives the value of the interval through which the free extremity must be at liberty to move.

89. *Effects of unequal Expansion and Contraction in Machines.*—In machines, and also in all large works of construction, in which the parts form a strictly rigid system, care must be taken as to the manner in which materials are combined, whose rates of expansion are very different, so as to secure, in the former case, the requisite freedom of motion, and in both to prevent the distortion of form which might result from neglect of this precaution. Thus if the socket, in the case of a stop-cock placed on a cast metal pipe destined to convey water at different temperatures, were made of cast metal, and the moveable part of brass, the unequal expansion of the two metals would either produce the effect of wedging it so tightly at the higher temperature as to prevent all power of opening or closing it, or if it moved freely at that temperature, it would be so loose at the lower as to leak, and allow the water to escape.

It is probably in a similar way that the iron axles of heavily laden goods' waggons on railways, when there is a failure in the supply of the lubricating material, become wedged in their journals by the effect of unequal expansion, and are sometimes violently twisted, or even wrenched off.

* Civil Engineers' and Architects' Journal, vol. xi. p. 161.

90. *Effects of Change of Temperature on Buildings.*—In the case of buildings, however, although the rates of expansion of some of the materials used in their construction differ as widely as those of any two metals, yet the total amount of the difference is so small as not to produce any sensible effect, where absolute rigidity does not exist, and where there is the least room for compression and extension. Thus Mr. Adie observes: "From the results given in the table it is evident that no danger is to be apprehended from a change of temperature affecting cast iron and sandstone in a very different degree, as their expansion, in so far as regards all building operations at least, may be regarded as the same. The difference between the expansion of bricks and malleable iron amounts only to .00042 for 90° F., or half an inch on 100 feet; and Roman cement, when in as damp a state as all buildings must be, will expand more than malleable iron, since it did so in my experiment, even after it had been dried for eleven hours at a temperature of 207°. This, therefore, will in a great measure serve for an explanation of the fact mentioned by Mr. Brunel, in describing his method of constructing arches, by suspending the courses of brick with straps of hoop-iron, viz., that he had had some anxiety as to the manner in which variations of temperature would affect his mode of operating, by expanding the metal more than the brick and mortar; but on examining his experimental arch, both in summer and in winter, not a crack was to be seen."*

* Transactions of the Royal Society of Edinburgh, vol. xiii. p. 367.

CHAPTER II.

ON THE RELATION BETWEEN THE TEMPERATURE AND THE STATE
OF BODIES, AND ON THE LAWS OF VAPOURS.

SECT. I.—LIQUEFACTION AND SOLIDIFICATION.

91. *Solidity dependent on Temperature.*—Observation and experiment prove to us in the case of all bodies, except the diamond and charcoal, that their existence in the solid or liquid state depends on their temperature. The point at which the change from one of these states to the other takes place varies considerably in different cases. The ordinary vicissitudes of atmospheric temperature suffice to produce it in some bodies, as water and oil; the alloys called fusible metals melt at temperatures near that of boiling water; tin, bismuth, and lead, are liquefied below a red heat; silver may be melted in a common fire urged by a bellows, gold and copper in a wind furnace, and iron has been melted in a small quantity in a draught furnace.

The most intense heat, however, capable of being produced in the best furnaces, fails to affect platina. This metal, however, yields to the heat of the oxy-hydrogen blow-pipe, and to that generated between the poles of a powerful galvanic battery. By means of the former apparatus Dr. Clarke reduced 100 grains of platina to a state of fusion, and kept it liquid for some minutes. Iridium is also melted in small globules by the same powerful sources of heat, as also lime, magnesia, and silica.

There are some substances, again, whose fusion cannot be effected unless they are operated on under great pressure. This is owing to their extreme volatility in the liquid form, which causes them to pass directly, as it were, from the solid to the gaseous state, under the ordinary atmospheric pressure. Thus Sir James

Hall succeeded in melting chalk (carbonate of lime) confined in the end of a gun barrel, under the pressure of the atmosphere of carbonic acid gas, disengaged by the action of heat, and converted it by this means into crystalline marble.

Carbon, it has been observed, whether under the form of diamond or of charcoal, has never been fused. When exposed to high temperatures in contact with oxygen, this substance undergoes combustion. Messrs. Hare and Silliman, indeed, announced that they had melted charcoal in small globules between the poles of a peculiar form of galvanic battery with large surfaces, called a *deflagrator*; but there is no doubt that those globules consisted, not of molten charcoal, but of some of the earthy bodies from which, in its purest state, charcoal is never entirely free. The *diamond*, when placed *in vacuo*, or in an atmosphere of carbonic acid, and exposed to the action of the sun's rays concentrated by a powerful lens, in some instances becomes black and carbonaceous, and acquires the property of marking the fingers or paper.

M. Jacquelin has recently shown that the same result follows from the action of a powerful galvanic battery. On placing a diamond between the poles of one of Bunsen's batteries, consisting of 100 couples, "it became luminous, softened (*se ramollissait*), acquired the property of conducting electricity (which the diamond does not possess), and passed into the state of genuine coke. Its density at the same time changed from 3.336 to 2.6778."*

There is no doubt, however, that carbon, under its various forms, presents only an apparent exception to the general law that all things are capable of being melted by "fervent heat," and that, if it could be operated on under the circumstances required by its chemical constitution, and exposed to a temperature sufficiently high, it would be reduced to the liquid state. We proceed now to consider some of the phenomena presented by bodies when passing from the solid to the liquid state, and conversely; and first of the former, as being in some respects the simpler.

* Ann. de Chimie et de Phys., 3^{me} Serie, tome xx. p. 467.

92. *Phenomena accompanying Liquefaction.* — When heat is applied to a solid body its temperature is raised, and continues to increase gradually and constantly, until a certain point is attained, when a change in its state commences; the temperature then ceases to rise, and remains unaltered until the change is completed.

The first point to be noticed in respect to the change from the solid to the liquid state is, that it always takes place at a fixed temperature for the same body; a temperature which, as has been stated, remains unaltered until the whole of the solid mass has been reduced to the fluid state.

M. Pouillet gives the following table of the point of fusion of various substances, estimated according to the centigrade scale.

TABLE of the Fusing Point of various Substances, in Centigrade Degrees.

Substance.	Fusing Point.	Substance.	Fusing Point.
English wrought iron,	1600°	1 atom lead, 4 tin, 5 bismuth,	118°.9
French soft iron, . .	1500	Sulphur,	114
Steel,	1400 to 1300	Iodine,	107
Cast metal,	1250 to 1050	2 lead, 3 tin, 5 bismuth,	100
Gold (pure),	1250	5 lead, 3 tin, 8 bismuth,	100
Gold (in coin), . . .	1180	4 bismuth, 1 lead, 1 tin,	94
Silver (pure),	1000	Sodium,	90
Bronze,	900	Potassium,	58
Antimony,	432	Phosphorus,	43
Zinc,	360	Stearic Acid,	70
Lead,	320	Wax (bleached), . .	68
Bismuth,	262	Wax (unbleached), .	61
Tin,	230	Margaric acid, . . .	55 to 60
Alloys,	Stearine,	49 to 43
5 atoms tin, 1 lead, .	194	Spermaceti,	49
4 tin, 1 lead,	189	Acetic acid,	45
3 tin, 1 lead,	186	Tallow,	33-33
2 tin, 1 lead,	196	Ice,	0.0
1 tin, 1 lead,	241	Oil of turpentine, . .	- 10
1 tin, 3 lead,	289	Mercury,	- 39
3 tin, 1 bismuth, . .	200		
2 tin, 1 bismuth, . .	167.7		
1 tin, 1 bismuth, . .	141.2		

M. Pouillet* states that he determined all the preceding fusing points above the temperature of a red heat, about 500° C., either by means of his air pyrometer, by the method of specific heats, or by the help of a magnetic pyrometer, described in the second volume of his "Elements de Physique."

The second point of importance in reference to the phenomenon of liquefaction is, that a considerable quantity of heat is absorbed by a body during this process, which produces no change in its temperature. In ordinary cases of fusion by heat this quantity is supplied by the source of heat, but if fusion is effected by chemical action, as in the mixture of salt and snow, or pounded ice, and other combinations which will be mentioned hereafter, this supply of heat is taken from the mixture itself, and from surrounding bodies, which are thus considerably reduced in temperature. The quantity of heat thus absorbed, as it produces no effect on the temperature of the melting body, which remains, as was mentioned before, at a constant temperature during the whole process of liquefaction, is called *latent* heat; and as it is necessary to the existence of the body in its liquefied state, it is called the *constituent* heat, or *caloric of liquidity* of the fluid.

The fuller consideration of the subject of latent heat is reserved for the second chapter of Book II.

93. *Phenomena accompanying Solidification.*—The return from the liquid to the solid state takes place at the same temperature as that at which the change from the solid to the liquid occurs. In some cases, indeed, liquids may be reduced below this point, and still retain their fluid state. Thus water, deprived of air by recent boiling, and slowly cooled, may be reduced to -6° C., without congelation; and if it is enclosed in a tube where it is exposed only to the pressure of highly rarefied air, and its surface is covered with a thin film of oil, it may even be reduced to -12° C.; the presence, however, under those circumstances, of the smallest fragment of ice, or the least *vibratory* movement, immediately determines its solidification, on which its temperature instantly rises to 0° . Water containing carbonic acid always

* Elements de Physique, vol. i. p. 299.

congeals at 0° ; the same is the case also with water which is at all turbid. It appears that the temperature to which it can be cooled below 0° , without congelation, is so much lower the smaller the diameter of the tube is in which it is contained. This may assist in explaining the fact, that the water contained in plants is not frozen except at very low temperatures.

As absorption of heat takes place during liquefaction, so a disengagement of it is effected on returning to the solid state. If it were not for this, the process of solidification would be instantaneous, and would take place the moment the fluid mass reached the temperature of liquefaction of the solid. But in consequence of this disengagement of heat, whenever a small portion of the superficial film of the liquid, which is probably below 0° , is solidified, the temperature of the surrounding particles is slightly raised, and must again be lowered before their solidification can take place.

As there are some solids which, as has been mentioned, have not been reduced to the liquid state, so, on the other hand, there are some liquids which no degree of cold yet attained has solidified. Among these are ether, alcohol, sulphuret of carbon, camphine, or rectified oil of turpentine, caoutchoucine, and several of the liquefied gases. These liquids, when submitted by Mr. Faraday* to a temperature of -166° F., still retained their liquid state, although alcohol, camphine, and caoutchoucine lost somewhat of their fluidity, and became thick like oil.

94. *Abrupt Change of Volume accompanying these Changes of State.*—Most fluids, on assuming the solid state, undergo an abrupt change of volume. In the case of mercury contraction takes place; in the case of water, cast iron, and bismuth, on the contrary, there is considerable expansion. The sudden contraction of mercury is strikingly exhibited by solidifying this metal in a mercurial thermometer. As the temperature is lowered, the mercury falls until it reaches -39° C., where it remains stationary for a considerable time, and then suddenly sinks to a point which would mark about -300° C. This phenomenon led early ex-

* Philosophical Transactions, 1845.

perimenters, who were not acquainted with its cause, to assign the temperature of -300° as the freezing point of mercury. Its cause, however, was ascertained, and the true freezing point of mercury fixed at -39° , by a series of experiments made by Mr. Hutchins at Hudson's Bay, in the winter of 1781, under the directions of Mr. Cavendish.*

The sudden *contraction* of mercury on assuming the solid state amounts to about one-twenty-third of its volume at its freezing point. The *expansion* of water, under the same circumstances, is much greater, being 0.07 of its volume at 0° . The force with which this expansion is effected is very considerable. Major Williams, during a severe winter at Quebec, filled a mortar with water, and closed its muzzle with a plug of wood driven home with a sledge-hammer, the temperature of the surrounding air being at -28° C. After awhile the water was frozen, and the consequent expansion took place with such force as to project the wood, with a loud explosion, to the distance of 400 feet. The effect of the expansion of water in freezing is also exhibited in severe winters, by the fracture of rocks in whose crevices water has lodged, and in the loosening of considerable masses from high cliffs, owing to the same cause. It is to this expansion also that the injurious effect of frost on plants is due, since, when the water contained in the capillary tubes of which they are composed is frozen, its expansion breaks up their envelopes, and entirely destroys their organization.

A reference to M. Despretz's Table in p. 63 will show the points of congelation of different saline solutions, and their temperature while this change is taking place.

SECT. II.—VAPORIZATION AND LIQUEFACTION.

95. *Definition and Measure of the Tension or elastic Force of Gases.*—The essential difference between the solid, liquid, and gaseous states consists in the physical connexion between the particles of bodies existing in those states. In solid bodies these

* Philosophical Transactions, vol. lxxiii. 1783.

particles are held together by a strong attractive force which powerfully resists their separation; as their temperature increases this force diminishes, very gradually indeed, and almost insensibly at first, but more rapidly as the point of fusion is approached, as is strikingly exemplified in the case of metals. In liquid bodies this attractive force almost entirely disappears, and the constituent particles of fluids possess a freedom of motion *inter se*, which is the more perfect the more nearly the state of the body approaches to that of perfect fluidity; while in gaseous bodies the particles appear endowed with a mutually repulsive force, which causes them to separate from one another, and tends to produce an almost indefinite enlargement of volume. In order to exist, therefore, in a limited space, a gas must be contained in a vessel enclosed in all directions, against the sides of which the repulsive action of its particles causes it to exert a pressure, which is called the *tension* or *elastic force* of the gas. The amount of this pressure on any given portion, as for instance on the unit of surface, may be represented by the weight which would retain that portion of the enclosing vessel in its place if it were detached from the remainder; or the *elastic force* of a gas, without any reference to the extent of surface on which it acts, may be expressed by the height of a mercurial column, at a given temperature, which it would support, as this height is independent of its section, or of the surface on which the gas may exert its pressure. To illustrate these methods of expressing the elastic force or tension of a gas, imagine a hollow cylinder, the area of whose section equals a square inch, closed above and below, and furnished with a moveable diaphragm or piston. Below this piston suppose the space in the cylinder occupied by a gas, and above it imagine a perfect vacuum. If the diaphragm were devoid of weight the repulsive force of the particles of the gas would drive the piston to the upper part of the cylinder, and the gas would occupy it entirely; but if the piston were either itself sufficiently heavy, or were loaded to a sufficient amount, the gas would be retained in a limited space, and its elastic force, under any given circumstances of temperature and density, might be expressed by the weight which it supported on a square inch of surface, in the

present case, the weight of the piston and its load. This weight, under the circumstances supposed, might be of any amount, however small; but if the upper cover of the cylinder were removed, and the surface of the piston exposed to the atmosphere, it could not be less than the weight of the piston + that of the atmosphere on its surface.

To understand the second method of expressing the elastic force of a gas, conceive a U-shaped barometer tube, closed at both ends, and containing a quantity of mercury in the lower or curved part. Imagine the space over the mercury in one arm a vacuum, and that in the other filled with a gas. The latter will exert a pressure on the surface of the mercury in contact with it, and will accordingly depress it, and raise the column in the other arm. The difference between the heights of the two columns will measure the elastic force of the gas, and the amount of its pressure on any surface will be equal to the weight of a column of mercury whose height equals this difference, and the area of whose section equals that of the given surface.

96. *Division of Gases into permanent Gases and Vapours.*—Gases are divided into the two classes of *permanent gases* and *vapours*. The former were originally so called under the impression that they existed permanently in the gaseous state, and could not be possibly reduced to the liquid form; while those which could be so reduced, and could be reconverted to the state of gas, were called vapours. Sir H. Davy and Mr. Faraday,* however, have shown that by the conjoined effects of great pressure and of a high degree of cold most of the permanent gases may be liquefied. The pressure was produced either, as Sir H. Davy suggested, by generating the gas in large quantities in a limited space, and thus making its own tension produce the requisite pressure, or by means of a powerful condensing pump. The cold was produced in Mr. Faraday's later experiments by means of a mixture of solid carbonic acid and ether. For the purpose of estimating the degree of cold produced by this means, Mr. Faraday employed a spirit thermometer, whose scale he graduated below 0° F., on the supposition of a uniform rate of con-

* Philosophical Transactions, 1823, pp. 160, 189; 1845, p. 155.

traction with the mercurial thermometer. He thus estimated the temperature of this mixture at -106° F. in air; on placing it, however, under the receiver of an air pump, and drawing off the air and vapour of the carbonic acid, the temperature fell to -166° F., and at this point the vapour had only a tension equal to 1.2 inch of mercury, and accordingly the mixture of acid and ether was not more volatile than water at 86° F., or alcohol at the ordinary temperature.

By these means Mr. Faraday succeeded in liquefying all the permanent gases except those undermentioned, which at the subjoined temperatures and pressures still retained the gaseous state.

Hydrogen	at -166° F. and 27 atmospheres.
Oxygen,	„ -166 „ 27 „
Do.	„ -140 „ 58.5 „
Nitrogen,	„ -166 „ 50 „
Nitric oxide,	„ -166 „ 50 „
Carbonic oxide,	„ -166 „ 40 „
Coal gas,	„ -166 „ 32 „

Several of the liquefied gases are further capable of being reduced to the solid state. Thus, hydriodic acid passes from the solid to the liquid state at -60° F.; hydrobromic at -124° ; sulphurous acid at -105° ; sulphuretted hydrogen at -122° ; carbonic acid at -148° , according to M. Thilorier,* who first effected its solidification; at from -72° to -70° , according to Mr. Faraday, at which temperature, whether in the solid or liquid state, it exerts a pressure of 5.33 atmospheres; oxide of chlorine at -75° ; protoxide of azote at -150° ; cyanogen at -30° ; and ammonia at -103° ; but olefiant gas, fluosilicic acid, phosphuretted hydrogen, fluoboric acid, muriatic acid, and arseniuretted hydrogen, remain fluid at -166° F.

The difference, then, between the permanent gases and vapours, is merely one of degree, and depends upon the temperature at which the change from the fluid to the gaseous state occurs. Those which exist in the fluid state under ordinary temperatures and pressures are called vapours, while those which require

* *Annales de Chimie et de Physique*, tome ix. (1835).

strong pressure and high degrees of cold to reduce them to the liquid form are called permanent gases.

97. *Relation between elastic Force and Density; Boyle's and Mariotte's Law.*—Towards the close of the seventeenth century it was announced by the Hon. Mr. Boyle,* that the *spring* of compressed air is directly proportional to the amount of its compression; in other words, that its elastic force is directly proportional to its density, or inversely to the volume occupied by the same mass at the same temperature. Thus if air occupying a volume V_0 sustains a pressure P_0 , twice this pressure will compress it into one-half of the original volume, three times the pressure into one-third of the volume, and so on; and conversely, the same mass of air at the same temperature, occupying one-half, one-third, &c., of its original volume, will sustain twice, thrice, &c., its original pressure. Mr. Boyle performed his experiments with a U-shaped glass tube, whose arms were of unequal length. In the shorter arm, which was closed at the extremity, was contained the air on which he was operating; the longer arm, which was open at the end, contained the column of mercury that produced the pressure and measured the elastic force. As Mr. Boyle neglected to dry the air contained in the tube, his experiments, which he pushed as far as a pressure equal to four atmospheres, showed only a very near approximation to the law as stated above, and which only applies to perfectly dry air.

About the same time, or shortly after, M. Mariotte,† a celebrated French physicist, arrived independently at the same result; and accordingly this law of the relation between the elastic force and density of air is called indifferently Boyle's or Mariotte's law.

It has been mentioned that the results of Mr. Boyle's experiments did not *strictly* establish his law of pressures, but only proved a near approximation to it. As the subject is one of the

* A Continuation of new Experiments, physico-mechanical, touching the Spring and Weight of the Air, &c. Oxford, 1668. Philos. Transact., vol. iii. p. 845 (1668).

† Histoire de l'Academie Royale des Sciences, vol. i. p. 271 (under the year 1679).

greatest importance in physics, it has been, accordingly, frequently submitted to the test of experiment, and the result has been a gradually increasing conviction of the accuracy of the law, as applied to *dry* air, and at a *constant temperature*.

This conviction acquired additional strength from the experiments made in 1830 by MM. Dulong and Arago, as members of a Commission appointed by the Academy of Sciences in Paris, to investigate the law of elastic forces of aqueous vapour at high temperatures. In the course of those experiments it was necessary to construct a *manometer*, an instrument which serves to measure the elastic force of a gas or vapour by the amount of compression which it produces in a given mass of air; and in graduating this manometer, which was done by means of a column of mercury, as in Mr. Boyle's experiment, the Commissioners arrived at the result, that his law is *strictly* true up to a pressure of twenty-seven atmospheres, and probably to a considerably higher limit.

For the purpose of ascertaining whether this law extends to other permanent gases, M. Pouillet* has compared them with air in this respect, by means of a very simple and ingenious apparatus, which enabled him to extend his comparison as far as a pressure of 100 atmospheres. The following are his results.

(1). As far as 100 atmospheres, oxygen, nitrogen, hydrogen, nitric oxide, and carbonic oxide, follow the same law of compression as atmospheric air.

(2). Sulphurous gas, ammoniac gas, carbonic acid, and protoxide of azote commence to be sensibly more compressible than air, when they have been reduced to one-third or one-fourth of their original volume; and there is little reason to doubt that this difference exists for even smaller changes of volume.

(3). Protocarburetted hydrogen and bicarburetted hydrogen, which do not liquefy at 8° or 10° C. under a pressure of 100 atmospheres, are yet sensibly more compressible than air.

As a proof of the variations which the compressibility undergoes, M. Pouillet gives the following Table.

* Elements de Physique, vol. i. p. 328.

TABLE of the Compressibility of the undermentioned Gases, as referred to Air.

Pressures.	Theoretic Volumes.	Carbonic Acid.	Protoxide of Azote.	Protocarburetted Hydrogen.	Bicarburetted Hydrogen.
1 atmos.	1.000	1.	1.	1.	1.
2	0.500	1.	0.996	0.998	0.994
4	0.250	1.	0.988	0.995	0.989
5	0.200	0.989	0.983	0.992	0.986
6.67	0.150	0.980	0.971	0.989	0.983
10	0.100	0.965	0.956	0.981	0.972
15.38	0.065	0.934	0.923	0.949	0.962
20	0.050	0.919	0.896	0.956	0.955
25	0.040	0.880	0.849	0.951	0.948
33.3	0.030	0.808	0.787	0.951	0.931
40	0.025	0.739	0.732	0.940	0.919
50	0.020	0.907	0.899
83	0.	0.850

The numbers in the last four columns were obtained by dividing the volume of the gas, at the corresponding pressure, by the volume which air would have had at the same pressure. We see that these quotients form a decreasing series in the case of each gas, and that the law of decrease is tolerably regular.

The carbonic acid liquefied at 45 atmospheres, the temperature being 10° C.; protoxide of azote at 43 atmospheres and 11° . The liquid appeared perfectly transparent. At 10° ammoniacal gas liquefied under a pressure of 5 atmospheres. The liquid had a greenish-yellow tint.

At 8° sulphurous gas liquefied under $2\frac{1}{2}$ atmospheres.

In all the cases of liquefaction which M. Pouillet observed, he found that it was always possible to augment considerably the pressure, without reducing the whole of the gas to the liquid state; and still he considered it as certain that neither atmospheric air, nor any of the more permanent gases, was mixed with that under examination. It may be remarked, in connexion with this subject, that in consequence of some irregularities which Mr. Faraday observed in the tension of the vapour of liquefied

olefiant gas and protoxide of azote, and also in the pressure to which the liquefaction of phosphuretted hydrogen appears due, that distinguished chemist was led to suspect that those gases were either not perfectly pure, after all the care which was taken in their preparation for his experiments, or that they are, in reality, compound bodies, consisting of different substances soluble in each other.

More recently, M. Regnault has investigated directly the accuracy of Boyle's law as applied to some of the permanent gases. He has observed* that in all previous methods it was the *same* mass of air or other gas which had been operated on, and which, under constantly increasing pressures, was compressed into volumes constantly diminishing, until they became so small that any deviation from the law in question was imperceptible. To avoid the errors arising from this cause, M. Regnault operated on different masses of gas at different pressures contained, in a glass tube of about three metres in length, and from 8^{mm} to 10^{mm} internal diameter. The gas, at the commencement of each experiment, occupied *q. p.* the same volume V_0 under different pressures P_0 ; the pressure was then increased until it attained an amount P_1 , which reduced the original volume about one-half. The volumes V_0 , V_1 , and corresponding pressures P_0 , P_1 , which were produced, as in the former experiments, by columns of mercury of different heights, were carefully ascertained, and on making all necessary corrections, for an account of which the student is referred to the original memoir, M. Regnault arrived at the result that Boyle's and Mariotte's law is not *strictly* true for any gas.

He found that the ratio $\frac{V_0}{V_1} \div \frac{P_1}{P_0}$, which, according to this law, should always be *equal* to unity, is *less* for air, nitrogen, and carbonic acid, and *greater* for hydrogen, and that the difference $\frac{V_0}{V_1} \div \frac{P_1}{P_0} - 1$ goes on increasing regularly with the pressure.

Thus when air occupying a volume equal to 1, under a pressure equivalent to 9336^{mm}.41 of mercury, was compressed into a

* Memoires de l'Institut, tome xxi. p. 369 (1847).

volume equal to 0.50009, its elastic force was represented by 18551^{mm}.09, and consequently the ratio $\frac{V_0}{V_1} \div \frac{P_1}{P_0}$ equalled 1.006366. M. Regnault remarks, that the elastic force of air, under a pressure of about twenty-five atmospheres, is a little more than one-seventh of an atmosphere less than it would be were Boyle's law strictly true.

The deviation is considerably greater in the case of carbonic acid. Thus when this gas, under a pressure of 6820^{mm}.22, was compressed into a volume bearing the ratio 1 : 3.5 to that originally occupied by it, its pressure was represented by 20284^{mm}.08, and the ratio $\frac{V_0}{V_1} \div \frac{P_1}{P_0} = 1.177293$.

Hydrogen, it has been mentioned, also deviates from Boyle's law, but in an opposite direction to air and the other gases examined. "While air and the other gases are more compressed than they ought to be, according to this law, hydrogen suffers a less compression, and its compressibility diminishes as the pressure increases. The elastic force of hydrogen, then, is analogous to that of a metallic spring which offers a resistance to compression, increasing with the force applied."*

In confirmation of these results, M. Regnault mentions the fact, that the value of the coefficients of expansion of air, nitrogen, and carbonic acid, as determined by the *direct method of dilatation* (54), is greater than that derived from the *method of elastic forces* (53), while the contrary is the case with hydrogen.

Temperature has a considerable influence on these results. Thus M. Regnault found that at 100° the compressibility of air differed much less from the ordinary law than at common temperatures.

Finally, M. Regnault is of opinion that Boyle's and Mariotte's law may be considered a "limit law," which is not rigorously true except when gases are infinitely dilated, and from which they deviate more and more according as they are in a state of greater condensation.

We now proceed to consider the phenomena attending the

* Memoires de l'Institut, tome xxi. p. 373.

formation and liquefaction of that class of gases specifically termed vapours.

98. *Vaporization by Ebullition.*—The transition from the liquid state to that of vapour, termed in general *vaporization*, takes place in two ways, which are respectively called *ebullition* and *evaporation*. In the former the vapour is formed through the whole of the liquid mass, in elastic bubbles, whence its name is derived; in the latter it is emitted merely from its surface. The former takes place at a certain temperature, determined by circumstances to which we will presently refer; the latter, in most liquids, at all temperatures. We will first consider the process of vaporization by ebullition.

If we heat water contained in an open vessel, and in free air, we see that, after it has attained a certain temperature, bubbles of vapour form on the bottom and sides of the vessel, are subsequently detached from them, and rise through the liquid mass until they reach the surface, where they mix with the surrounding medium.

The temperature at which water thus enters into a state of ebullition depends (I.) on the pressure to which it is submitted; (II.) on the presence or absence of foreign bodies, whether (1) in solution, or (2) in contact. We will treat of these several conditions separately.

99. I. *Influence of Pressure on Temperature of Ebullition.*—It has been observed that at the tops of high mountains, and on other elevated situations, where the atmospheric pressure is considerably diminished, water boils at temperatures continually decreasing as the altitude increases. Thus at Quito, standing at an elevation of about 9540 feet, where the barometer has a mean height of 20.7 inches, water boils at $90^{\circ}.1$ C., or $194^{\circ}.18$ F. At the Hospice of St. Gothard, at a height of 6800 feet, and under a mean pressure of 23 inches, the boiling point is $92^{\circ}.9$ C., or $199^{\circ}.22$ F. Water boiling in free air, then, is not equally hot at all places on the earth's surface, and hence is not everywhere equally effective for the cooking of food, and other domestic purposes.

The effect of diminished pressure in reducing the boiling point of water is exhibited in a very simple and striking manner by boiling a small quantity of this fluid in a Florence flask over a spirit

lamp. If, when all the air has been expelled from the upper part of the flask, and replaced by steam, we close its aperture tightly with a cork, and remove it from the lamp, the ebullition is renewed, and is rendered still more violent by pouring some water on the outside of the flask, which produces a partial condensation of the steam within, and a consequent diminution of its pressure. And the water may by this means be kept in a state of ebullition at temperatures considerably below 100 C. The same effect is produced by placing water which has just ceased boiling under the receiver of an air-pump, and gradually diminishing the pressure on its surface.

The relation between the pressure and the boiling point is, however, more accurately exhibited by means of the following apparatus. In a retort, *A* (fig. 60), a quantity of water is heated, whose temperature is indicated by the thermometer, *t*; attached to the neck of this retort is the tube, *TT'*, whose other end opens into the globe, *B*, which is connected with an air-pump by means of the tube, *tt*, furnished with a stop-cock, *r*. This globe is also connected with a mercurial gauge, for the purpose of determining the pressure within it. Round the tube, *TT'*, is a cylinder containing cold water, which flows in a constant stream through the pipe, *E*, into a receiver, *M*. The effect of this current is continually to condense the vapour rising from *A*, and thus prevent its accumulating and increasing the pressure on the surface of the water. The manner of operating with this apparatus is first to produce a determined pressure in *B*, and then to raise the temperature of the retort until the water enters into a state of ebullition, when its boiling point, corresponding to this pressure, is indicated by the thermometer *t*.

When vapour forms in the body of a liquid mass, it is plain that its elastic force is equal to the pressure on the surface of the liquid, increased by the pressure of the stratum of liquid over the point where the vapour is formed. According as the bubble approaches the surface this pressure diminishes, and ultimately becomes equal to that of the external medium, and hence the apparatus above described serves also for the purpose of determining the elastic force of vapours produced from water at diffe-

rent temperatures. We will return to this subject in the last section of this chapter, where we will also notice the application which has been made of the connexion between the boiling point of water and its pressure to the measurement of mountains.

As the boiling point of water depends on the pressure to which it is exposed, it follows that in deep vessels, when the whole of the liquid which they contain is in a state of ebullition, the lower strata of water must have a higher temperature than those nearer the surface. It follows, also, that if we increase by any means the pressure on the surface of water, its boiling point will be at the same time raised, and thus its temperature, which in free air can never much exceed 100° C., may be raised to a considerably higher limit. This effect may be produced by heating it in a strong steam-tight vessel, of moderate dimensions, as the constantly accumulating vapour collected in the upper part of such a vessel will exert a rapidly increasing pressure upon the surface of the water, and thus, as we have seen, retard its ebullition.

A vessel constructed on this principle, and called Papin's digester, is sometimes used for raising water to temperatures above the ordinary boiling point. It was invented by the person whose name it bears, a distinguished physicist of the seventeenth century, and is simply a strong metallic boiler, with a steam-tight lid, and furnished with a safety valve, which may be loaded to any required pressure short of that which would risk the rupture of the vessel. Such an apparatus is necessary for ordinary culinary operations at high levels, where, as we have mentioned, the temperature at which water boils in free air is insufficient for such purposes, and is besides useful for exhibiting generally the solvent power of water at high temperatures.

If water is boiled in a vessel furnished merely with a small aperture for the escape of the steam, the pressure, and consequently the boiling point, will depend on the area of this aperture as compared with the quantity of steam generated in a given time, which, as experience shows, varies as the area of the heating surface. M. Pouillet gives the following table of the approximate value of the boiling point in connexion with this ratio.

Temperature of Water in Boiler.	Ratio of Orifice to Heating Surface.
100° C.	1 ÷ 1000 and upwards.
105	1 ÷ 5000
115	1 ÷ 10000
138	1 ÷ 20000

100. *Phenomena presented by Liquids raised to very high Temperatures in a limited Space.*—On continuing to raise the temperature of a liquid contained in a closed space of limited capacity, the density of the vapour which is formed, and consequently its elastic force, increases very rapidly, and retards, as we have remarked, the point at which it enters into a state of ebullition. In the case of some liquids enclosed in a space but little larger than their own volume, M. le Baron Cagniard de la Tour* observed that, on being raised to a certain temperature, they passed instantaneously and totally into the state of vapour. The apparatus he made use of consisted of a short siphon of strong glass tube, both ends of which were closed, and the curved part filled with mercury; above the mercury in one arm was contained the liquid on which he operated, and in the other atmospheric air, whose compression served to measure the elastic force of the vapour furnished by the liquid. The temperature of the liquid was raised either by a bath of fixed oil or a blow-pipe. By means of this apparatus M. Cagniard de la Tour observed, that at a temperature of 150° C. ether passed completely into the state of vapour, occupying a space somewhat less than twice the original volume of the liquid, and having a pressure of thirty-seven atmospheres; similarly, at a temperature of 207°, alcohol underwent the same change, and its vapour, occupying less than three times the original volume of the liquid, had an elastic force equal to 119 atmospheres. Sulphuret of carbon also at 220° passed into the state of vapour, having a force equal to seventy-eight atmospheres. He succeeded in producing the same result in the case of rectified essence of petroleum, but failed in that of water. At the high

* *Annales de Chimie et de Physique*, tome xxi. pp. 127, 178 (1822); xxii. p. 410; xxiii. p. 267 (1823).

temperatures necessary for the experiment, this latter liquid acquired so powerful a solvent property as to attack the alkali of the glass, which communicated to it a tint of greenish hue, that deepened as the temperature increased, until on one occasion it became nearly black. On being cooled again the water recovered its original transparency. In all the cases in which the heat was pushed sufficiently far, the tubes exploded before the water was completely reduced to vapour, partly owing, no doubt, to the disintegration of the glass, as well as to the enormous elastic force of the aqueous vapour. The experiment is, in all cases, one of considerable risk, and requires to be conducted with all the precautions which are adopted where similar danger is apprehended.

Connecting this phenomenon with the liquefaction of gases, Mr. Faraday* makes the following ingenious observations:

“M. Cagniard de la Tour has shown that at a certain temperature a liquid, under sufficient pressure, becomes clear, transparent vapour or gas, having the same bulk as the liquid. At this temperature, or one a little higher, it is not likely that any increase of pressure, except, perhaps, one exceedingly great, would convert the gas into a liquid. Now the temperature of 166° below 0° , low as it is, is probably *above* this point of temperature for hydrogen, and perhaps for nitrogen and oxygen; and then no compression, without the conjoint application of a degree of cold below that we have as yet obtained, can be expected to take from them their gaseous state. Further, as ether assumes this state before the pressure of its vapour has acquired thirty-eight atmospheres, it is more than probable that gases which can resist the pressure of from twenty-seven to fifty atmospheres, at a temperature of 166° below 0° , could never appear as liquids, or be made to lose their gaseous state, at common temperatures. They may, probably, be brought into the state of very condensed gases, but not liquefied.”

101. II. *Influence of foreign Bodies, whether in Solution or in Contact, on the Temperature of Ebullition; Theory of M. Magnus.*—This subject has been examined with particular care by M. Mag-

* Philosophical Transactions, 1845, p. 171.

nus* and M. Donny,† and we purpose to lay before the student the views of those writers on the process of ebullition in this and the following paragraph.

M. Magnus, in the course of his experiments on the elastic force of vapours, remarked, what had been observed by previous physicists, that water which has been well boiled does not generally pass into the form of steam in glass vessels until it has acquired a temperature considerably above that due to the force of its vapour, and that the formation of steam then takes place suddenly and with great violence. From this it follows, that the force requisite for the disengagement of the steam is greater than its expansive force subsequently, and the difference of these forces M. Magnus refers to the attraction of cohesion existing between the particles of the liquid, which requires to be overcome at the moment of formation of the steam, in addition to those pressures which the vapour itself subsequently sustains.

Hence he concluded, that whatever would tend to affect the cohesive force of those particles, either generally throughout the mass, or at certain points, would affect the temperature of ebullition.

(i.) On this principle, he remarks, we can understand why salts in solution *raise* the boiling point of water. For the cohesion between water and salt being stronger than that between the particles of water among one another, a stronger force, and consequently a higher temperature, is requisite to overcome the cohesion in the case of solutions than of pure water. Moreover, the presence of a salt lowers the elastic force of vapour in contact with water, as is proved by introducing a salt—for instance, chloride of sodium—into the water contained in the vacuum of a barometer, when the expansive force of the vapour is immediately diminished. And therefore the temperature of a saline solution must be higher than that of pure water, to maintain vapour of equal elastic force.

(ii.) We can also understand how the presence of bodies held suspended in a liquid, or the sides of the containing vessel, if these

* Poggendorff's *Annalen*, 1844, No. 2 ;
Taylor's *Scientific Memoirs*, vol. iv. p. 235.

† *Ann. de Chim. et de Phys.*, tome xvi.
p. 167 (1846).

have a less attraction for the particles of the fluid than the latter have for one another, will *lower* the boiling point. And this will always be the case if physical contact does not exist between such bodies and the liquid, that is, if they are not completely *moistened* by it. Accordingly, sawdust or insoluble powders, diffused through the fluid mass, and the sides of a metallic vessel, which, as is well known, are never completely moistened at all points by water, lower the temperature of ebullition to that of the vapour. But if the water is boiled in a glass vessel, especially if the sides of the latter are perfectly cleaned by heating sulphuric acid in it up to 150° C., and then rinsing it with distilled water, by which means the contact is rendered more perfect, and the cohesive force of the glass on the particles of water stronger, the boiling point will rise to 105° or 106° . Now, as M. Magnus remarks, the action of the sides of the vessel, and of solid bodies in general, may diminish but cannot raise the temperature of ebullition; for if the force of attraction of such bodies for the fluid molecules were stronger than that of the latter for one another, the only effect would be that the ebullition would commence at the centre of the liquid, and not at the sides. Accordingly, no liquid can assume a higher temperature than that at which the expansive force of the vapour suffices to overcome the pressure and cohesion of the liquid. And the *highest* temperature of ebullition observed in the case of a pure liquid under a given pressure, is its *true* boiling point at that pressure, and is the same as that which it would indicate if it could be boiled in vessels formed, as it were, of the same liquid, or in vessels the sides of which would retain it everywhere with the same force as its particles attract one another.

The *true* boiling point of water, therefore, according to M. Magnus, is about 105° C.; and as its vapours at 100° have an elastic force equal to the atmospheric pressure, it follows that the difference between this force and that due to vapours at 105° , or about one-fifth of an atmosphere, is the measure of the cohesive attraction of the particles of water.

102. *Theory of M. Donny.*—Such are the views of M. Magnus on this subject, and the facts from which he derives them. M. Donny, however, has been led, by his investigation into the

force of cohesion of liquids, to conclude that the boiling point of water, as defined above, that is, the temperature at which water, perfectly free from all foreign bodies, would pass into the state of vapour throughout its mass, is considerably higher than even the highest limit assigned by M. Magnus. Having observed that a column of sulphuric acid, well freed from air, of 1^m.255 in height, remained suspended in the closed branch of a manometer, which he had adapted to an improved air-pump of his own construction, when the pressure on the surface in the open arm did not exceed that due to 5^{mm} of the acid, M. Donny was led to the conclusion that the cohesion of the particles of liquids to one another, and to solid bodies, must be much greater than had been previously suspected; and that the apparent feebleness of this power, when measured by direct experiments, was owing to the extreme mobility of the particles of fluids and the presence of gaseous bodies diffused through their mass. He was accordingly led to examine the boiling point of water when deprived, as far as possible, of air; and found that, when enclosed in a tube of peculiar form, and subjected, through the whole of the experiment, to a very feeble pressure on its surface, it might be raised to temperatures of 113°, 121°, 128°, and 131° C., without entering into a state of ebullition. At last, on plunging it in a bath of chloride of calcium, which rose, in about two minutes and a half, from 132° to 138°, when the portion of the water submitted to the action of the heat had acquired a temperature of about 135°, it was instantaneously converted into vapour, and projected with considerable force the remainder of the column into the balls terminating the tube. From this M. Donny concludes, that the mutual force of cohesion of the particles of water is equal to a pressure of about three atmospheres, and in this strong cohesive force finds an explanation of the phenomenon called "soubresaut," or jumping motion, sometimes observed in liquids when in a state of ebullition, as well as, probably, of those fatal explosions which occur so frequently in steam boilers, and whose origin continues to perplex engineers and physicists. For, says M. Donny, "by the effect of boiling, liquids lose the greater part of the air which they hold in solution, consequently the molecular attraction commences to manifest itself in a sensible

manner, and permits the liquid to attain a temperature considerably above its normal boiling point; this elevation of temperature determines the appearance of new bubbles of air, the liquid then separates abruptly with a "soubresaut," a large quantity of vapour forms, and consequently a reduction of temperature ensues, which restores a momentary calm to the liquid. Presently the same causes reproduce the same effects, and the phenomenon is renewed with increased violence;" and, as M. Donny shows, may eventually result in a violent explosion, when occurring under circumstances analogous to those of a steam boiler.*

From these facts, proving the strong cohesive attraction of liquid particles, joined to the well-known tendency of all fluids to assume the vaporous state at all temperatures, M. Donny concludes that the superficial stratum of liquids possesses a *peculiar* property in this respect, and has hence been led to form the following theory of ebullition. "The elevation of temperature of a liquid produces the formation of small bubbles of air in the hottest portions of its mass, and consequently on the side of the containing vessel nearest the source of heat; each of these bubbles presents to the liquid molecules which surround it a surface which facilitates the vaporization of those molecules; and when the tension of the vapour becomes sufficient to counterbalance the pressure to which those bubbles are exposed, there is no further resistance to the development of the vapour, which then forms currents that traverse the liquid, and produce the phenomenon of ebullition.

"I think, then," continues M. Donny, "we are justified in concluding that ebullition is nothing but a kind of rapid evaporation, which takes place at those internal surfaces of a liquid that bound the bubbles of aeriform fluid which are formed through its mass."

We will conclude these remarks on the phenomenon of ebullition with the observation of M. Magnus, that "there does not exist an older physical experiment, nor one more frequently re-

* These "soubresauts," which are more violent in the case of fluids boiled in glass vessels, and especially in the case of sul-

phuric acid, may to a considerable extent be prevented by throwing scraps of metal, platina, &c., into the boiling liquid.

peated, than that of boiling water; but nevertheless what occurs in the process was not sufficiently known, and even now much still remains unexplained."

103. *Vaporization by Evaporation.*—If a small quantity of water or other liquid, volatile at ordinary temperatures, is placed in a shallow vessel in free *air*, it is found to pass gradually into the state of vapour, and disappear completely after a space of time depending on the quantity of liquid, on the extent of surface exposed to the air, on the amount of vapour of the same liquid existing in the surrounding medium, and on the common temperature. If the liquid is *in vacuo*, and the vapour is removed as fast as it is produced, the result is the same, but the rate of evaporation is infinitely more rapid.

The connexion between these several conditions will be better understood after we have investigated the laws of the elastic force and density of vapours in relation to their temperature, which we purpose to do in the following section.

104. *Heat absorbed in the Formation of Vapour.*—We have observed that in the transition from the solid to the fluid state there is an absorption of a considerable quantity of heat, forming the constituent or latent heat of the fluid; and a similar phenomenon manifests itself, in a still more striking manner, in the passage from the fluid to the vaporous state. If this change is effected by the way of *ebullition*, the latent heat of the vapour is supplied by the fire or other source of heat producing the ebullition, all the heat from which is expended in the formation of vapour, and the temperature of the boiling liquid consequently remains unaltered. But if the vaporization is effected by *evaporation*, the heat requisite for the formation of the vapour is taken from the liquid itself and from surrounding bodies; and if the rate of evaporation is very rapid, owing to the extent of surface, and the previous absence of vapour in the surrounding medium, and if, at the same time, the amount of heat radiated from neighbouring bodies is very small, the temperature of the liquid is rapidly and considerably lowered. Thus in clear summer nights in India, where the air is remarkably free from aqueous vapours, and the absence of clouds prevents the return by reflexion of the heat radiated from the earth, water

in shallow vessels is frequently frozen. And in the same way if water is placed under the exhausted receiver of an air-pump, in a shallow vessel, over a dish of strong sulphuric acid, which absorbs and condenses the aqueous vapours as fast as they are formed, the same result is obtained.

We will return to this subject in the second chapter of Book II.

105. *Leidenfrost's Phenomenon*.—Before proceeding to the investigation of the laws of vapours, we must notice a singular phenomenon connected with the vaporization of liquids projected on the surface of solid bodies raised to a very high temperature. Although this phenomenon must have been frequently noticed previously, yet it was examined scientifically for the first time by Leidenfrost, who published his observations in 1756, in a thesis entitled "*De Aquæ communis nonnullis Qualitatibus*," and is hence generally called Leidenfrost's phenomenon. It is this. If a drop of water or other liquid is thrown upon a surface raised to a very high temperature, the liquid does not moisten or diffuse itself over the surface, but forms a flattened ellipsoidal mass, which, if the drop is sufficiently small, assumes the spheroidal form, revolves rapidly round a shifting axis, and, the heat of the surface being kept up, evaporates with extraordinary slowness, without ever entering into a state of ebullition.

This phenomenon, which appears so completely opposed to all the known facts of the communication of heat, as to merit the appellation of the *caloric paradox*, has been examined by several physicists.* We will briefly mention the principal facts connected with it, as elicited by their researches.

There are two peculiarities connected with this phenomenon; the *first* is the fact that the liquid does not moisten the hot surface, but assumes, when in sufficiently small masses, the spheroidal form; the *second* is, the absence of equality of temperature be-

* De Saussure, *Journeys in the Alps*, vol. iii.; Klaproth, *Annales de Chimie et de Physique*, tome xxxv. p. 325 (1827); Fischer, *Poggendorff's Annalen* (1830), No. 8; Lechevalier, *Journal de Pharmacie*, tome xvi.; Buff, *Poggendorff's Annalen*

(1832), No. 8; Baudrimont, *Ann. de Chim. et de Phys.*, tome lxi. p. 319 (1836); Laurent, *Ibid.*, tome lxii. p. 327 (1836); Boutigny, *Ibid.*, tome ix. p. 350 (1843), tome xi. p. 16 (1844); Boutan, *Mem. Acad. de Rouen* (1848), p. 48.

tween the liquid drop and the hot plate. Buff appears to have been the first to point out the connexion between these two conditions. He observes that if a clean silver spoonful of water is placed over a lamp, it may be held without inconvenience while the water is boiling, until the last drop is vaporized, the heat communicated to the metal being in this case transferred to the fluid, and absorbed in the latent state by its vapour. But if the inside of the spoon is covered with a coating of lampblack, which does not admit of being moistened by water, the spoon, in this case, becomes intolerably hot before the water boils. Nor is this owing to the non-conducting nature of the lampblack, for if another carbonaceous coating, which admits of being moistened by water, is substituted for it, this effect is not produced. It follows, therefore, that the non-moistening of the surface, and consequent assumption of the spheroidal form by small masses, is intimately connected with the non-establishment of thermic equilibrium. We will notice the facts connected with these two conditions separately.

106. I. *Assumption of spheroidal Form by small Masses of Liquids projected on hot Surfaces.*

(i.) M. Boutigny is of opinion that all liquids are capable of assuming this state, and thinks it probable that the volumes of their spheroids are inversely proportional to their specific gravities, so that the masses of different liquids capable of assuming this state are equal. He ascertained also that the velocity with which the liquid is projected does not affect the result, for, on throwing a quantity of distilled water from the cupola of the Pantheon of Paris on a platina dish, seven inches in diameter, maintained at a red heat, on the floor of the building, the total height being 230 feet, the water in falling broke up into drops of different sizes, which consequently reached the ground with very different velocities, but all which struck the platina vessel passed immediately into the spheroidal state.

(ii.) The same physicist ascertained that the lowest temperature of the surface capable of producing this state varies with the volatility of the liquid, and is higher the higher the boiling point of the latter. For water he determined the requisite temperature of the plate to be 171° C., for alcohol 134° , and for ether 61° .

M. Marchand* has shown that it also varies with the temperature of the projected liquid and the nature of the surface. Thus for water at 100° C. the requisite temperature of platina is 120° , and of glass 180° ; for water at 0° those temperatures are 400° and 800° respectively.

(iii.) The spheroid *apparently* does not touch the plate. If the surface is plane, the eye placed on a level with it can see the light of a taper between the drop and the plate. Moreover, M. Boutigny has shown that a drop of nitric acid in the spheroidal state has no action on a silver or copper surface, nor has one of dilute sulphuric acid any action on iron or zinc, while if a cold wire of those metals is introduced into the drop it is immediately and powerfully attacked. Further, the same writer mentions, that if a large drop of water, sufficient to form a very flattened ellipsoid, is projected on a silver capsule nearly plane, and if an iron cylinder of about .4 inch diameter is raised to a white heat, and introduced into the drop, the latter assumes a form which, "à tort ou à raison," he compares to Saturn's ring.

(iv.) If sand, iron filings, pounded glass, &c., are mixed with the fluid forming the drop, they do not sink to the plate, as might have been expected from their greater specific gravity, but are held suspended by the spheroid, and according as the fluid evaporates these particles approach more and more, until they are finally deposited on the surface in the form of a hemisphere or a cup, or a disk pierced or not with a central aperture.

(v.) The most probable explanation of the non-moistening of the surface appears to be, that it results from a change in the direction of the resultant of the capillary forces, owing to the increased distance of the molecules of the solid body from one another, in consequence of the action of heat. Some physicists are of opinion that the detachment of the drop from the plate may be accounted for by supposing it to rest on a cushion of its own vapour; this, however, appears extremely improbable, in consequence of the low density of the latter. It appears more probable that the drop is in a state of perpetual and rapid oscillation to and from the surface; and M. Boutan suggests that this

* Berzelius, *Rapport* (1843), p. 17.

vibratory movement may be owing to the expansion and contraction of solids arising from change of temperature not taking place uniformly, but by a rapid succession of intermittent motions, and that the musical tones produced in the experiment noticed by Mr. Trevelyan result from the same cause.

107. II. *Non-Establishment of thermic Equilibrium in spheroidal State.*

(i.) In order to ascertain the temperature of the liquid in the spheroidal state, M. Boutigny made use of a very small thermometer, the bulb of which he immersed in the drop. He thus arrived at the result "that bodies in the spheroidal state remain constantly at a temperature inferior to that of ebullition, whatever may be the temperature of the vessel which contains them." This temperature he determined to be, for water, $96^{\circ}.5$ C.; for alcohol, $75^{\circ}.5$; for ether, $34^{\circ}.25$; for chloride of ethyle, $10^{\circ}.5$; and for sulphurous acid, $-10^{\circ}.5$.

M. Boutigny's method of determining these temperatures is liable to serious objections, and does not appear capable of giving accurate results. It can only be relied on as giving a limit which the temperature of the drop cannot exceed.

The method adopted by M. Boutan appears susceptible of much greater accuracy. It depends upon the principle that if a bar of any metal is placed in metallic contact at its two extremities with bars of a different metal, and if the points of contact are maintained at different temperatures, an electric current is produced in the metals when forming part of a closed circuit, the intensity of which varies with the difference between those temperatures, and is capable of being rendered sensible by its action on a magnetic needle. The apparatus employed by M. Boutan consisted of a very fine platina wire, attached at its two extremities to pieces of iron wire equally fine. The attachment was effected by raising the temperature of the iron to the point at which it united with the platina, without any intermediate solder. The extent of the attachment was not above 1^{mm} in length. This apparatus was employed as follows. The free extremities of the iron wire were connected with the binding screws of a sensitive galvanometer, of the kind employed by M. Melloni in his experiments on radiant heat, and the two points of contact of the wires

placed, one in the liquid spheroid, the other in a quantity of mercury in which was plunged the bulb of a very sensitive mercurial thermometer. Any difference of temperature between the spheroid and mercury was exhibited by the deflection of the galvanometer needle; and by raising or lowering the temperature of the mercury until the needle resumed its normal position, the two temperatures might be brought to a state of perfect equality, and thus the temperature of the drop ascertained with a great degree of accuracy.

M. Boutan gives the following general account of the results of his experiments:

"I have ascertained that the supposition of the liquid in the spheroidal state having a constant temperature throughout its mass is erroneous. When the mass is at all considerable, the different layers of the liquid have different temperatures. It is only when the drops are small, and the vessel has attained a stationary temperature, that the calorific equilibrium is thus established in the drop. I have also succeeded in ascertaining the influence exerted by the presence of a solid body plunged in the liquid mass, and which is at the same time in contact with the heated surface. The temperature of the liquid immediately rises, but still remains below its boiling point, although the body immersed may contain within a cavity in it the same liquid in a state of ebullition. Only, bubbles of vapour form on the surface of metallic bodies introduced into the drop, to which they adhere firmly, and from which they are detached in small numbers. Finally, I have, by the aid of the same instrument, succeeded in making a considerable number of numerical determinations, which I have not yet been able to complete, and which I shall shortly have the honour of communicating to the Academy. These numbers appear to me to furnish the first elements necessary to connect the theory of calefaction with the well-established laws of caloric."*

(ii.) With respect to the temperature of the vapour, M. Boutigny is of opinion that it is the same with that of the enclosing vessel, as, indeed, is probable, without evidence to the contrary.

(iii.) The experiments made with anhydrous sulphurous acid

* Mem. Acad. de Rouen, p. 50.

are so extraordinary as to deserve special notice. M. Boutigny having raised a platina capsule to a white heat, poured on it some grammes of anhydrous sulphurous acid. Watching the neck of the vessel containing it, the acid was observed to be there in a state of ebullition, which ceased when it fell upon the capsule, and assumed the spheroidal form. Its rate of evaporation was incredibly slow, and it presented no signs whatever of ebullition. (This liquid, it is to be observed, boils at about 10°). On pouring some distilled water, drop by drop, into the sulphurous acid while in this state, it was instantly congealed, even while the capsule remained at a white heat. And finally, on introducing for about half a minute the ball of a small *mattress* containing one gramme, about fifteen grains, of distilled water, into the sulphurous acid in the spheroidal state, and then removing it, it was found, on being broken, to contain a small fragment of ice.

On obtaining these results, says M. Boutigny, "J'éprouvai ce jour-là une de ces jouissances de laboratoire qui ne sont connues que des physiciens et de chimistes...."

The congelation of the water under those circumstances is easily explained. Its vapour is absorbed by the anhydrous acid as quickly as it is formed, and the rate of evaporation is consequently so rapid as to cool the temperature of the water, and finally congeal it in the manner explained in (104).

Mr. Faraday also mentions,* in a letter to M. Boutigny, that he introduced into a platina capsule, maintained at a red heat, some ether and solid carbonic acid, and into the spheroid formed by these substances plunged a small metallic capsule containing about thirty-one grammes of mercury, which was solidified in two or three seconds.

(iv.) De Saussure was of opinion that there is a maximum slowness of evaporation corresponding to a certain temperature of the hot surface. Klaproth, who made his experiments with water in an *iron* spoon, arrived at the conclusion that the rate of evaporation is the slower the higher the temperature of the vessel; but M. Boutigny, operating with a *platina* capsule, has arrived at the very opposite result, and has proved that the rate of eva-

* Annales de Chimie et de Physique, tome xix. p. 383.

poration *increases* with the temperature of the surface. The paradoxical conclusion arrived at by M. Klaproth probably resulted from the rapid oxidation, during the experiment, of the iron vessel which he employed. The time requisite for the evaporation of 0.1 gramme of water in the spheroidal state, according to M. Boutigny, when the surface is heated to 200° C., is fifty times more than that required to produce the same effect by boiling.

The slowness of evaporation of liquids in the spheroidal state is most striking in the case of those which are most volatile, such as sulphurous and carbonic acids, which, evaporating with extraordinary rapidity under ordinary circumstances, appear comparatively fixed in the spheroidal state. The rate of evaporation varies, within certain limits, with the form of the capsule, its polish, its capacity, and its thickness.

(v.) Assuming the temperature assigned by M. Boutigny to water in the spheroidal state to be even approximately correct, it follows, from the preceding remarks on the rate of evaporation, that the density of the vapour given off by it is far inferior to that due to its temperature, and that accordingly there is neither equilibrium of temperature between the surface and the liquid, nor equilibrium of tension between the liquid and its vapour.

(vi.) We have seen (102) that M. Donny suggests that the explosive ebullition of water, when freed from air by long boiling, may be one cause of steam-boiler explosions. M. Boutigny conceives that the assumption of the spheroidal state by large masses of water in those boilers may also produce the same effect. "When the steam passages are closed," says this writer, "the temperature of the boiler must rise; . . . the water is then exposed to the action of two forces which neutralize each other, *scil.* the pressure exercised by the vapour on the surface of the water, and the repulsive force of the boiler acting underneath. If a valve is now opened, the steam rushes out with considerable velocity, a partial vacuum is formed, and the water, repelled by the hot metal below, and attracted, so to speak, by the vacuum, is driven to the upper part of the boiler, and, falling back immediately in obedience to the laws of gravity, assumes the spheroidal state. It then furnishes but little vapour, thermic equilibrium no longer exists, and

an explosion is imminent. This may be hastened by two causes: first, by the addition of a certain quantity of cold water; secondly, by the lowering of the fires. The water in boilers may also pass into the spheroidal state in consequence of a deficiency in the supply, either from the negligence of the engineers, or defect in the feed-pumps, &c.*

(vii.) It remains only to mention some of the explanations given of the non-establishment of thermic equilibrium between a liquid in the spheroidal state and the containing vessel. The first is, that the heat in this case traverses the liquid without being absorbed by it. This, at first sight, appears probable, as we know that liquids and other bodies are capable of being permeated by radiant heat, as diaphanous bodies are by light. However, some of the facts mentioned above show that this principle does not apply to the present case, since the presence of particles capable of arresting and absorbing the rays of heat, such as charcoal, sand, &c., fails to raise the temperature of the spheroid. Such a view is also inconsistent with the formation of ice in the drop of sulphurous acid, as described in page 155.

The second hypothesis advanced to account for this phenomenon is, that the rays of heat do not enter the spheroid at all, but are reflected from its surface, and although there is no very obvious reason assignable why this should be the case, yet it appears, on the whole, the most probable explanation offered hitherto. It is due to M. Boutigny, who mentions, in confirmation of his view, that if a drop of nitric acid in the spheroidal state is made to describe a curve on a silver capsule, the points of the metal under the spheroid will appear of a brighter red than the rest of the vessel, if the experiment is performed in a dark room, and if the course of the liquid is afterwards examined with attention, it will be found mammellated, exhibiting traces of incipient fusion. "This remark proves sufficiently," observes our author, "that heat is reflected by bodies in the spheroidal state."†

* Annales de Chim. et de Phys., tome xi. p. 25.

† Ibid. tome ix. p. 368.

It is to be observed that, in the preceding account of Leidenfrost's phenomenon, we

have used the expression *spheroidal state* as equivalent to *existence in the spheroidal form*. This remark is rendered necessary by the circumstance, that it has been proposed to consider the condition of bodies

We will conclude these remarks by observing, that some French writers have called this phenomenon the *calefaction of liquids*, and say that a liquid is *calefied* when it exists in the spheroidal form.

SECT. III.—LAWS OF VAPOURS.

108. *General Properties of Vapours.* A. *In vacuo.*—The laws of the formation of vapours, and of the relations existing between their elastic force and temperature, were first successfully investigated, about the commencement of the present century, by Mr. Dalton. The results of his experiments were laid before the Literary and Philosophical Society of Manchester in the year 1801, and appeared in a volume of their Memoirs published in the following year. The apparatus employed by Mr. Dalton in his investigations of the general properties of vapours, was constructed as follows.

ab (Fig. 48) represents a long barometer tube, inverted in a tall cylindrical vessel full of mercury. The height of the mercurial column in the tube is measured by a graduated rule, *r*, whose point is brought to coincide with the surface of the mercury in the reservoir. Preparatory to experiment, the inside of the tube is moistened with the liquid whose vapour it is intended to observe, and the tube being filled with mercury lately boiled is inverted into the reservoir. The fluid moistening the sides of the tube, being lighter than the mercury, rises to the top, where it forms a thin film on its upper surface. The mercury is then found to be depressed below the level at which it would stand with a vacuum above it, by a quantity measuring the elastic force

described in the preceding pages as constituting a fourth state, in addition to the solid, liquid, and gaseous. M. Boutigny even suggests that we only know metals in the solid, *spheroidal*, and gaseous states, and that they pass *immediately* from the first to the second of these, as ice assumes the spheroidal form instantaneously when dropped on a surface sufficiently hot.

His principal reason for this strange hypothesis is, that metals, on cooling, take the form of the vessel they occupy more accurately than in the liquid state; that they diffuse themselves over it, *moisten* it, as it were, or *metallize* it rather; and besides that their temperature rises on solidification.

of the vapour emitted from the fluid. The space occupied by vapour may be altered by raising or lowering the tube in the reservoir, and changes of temperature may be produced by surrounding the upper part of the tube by a glass cylinder, *cd*, about two inches in diameter and fourteen inches in length. This cylinder is closed at both ends with cork, through which the tube passes, and may be filled with water at different temperatures. On altering the temperature and volume occupied by the vapour we obtain the following results:

(i.) If, while the *temperature remains constant*, we increase the volume, then

(α .) As long as there is liquid in excess above the mercury to furnish fresh vapour for the increased space opened for its reception, the elastic force, as measured by the depression of the mercury, and consequently the density of the vapour, remain constant. The density of vapour *in contact with its liquid* depends, therefore, solely on its temperature, and is independent of the volume occupied by it. This density is called the *maximum density at the given temperature*.

(β .) When the liquid above the mercury is all evaporated, the elastic force and density diminish on increase of volume, as in permanent gases; and if the volume be again reduced, the elastic force and density increase until the latter attains the maximum due to the temperature. The effect of any further diminution of volume is to reduce a portion of the vapour to the liquid state.*

The changes of density and elastic force in vapour separated from its liquid are generally assumed to follow Mariotte's law, up to the point of maximum density; this is true while the vapour is still at some distance from this point, but on approaching it the density is found to increase more rapidly than the elastic force. This is probably owing to a partial condensation of the particles of vapour through its mass. We will return to this subject in the section on the density of vapours.

* As we have assumed the temperature to remain *constant*, the preceding results are only true after the heat developed by condensation or absorbed by expansion, on change of volume, and which causes a

momentary change of temperature, has been dissipated, or restored by surrounding bodies. This remark is also to be attended to in the application of Mariotte's law, either to permanent gases or vapours.

(ii.) If the *space* occupied by the vapour above the mercury *remains unaltered*, then

(a). As long as there is liquid in excess, if the temperature is raised, the density of the vapour already formed is increased by fresh vaporization, and the elastic force is consequently increased in a much more rapid ratio than it would be in a permanent gas, by the same change of temperature. Conversely, if the temperature be lowered, a portion of the vapour is condensed, and reconverted to the liquid state; its density is diminished, and its elastic force reduced more rapidly than in a permanent gas.

(β). When the liquid is exhausted the elastic force increases on increase of temperature, as in the case of gases, and diminishes on its decrease, according to the same law, until the temperature reaches the point for which the density of the gas is the maximum; any further reduction of temperature is accompanied, as was stated above, by a partial condensation of the vapour.

As Mariotte's law is assumed to apply to the changes of density and elastic force in vapours separated from their liquids, so Gay-Lussac's law of the equal expansion of all gases and vapours under all pressures, for the same change of temperature, is generally considered applicable under the same circumstances. The investigations of MM. Regnault and Magnus on the expansions of gases, however, render it very improbable that the coefficient of expansion of any given vapour is the same under all pressures, or is the same with that of air for different vapours at a given pressure. But while we are ignorant of the true law which connects the elastic force with the density, when the temperature is constant, and of the relation between the increase of volume and temperature under all pressures, we are obliged, in our calculations, to assume the applicability of Mariotte and Gay-Lussac's laws to the case of vapours separated from their liquids. In the following investigation, accordingly, we shall proceed upon this assumption, reminding the student, however, that all results derived from it can be only considered approximately true.

109. *General Properties of Vapours.* B. *In a Space filled with a permanent Gas.*—If the space over the liquid in the barometric tube (Fig. 48) is occupied with air or any other permanent gas, the laws of the formation of vapour, and of the relation between

the density, tension, and temperature, are exactly the same as *in vacuo*. The only difference between the two cases is this, that while *in vacuo* the quantity of vapour requisite to saturate the space open for its reception is formed *instantaneously*, time is required for its production in a space occupied by a permanent gas. The density and elastic force, however, of the vapour formed, are the same as *in vacuo*, and depend merely on the temperature; and accordingly the density and tension of the *mixture* of dry gas and vapour are equal to the sums of the separate densities and tensions respectively.

110. *Relation between the elastic Force and Temperature of Vapours at their maximum Density; absolute Density of Vapour.*—If we raise the temperature of a vapour separated from its liquid, and then diminish the space occupied by it, until the density becomes the maximum due to the increased temperature, the relation between the elastic force at the former temperature and density, and at the latter, is the same as in the case of a permanent gas. Hence, calling the elastic forces of the vapour at the two temperatures f, f' , and the corresponding densities d, d' , we have*

$$\frac{f}{f'} = \frac{d}{d'} \cdot \frac{1 + at}{1 + at'}$$

a being the coefficient of expansion of the vapour for 1°C. ,

* For, in the case of a permanent gas, suppose the initial elastic force, density, and temperature, to be f, d , and t , and after any change of temperature and volume, let those quantities become f', d' , and t' . Now, suppose, in the first instance, the *density* alone to change, and become d' , call the corresponding elastic force ϕ , we have, by Mariotte's law, as the temperature remains constant, $\phi : f :: d' : d$, and therefore $\phi = f \frac{d'}{d}$. Next, suppose the *temperature* to change from t to t' , the elastic force, ϕ , remaining unaltered, the volume of the gas is thus increased in the ratio $\frac{1 + at'}{1 + at}$, and accordingly its density diminished in the ratio $\frac{1 + at}{1 + at'}$, so that the density d' be-

comes $d' \frac{1 + at}{1 + at'}$. If, now, the temperature t' remaining unchanged, we compress the gas by a quantity equal to that by which it was expanded, so that it shall return to its original volume and density d' , we have again, by Mariotte's law, the new elastic force $f' : \phi :: d' : d' \frac{1 + at}{1 + at'}$, therefore $f' = \phi \frac{1 + at'}{1 + at}$, and substituting its value for ϕ , $f' = f \frac{d'}{d} \cdot \frac{1 + at'}{1 + at}$, as in the text. And further, if in this equation we put $t' = 0$ and $\frac{f'}{d'}$, the ratio of the elastic force to the density at 0° , $= k$, we have $f = kd(1 + at)$.

which we will assume, according to Gay-Lussac's law, to be the same as for dry air, assigning to the latter, however, the value 0.00366 determined by M. Regnault, instead of that given by M. Gay-Lussac.

Since we have in general, in the case of gases, the relation

$$f = kd(1 + at),$$

where k is a constant depending on the nature of the gas, being the ratio of its elastic force to its density at 0° , it follows that for two *different* gases taken at the *same* temperature t , and under the *same* pressure f , assuming a to be constant for all gases, we have the relations

$$f = kd(1 + at),$$

$$f = k'd'(1 + at);$$

hence

$$\frac{d}{d'} = \frac{k'}{k} = \text{const.};$$

in other words, the ratio of the densities of any two gases under the same pressure, and at the same temperature, is constant for those gases, whatever be the temperatures and the pressures. And as we have seen above that the relation between the elastic force of a vapour and its density and temperature is the same as in the case of a gas, it follows that the ratio of the density of any vapour to the density of dry air, under the same pressure and at the same temperature, is constant for that vapour. This ratio is called the *absolute density* of the vapour, which may, accordingly, be defined to be *the ratio which the weight of a given volume of vapour bears to the weight of the same volume of dry air, under the same pressure and at the same temperature.*

111. *Density of a Vapour referred (1°) to dry Air, at a determined Temperature and Pressure, or (2°) to its own Liquid at a given Temperature.*—If we represent this absolute density of a vapour by the symbol m , and by d and A the densities of vapour and dry air at the same temperature t° and pressure f , referred to any fixed standard, we have

$$d = mA;$$

but (110)

$$\frac{A}{A'} = \frac{f}{f'} \cdot \frac{1 + at'}{1 + at},$$

A' being the density of dry air at t° and f' . Hence

$$d = mA' \frac{f}{f'} \cdot \frac{1 + at'}{1 + at},$$

which gives the ratio of the density of a vapour, at any temperature t and elastic force f , to the density of dry air at any assigned temperature and pressure.

(1°). If we adopt as the standard of density the density of dry air at the temperature of 0° C., and under a pressure equivalent to a column of mercury 760^{mm} in height, we have

$$d = m \frac{f}{760} \cdot \frac{1}{1 + at},$$

where the force f of the vapour should be expressed by the height in millimetres of the mercurial column which it would sustain.

(2°). It is, however, more convenient, in some cases, to assume as the standard of density the density of the liquid from which the vapour is produced, at some determined temperature. In the case of aqueous vapour this temperature is generally either 0° C., or that of the maximum density of water, viz., 4° C.; and the density of dry air at the temperature of 0° , and under the pressure of 760^{mm}, referred to the latter standard, being 0.001293187,* it is necessary to substitute these values for A' , t , and f' , and also for m its value, which, as we shall see hereafter, is 0.622. By these substitutions we finally obtain

$$d = 0.000001058 \frac{f}{1 + at},$$

if the elastic force is expressed in millimetres of mercury, and the temperature in centigrade degrees. But if we express the elastic force in English inches, and the temperature in degrees Fahrenheit, the formula becomes

$$d = 0.000026812 \frac{f}{1 + a'(t - 32)},$$

a' being equal to 0.00203. Hence if we represent by ρ the ratio of the volume of vapour, at t° Fahr., and f (expressed in inches

* For, according to M. Regnault, the weight of one litre of dry air at 0° and 760^{mm} is equal to 1.293187 grammes,

while the weight of the same volume of distilled water at 4° C., is equal to 1000 grs. — *Memoires de l'Institut*, tome xxi.

of mercury), to the volume of water at 39° F., from which it was produced, we have

$$\rho = 37296.7 \frac{1 + \alpha'(t - 32)}{f}.$$

112. *Connexion between elastic Forces and Densities.*—It follows that if we know the density of a vapour, referred to a given standard, at any one temperature t' , and the elastic forces at all temperatures, we can determine, by means of the preceding equations, the density, referred to the same or any other fixed standard at any assigned temperature; and conversely, if we know the elastic force at one temperature, and the densities at all, we can calculate, from the same equations, the elastic forces at all temperatures. For knowing the density of a vapour at t° , and its elastic force f' , and dividing the former by the density of dry air, at the same temperature and elastic force, referred to the same standard, we obtain the *absolute density*, m , and then, by the method indicated in the preceding paragraph, we can obtain the density, referred to any fixed standard, at any assigned force and temperature, or the elastic force at any assigned density and temperature.

The series of elastic forces and densities corresponding to different temperatures, therefore, being thus related, if either be determined by experiment, the other can be ascertained by calculation. Such calculated values, however, being obtained on the assumption of the applicability of Mariotte's and Gay-Lussac's laws to the case of vapours, can only be regarded as approximately true; and to obtain accurate results, both the elastic forces and the densities corresponding to different temperatures should be made the object of direct experiment.

We proceed to point out, in the following sections, the principal methods which have been adopted by various physicists for the experimental investigation of those quantities, and first of the methods of determining the elastic forces of vapours of various liquids, at their maximum density for the temperature.

SECT. IV.—EXPERIMENTAL RESEARCHES ON THE ELASTIC FORCE OF VAPOURS.

113. *M. Ziegler's Experiments.*—The earliest experiments on this subject appear to have been made by Ziegler, a Swiss, who

published them at Basle in 1769, in a treatise entitled "*Specimen physico-chemicum de Digestore Papini, ejus Structura, Effectu, et Usu, primitias Experimentorum novorum circa Fluidorum a Calore Rarefactionem et Vaporum Elasticitatem exhibens.*" His apparatus consisted of a kind of digester formed of copper, and strengthened externally by iron rings or hoops. In the lid was fastened a short tube, open above and closed below, which contained oil, mercury, or fusible metal, and a thermometer placed in this indicated the temperature of the water in the digester. The elastic force of the vapour was measured, up to a certain amount, by the column of mercury, whose weight balanced its pressure; beyond that, by a weighted lever arm acting on a safety valve in the lid of the digester. As M. Ziegler allowed the space above the water at the beginning of the experiment to remain full of air, the elastic force of the latter was added to that of the vapour, and accordingly his experiments were made, not on pure vapour, but on a mixture of vapour and air.

114. *M. Betancourt's Experiments.*—Shortly after M. Ziegler's experiments, but without any previous knowledge of them, M. Betancourt, an ingenious Spanish physicist, investigated the elastic force of the vapours of water and spirits of wine, and laid his results before the Academy of Paris, who published his memoir in the year 1790, in the "*Memoires des Sçavans Etrangers.*" His apparatus consisted of a small copper boiler, about four inches in width by fourteen deep, which contained in its lid three openings; through one of these the liquid was introduced, the stem of the thermometer, whose bulb was immersed in the vapour or fluid in the boiler, passed in a steam-tight collar through the second, and the third was connected with a manometer for measuring the elastic force of the vapour. In the upper part of the side of the boiler was inserted a tube with a stop-cock, by means of which communication could be made with an air-pump, for the purpose of exhausting the air from the apparatus.

Having formed as perfect a vacuum as possible, and cooled down the fluid to 0° by means of melting ice, M. Betancourt observed, in the case of water, a difference of level in the mercury of the manometer, amounting to 0.375 of a French inch.

Knowing that the vacuum in the apparatus was not perfect, and having no means of ascertaining the exact amount of air remaining, and feeling convinced that the elastic force of aqueous vapour at 0° must be very small, M. Betancourt came to the conclusion that he would approximate most nearly to the truth by attributing the whole of the force at 0° to the remaining air. He accordingly subtracted the height of the column which measured it, namely, .375 inch, from the heights corresponding to all higher temperatures. It appears, however, as we shall see presently, from Mr. Dalton's investigations, that the elastic force of aqueous vapour at 0° is equal to .2 of an English inch, or .188 of a French inch, and that consequently the difference between this and .375 inch is all that M. Betancourt should have attributed to the presence of air. In order to compare his results, therefore, with M. Dalton's, we should add the quantity .188 to the heights of the mercurial column, as given by him, corresponding to different temperatures.

115. *M. Volta's Experiments.*—In 1793 Volta published in Brugnatelli's Journal some experiments on the dilatation of air, which he determined to be $1 \div 213$ of its volume at 0° for each degree of Reaumur, a result agreeing very closely with that subsequently assigned by M. Gay-Lussac. He also made some investigations on the elastic force of aqueous vapour, which were subsequently given by Sr. Moretti, in a Supplement to his translation of Klapproth and Wolff's Chemical Dictionary. Volta's apparatus was similar to that subsequently employed by Dalton. He assigned the force of two French lines, or 0.167 of a French inch, to aqueous vapour at 0° , a force not differing very much from that given by the English physicist. From his observations of the forces at other temperatures, Volta derived the law, that the *increments* of the elastic forces, starting from 0° , form a simple geometric series, whose first term is 0.36 and ratio 1.11, the corresponding temperatures constituting an arithmetic series, whose first term is 0° , and common difference 2° R., the forces being expressed by the height of mercury in lines which the vapour would sustain at 0° . To derive from this the corresponding elastic force at the actual temperature t , the force as determined above must be multiplied by $\left(1 + \frac{t}{213}\right)$, according to the value assigned by Volta for

the coefficient of expansion of air, supposed also to be the same for vapour. Thus the force at 0° being 2, that at 2° R. = $2 + 0.36$, that at 4° = $2 + 0.36 + 0.36 \times 1.11$, &c.

116. *Dr. Robinson's Experiments.*—The *Encyclopædia Britannica* (1797) contains, under the article "Steam," furnished by Dr. Robinson of Edinburgh, a table of the elastic forces of aqueous vapour for every 10° F., from 32° to 280° F. His apparatus was similar to M. Betancourt's, and, like him, he also considered the force of vapour at 32° F. as equal to 0; by adding its true force at this temperature, his results approach more nearly to those of Dalton below 212° .

117. *Mr. Dalton's Experiments.*—Mr. Dalton's researches were communicated to the Manchester Society in the year 1801, and published in their *Memoirs* in the following year. He extended his observations on aqueous vapour from 32° F. to 212° F. As far as 150° he employed the apparatus described in page 158, raising the temperature of the water in the surrounding cylinder to that temperature; for higher temperatures he used a tube bent as a syphon, having the shorter arm sealed and filled with mercury. Into this he introduced a small quantity of water above the mercury, and surrounded it with a double cylinder of tin, for holding the hot water or oil, as in Fig. 50. The force of the aqueous vapour depressed the mercury by a quantity which could be ascertained by the equal amount of elevation observed in the longer arm. Mr. Dalton employed this latter form of apparatus also to determine the elastic forces of vapours from other liquids than water, which, at temperatures below 150° , equalled or surpassed the atmospheric pressure, and for which, accordingly, the first form was unsuitable. The observations made by these two forms of apparatus Mr. Dalton verified by the method referred to in page 141, based on the principle, that when liquids enter into a state of ebullition, the elastic force of their vapours equals the pressure to which they are submitted. The results obtained by that method agreed exactly with those derived from the two former.

After repeated experiments and careful observation of the results, as Mr. Dalton states, he arrived at the conclusion, that the elastic forces of aqueous vapours, for temperatures ascending

from 32° by intervals of $11\frac{1}{4}^{\circ}$ F. or 5° R., are represented by the terms of a geometric series, whose ratio is not constant, but diminishes for each term by a quantity whose mean value is 0.015, or rather 0.01567, in proof of which he gives the following Table.

Temperature.	Forces in English Inches.	Ratios.
32°	.200	1.485
$43\frac{1}{4}$.297	1.465
$54\frac{1}{2}$.435	1.45
$65\frac{3}{4}$.630	1.44
77	.910	1.43
$88\frac{1}{4}$	1.290
$99\frac{1}{2}$	1.820	1.41
$110\frac{3}{4}$	2.540	1.40
122	3.500	1.38
$133\frac{1}{4}$	4.760	1.36
$144\frac{1}{2}$	6.450	1.35
$155\frac{3}{4}$	8.550	1.33
167	11.250	1.32
$178\frac{1}{4}$	14.600	1.30
$189\frac{1}{2}$	18.800	1.29
$200\frac{3}{4}$	24.000	1.27
212	30.000	1.25

By means of the law announced above, which he assumed to hold good below 32° and above 212° , Mr. Dalton extended his table, by calculation, beyond the limits of his experiments, to -40° F. and $+325^{\circ}$ F., taking as the successive ratios below 32° , 1.500, 1.515, &c., and above 212° , 1.235, 1.220, &c.

On comparing the elastic forces of vapours of different liquids at various temperatures, Mr. Dalton was led to the conclusion, that the vapours of all liquids have equal elastic forces at temperatures equidistant from those of ebullition,—points at which they have a common force equal to the atmospheric pressure. This law has been shown by M. Despretz to be approximately true, within certain limits, for water, ether, and alcohol, but to fail in the case of essence of turpentine and other liquids. Although only approximately true, however, it serves to explain why liquids which boil at high temperatures, as mercury and sulphuric acid,

produce vapours which are scarcely sensible at ordinary atmospheric temperatures; for if the law were strictly true, the force of mercurial vapour at 15° C., which is 345° C. below its boiling point, should be equal to that of water at a temperature equal to $100 - 345 = -245^{\circ}$ C.; and the force of vapour of sulphuric acid at the same temperature, which is 295° C. below its point of ebullition, should equal that of water at $100 - 295 = -195^{\circ}$ C.: but we have reason to believe that the elastic force of vapour of water is insensible long before it reaches so low a temperature as either of those.

118. *M. Gay-Lussac's Experiments.*—After Mr. Dalton's experiments, which, as we have seen, extended from 0° to 100° C., we have next to notice some made by M. Gay-Lussac for the purpose of ascertaining the force of vapours at temperatures below 0° . His apparatus consisted of a barometer tube bent as in Fig. 49, into the upper portion of which was introduced a small quantity of the liquid whose vapour was to be examined, as in Mr. Dalton's experiments. The requisite temperature was produced by plunging the bent portion of the tube into a freezing mixture, whose temperature was indicated by the small thermometer in A. The pressure of vapour then in the vertical tube was equal to that in the coldest part of the bent portion, for the different strata of vapour from the surface of the liquid to the mercury, resting one upon another, could none of them sustain a greater pressure than that due to the coldest stratum, namely, the one in contact with the liquid. Practically, in fact, if vapour of a higher tension existed, or was generated from a small quantity of liquid in the vertical part of the tube, it would expand into the bent arm AB, where the pressure was less, and there would be constantly condensed, until all the liquid in the vertical part was evaporated; and moreover, all through this process the pressure on the surface of the mercury would only be equal to the reaction of the stratum of vapour of lowest tension. By means of this apparatus M. Gay-Lussac determined the force of vapour of ice at $-19^{\circ}.59$ C., which he found to be equal to $1^{\text{mm}}.353$ or 0.0533 of an English inch, which would differ very little from the value determined by Dalton's law, since for the temperature -18.75 C. we have by

that law the force .06570, thus proving that the continuity of the law of the elastic force of aqueous vapour is not interrupted by the solidification of water.

119. *Mr. Ure's Experiments.*—In the year 1818 Mr. Ure published* a series of experiments on the elastic forces of different vapours through an extensive range of temperatures. The apparatus which he made use of was similar to that form of Mr. Dalton's represented in Fig. 50, except that he measured the elastic force of the vapour above the mercury in the closed portion, by the height of the column which it was necessary to add in the open branch *c*, to maintain a constant level *ab* in the closed one. In this way the experiments were capable of being conducted with greater facility at different temperatures, as the portion of the tube to be heated or cooled was of constant length, and the same form of apparatus answered for all temperatures.

Mr. Ure sought to represent his results by a law similar to Mr. Dalton's. He gives as the successive ratios for intervals of 10° F. starting from 210° upwards, 1.23, 1.22, 1.21, decreasing by the common quantity 0.01, and for the divisors descending from 210°, 1.23, 1.24, 1.25, &c. To preserve the continuity of the law, however, as 1.23 is the factor for the change from 210 to 220, 1.24 should be the first divisor descending from 210 to 200, 1.23 the next, and so on.

Mr. Ure also extended his researches to the vapours of alcohol, ether, petroleum, and oil of turpentine.†

120. *M. Despretz's Experiments.*—In a memoir read to the Institute in November, 1819, M. Despretz undertook to examine specially the correctness of Dalton's law relative to the equality of elastic forces of vapour at temperatures equidistant from their boiling points. This law he found, as we have mentioned above, to be only approximately true in the case of alcohol and ether, as compared with water, within certain limits, and to fail altogether with turpentine and other liquids. In proof of this he gave the following results.

* Philosophical Transactions, 1818, Part ii., p. 338.

† Ibid., p. 358.

Name of Liquid.	Temperature. Elastic Force = 0 ^m .76.	Temperature. Elastic Force = 1 ^m .14.	Increase of Tempera- ture.	Temperature. Elastic Force = 0 ^m .38.	Decrease of Tempera- ture.
Water, . . .	100°.0	110°.0	10°.0	84°.6	15°.4
Alcohol, . .	78 .7	89 .4	10 .7	63 .8	14 .9
Ether, . . .	35 .5	47 .5	12 .0	(17 .77)	(17 .7)
Turpentine,	156 .5	174 .1	17 .6	134 .4	22 .1

From this we see that the numbers in the fourth and sixth columns, which, if Dalton's law were true, should be equal, agree very closely for alcohol and water, less nearly in the case of ether, but are altogether discordant for turpentine. M. Despretz found that this law signally fails also in the case of a liquid formed of chlorine and olefiant gas, and M. Marx* of Brunswick has found a similar result in the case of sulphuret of carbon.

121. *Experiments of MM. Dulong, Arago, &c.*—The next series of experiments relative to the force of aqueous vapour, to which we will direct the attention of the student, was a most important one, made in the year 1829 by a Commission of the Academy of Paris, consisting of MM. Prony, Arago, Girard, and Dulong.† The object of this series of experiments was to determine the elastic forces of vapour at higher tensions than had yet been observed, and the Commissioners accordingly pursued their investigations as far as the temperature 224°.15 C., the elastic force corresponding to which they found to be equal to 23.934 atmospheres, or a column of mercury at the temperature of 0° of 18.19 metres, or 716.07 inches in height. The following is a brief description of the apparatus employed by the Commissioners.

This apparatus was composed of two distinct parts, the one intended for the production of the vapour, and the determination of its temperature; the other for the measure of its elastic force. The former consisted of a strong copper boiler, provided with a safety valve of such construction, that when it was raised in the least by the excess of pressure of the vapour, a sliding weight on the arm of its lever opened it completely, and gave free egress to the steam. In the upper plate of the boiler were secured two

* Neue Jahrbuch der Ch. und Phys., tome ii.

† Annales de Chimie et de Physique, tome xliii. p. 74 (1830).

gun barrels, open above and closed below, one reaching nearly to the bottom of the boiler, the other only extending about one-fourth of its depth. These barrels were filled with mercury, and were intended to hold the thermometers, which could not have been exposed unguarded to the pressure of the steam, without undergoing an alteration in the form and volume of their reservoirs which would have vitiated their indications. In order to make the necessary correction for the portion of the instruments unassimilated to the interior temperature, the stems above the boiler were bent, as in Fig. 52, at a right angle, and a current of water at a known temperature was kept constantly in motion along them. The vapour ascended through the tube dd' , where it acted on the extremity of a column of water in the inclined pipe dd'' , which transmitted its pressure to the apparatus designed for measuring its force. This column of water was kept at a constant temperature by means of the stream flowing round it from the reservoir v , and the vapour, on reaching its extremity at u , was constantly recondensed, and fell back into the boiler.

The apparatus for measuring the force of the vapour consisted of a compressed-air manometer, mn , the Commissioners having previously verified Mariotte's law by direct experiment as far as a pressure of twenty-seven atmospheres. The vapour exerted its pressure through the column of water in the pipe $dd'd''$, on the surface of the water in the reservoir, and so through the mercury in the tube mn , on the compressed air in its upper portion. The variable height of the mercury in the iron reservoir f was seen in a glass tube k , whose lower extremity communicated with the reservoir, and its upper with the water in the pipe $dd'd''$. The air and column of mercury in the manometer were kept at a constant temperature by a current of water flowing round it.

The experiments with this apparatus were thus made. The water in the boiler was maintained in a state of ebullition for fifteen or twenty minutes, with the safety valve open to get rid of all air both in the boiler and in the water itself. The valve was then closed, the supply of water round the pipe dd'' , was regulated, the furnace was supplied with a determinate quantity of fuel, and when the ascent of the thermometer and of the mercury in the manometer became very slow, their indications were ob-

served and registered, until they reached their maximum. The observations made at this last point alone were employed as the basis of the calculation of the forces and temperatures, those preceding and following serving only to guard against errors in the readings.

The following table gives the results of the more trustworthy of the Commissioners' experiments, after all necessary corrections.*

Elastic Force in Metres of Mercury at 0°.	Elastic Force in Atmospheres of 0 ^m .76.	Corresponding Temperature observed.
1.62916	2.140	123°.7 C.
2.1816	2.8705	133.3
3.4759	4.5735	149.7
4.9383	6.4977	163.4
5.6054	7.3755	168.5
8.840	11.632	188.5
13.061	17.185	206.8
13.137	17.285	207.4
14.0634	18.504	210.5
16.3816	21.555	218.4
18.1894	23.934	224.15

From these results, by a formula of interpolation which will be afterwards given, they calculated the following tables.

TABLE of the elastic Force of Steam from 1 to 24 Atmospheres, according to the French Commissioners.

Elastic Force.	Temperature.	Elastic Force.	Temperature.	Elastic Force.	Temperature.	Elastic Force.	Temperature.
1	100°	5	153.1	10	181.6	18	209.4
1½	112.2	5½	156.8	11	186.0	19	212.1
2	121.4	6	160.2	12	190.0	20	214.7
2½	128.8	6½	163.5	13	193.7	21	217.2
3	135.1	7	166.5	14	197.2	22	219.6
3½	140.6	7½	169.4	15	200.5	23	221.9
4	145.4	8	172.1	16	203.6	24	224.2
4½	149.1	9	177.1	17	206.6

* Ann. de Chim. et de Phys., tome xliii. p. 108.

TABLE from 25 to 1000 Atmospheres.

Elastic Force.	Temperature.	Elastic Force.	Temperature.	Elastic Force.	Temperature.	Elastic Force.	Temperature.
25	226.3	45	259.5	300	397.6	700	478.4
30	236.2	50	265.9	400	423.5	800	492.4
35	244.8	100	311.3	500	444.7	900	505.1
40	252.5	200	363.5	600	462.7	1000	516.7

122. *Experiments of American Commissioners.*—About the year 1836 a Committee of the Franklin Institute of Pennsylvania* was appointed, at the instance of the Government of the United States, to inquire into the causes of steam boiler explosions. In the course of their inquiries they instituted a series of experiments on the elastic force of steam at high pressures. The apparatus which they employed was similar in its construction and plan to that of the French Commissioners. Their experiments, of which the following table gives the *mean results*, as determined from a graphic representation of the *actual* results, were carried as far as a pressure of ten atmospheres. For the purpose of comparison with the table of the French Commissioners, we give the temperatures in both Fahrenheit and Centigrade degrees.

TABLE of elastic Forces of Steam, from one to ten Atmospheres, according to the American Commissioners.

Pressure in Atmos.	Temperature.		Pressure in Atmos.	Temperature.		Pressure in Atmos.	Temperature.	
	Fahren.	Centigr.		Fahren.	Centigr.		Fahren.	Centigr.
1	212°	100°	4½	298° .5	148° .0	8	336	168° .9
1½	235	112.8	5	304 .5	151 .4	8½	340.5	171. 4
2	250	121.1	5½	310	154 .4	9	345	173 .9
2½	264	128.9	6	315 .5	157 .5	9½	349	176 .1
3	275	135.0	6½	321	160 .5	10	352.5	178 .0
3½	284	140.0	7	326	163 .3			
4	291.5	144.2	7½	331	166 .1			

* Journal of the Franklin Institute, vol. xvii. p. 289 (1836).

123. *M. Magnus' Experiments.*—We have next to notice a memoir on the expansive force of steam, by Professor Gustav Magnus, which appeared in Poggendorff's *Annalen*, No. 2, for 1844, and of which a translation has been published in the fourth volume of Taylor's *Scientific Memoirs*. In this memoir the author, after noticing the defects in former methods, arising from the difficulty of determining with accuracy the true temperature of the vapour, and also the correct pressure, owing to the unequal heating, and consequent partial expansion of the mercurial column, proceeds to explain the method which he adopted to obviate these difficulties.

In this method the vapour was generated in a U-shaped vessel, *adeb* (Fig. 53), about four inches long, one end of which was closed, and blown into a ball to increase the space for vapour, the other was united with a long glass tube, *bc*, by means of which connexion was made with an air-pump. The closed end having been filled with mercury which had been well boiled, some water which had been boiled violently for half or three-quarters of an hour was poured into the open end, and while still warm a small quantity of it was conveyed, by inclining the tube, over the mercury in the bulb, and the remainder of the water in the open end was then removed. To measure the elastic force of the vapour generated in the bulb at any temperature below 100° C., the open end of the tube was connected with the air-pump, and the air contained in it was rarefied until the mercury stood nearly at the same level in both branches. The elastic force of the vapour was then measured by the height of the mercury in the barometer gauge attached to the air-pump, increased or diminished by the small difference of level in the U-shaped vessel, both heights being corrected for temperature, the thin stratum of unvaporized water being also taken into account, and expressed by its equivalent height of mercury. The temperature was ascertained by enclosing the whole of the U-shaped vessel in a case formed of sheet iron, about fourteen inches long and wide, and ten high, surrounded by three similar cases, so that between each two there existed a stratum of air five-eighths of an inch thick on all sides. These cases were suspended in one another to avoid all metallic contact. The outer case was heated by two argand lamps, which,

when placed at a constant distance, and burned with a moderate flame, M. Magnus found to yield a constant quantity of heat, and thus to preserve the inner case at an invariable temperature. The temperature was measured by an air thermometer whose reservoir, *xyz*, was fork-shaped, and enclosed the vessel containing the vapour; its stem passed through the metallic cases on one side, as the tube leading to the air-pump passed through the other. Two mercurial thermometers, *v*, *w*, also were introduced through the lids of the cases into the inner space.

124. *M. Regnault's Experiments*.—It only remains for us now to notice the very valuable memoir of M. V. Regnault on the "Elastic Forces of aqueous Vapour," first published in the *Annales de Chimie et de Physique* for July, 1844,* and since more fully in the *Memoirs of the Institute*.† "To establish a physical fact," says this ingenious and accurate physicist, "we must not confine ourselves to a single method of investigation. It is necessary to employ various methods, and even to repeat those made use of by former experimenters, unless they are absolutely faulty; and we must show that all, when used with proper precautions, conduct to the same result, or if this be not the case, we must point out by direct experiment the causes of error in the defective methods." Acting on this principle, M. Regnault repeated the methods of Dalton, Ure, Magnus, Dulong, and Arago, with such modifications as the improved state of experimental science and his own skill and experience suggested, and has pointed out the defects under which they labour, and the limits within which their results may be relied upon.

Mr. Dalton's method may be described as consisting essentially in determining the heights of the mercurial column in two barometer tubes, the chamber of one being occupied with vapour, and the other being a vacuum. In this method the temperature of the vapour is determined by that of a water bath surrounding the chamber, and either the whole or a part of the mercurial column is maintained by the same means at the same temperature. M. Biot has remarked that the chief defect in this method arises

* *Ann. de Chim. et de Phys.* (3^{me} Serie), tome xi. p. 273 (1844).

† *Memoires de l'Institut*, tome xxi. p. 465 (1847).

from the fact, that it is impossible to maintain the water surrounding the tubes at a constant temperature through all its depth, if this depth is considerable, and its temperature differs much from that of the surrounding medium. Mr. Ure attempted to remedy this defect by limiting the space occupied by the vapour, and thus reducing the depth of the bath, and in this respect certainly his modification of Mr. Dalton's method was a decided improvement. M. Regnault has shown that if the whole of the tubes be surrounded by water, for the purpose of maintaining the columns at the same known temperature, Mr. Dalton's method is capable of giving accurate results between the limits $+10^{\circ}$ C., and $+30^{\circ}$ C., provided the water be incessantly and rapidly agitated, the agitation being merely interrupted for a moment to observe the heights of the mercurial column. Above the higher limit, however, the separation of the liquid into strata of unequal temperature commences the instant the agitation ceases, and the observations are accordingly rendered uncertain.

Where it was intended only to raise the chambers and a part of the mercurial column to the temperature of the vapour, M. Regnault made use of the following form of apparatus.

(i.) Two barometers, as similar as possible, of about fourteen millimetres internal diameter, are arranged side by side on a frame, PP' (Fig. 55). These barometers pass through two tubular openings in the bottom of a vessel, vv', of galvanized sheet iron, and are secured by means of caoutchouc collars. The vessel vv', whose horizontal section is given in Fig. 57, has on one side a rectangular aperture, round which is fixed an iron frame. A plate of glass with perfectly parallel faces is secured to this frame by means of a similar frame attached to the former by screws. A slip of caoutchouc, of the form of the contour of the aperture, is placed between the glass and the frame EFGH, and renders the joint perfectly water-tight. The two barometers are plunged in the same reservoir u. The capacity of the vessel vv' is about forty-five litres. This vessel is filled with water which is continually agitated, and its temperature is given by a very sensitive mercurial thermometer immersed in it, which is observed by means of a small horizontal telescope, L. The height of the column in the barometer is read off by means of a kathetometer,

the agitation of the water being stopped for an instant at the time of the observation of each column.

Observations are made with great precision at the temperature of the surrounding air; to observe the force at higher temperatures a small quantity of water is removed from the vessel by means of a syphon, and replaced with a corresponding quantity of hot water. A spirit lamp is then placed under the vessel, and its distance from it, as well as the height of its wick, is so arranged, that the temperature of the water, which is still kept in a state of brisk agitation, finally becomes constant. This condition is easily attained after some trials, and if the temperature does not surpass 50° C., it may be maintained stationary and uniform for any length of time, provided only that the agitation of the water is brisk and constant. Three or four observations were made every time that the temperature was rendered stationary, an interval of eight or ten minutes being left between each. In this method of operating, the portions of the columns outside the vessel vv' are in circumstances completely identical, and the difference of height of the portions within, which are at the temperature of the bath, being reduced to 0° , measures exactly the tension of the vapour, allowance, of course, being made for the pressure of the film of water. It is unnecessary to point out how much more accurate this method of ascertaining the temperature corresponding to observed forces is, than either M. Betancourt's or Mr. Dalton's.

(ii.) A second series of experiments M. Regnault made with the following apparatus. A balloon, Λ (Fig. 55, 56), whose capacity equals 500 cubic centimetres, contains a little glass vessel full of water recently boiled. The balloon is soldered to a curved tube cemented into a tubular piece of copper with three branches, d, e, f . In the branch e is cemented a tube soldered to the upper part of the barometer h , and in the branch f a tube communicating with an air-pump, by means of the desiccating apparatus MN , filled with powdered pumice steeped in sulphuric acid. The tube o is a perfect barometer, as before. The apparatus being arranged as in the figures, a vacuum was made forty or fifty times successively, and each time the air slowly re-admitted; by this means the interior of the balloon and barometric chamber was com-

pletely dried. The vacuum was then finally made as perfectly as possible, and the tube *fl* sealed by a blow-pipe. At first M. Regnault was unable to reduce the vacuum below two millimetres, but his air-pump having been cleaned, he subsequently succeeded in bringing it frequently below 1^{mm} . The balloon was then surrounded with melting ice, and the difference of the columns in the two tubes gave the elastic force at 0° of the remaining air. The ice having been removed, the vessel *vv'* was filled with water, and its temperature being sufficiently raised, the little glass vessel was burst by the expansion of the water it contained, and the balloon and chamber filled with aqueous vapour, whose force at the corresponding temperatures was observed as before.

This method answers very well for temperatures below that of the surrounding medium, and for 10° or 15° C. above it; it also answers for determining the force of aqueous vapours in air of any density, within those limits of temperature.

(iii.) Those two forms of apparatus do not answer for tensions above 200 millimetres; beyond this force M. Regnault employed the following, which in principle is similar to Professor Magnus's, and which he also employed in the case of liquids more volatile than water.

A syphon-shaped tube, *abc* (Fig. 59), of about 15^{mm} internal diameter, terminates in a fine curved tube, *ce*. The closed branch *ab* is filled with mercury, which is carefully boiled to expel all air and moisture. When the mercury is cool a small quantity of volatile liquid is introduced into the branch *bc*, and boiled for some minutes; the tube is then inclined, and a little of the liquid, yet hot, is passed up into the branch *ab*; the branch *bc* is then completely dried. The tube is now fixed in a perfectly vertical position in the vessel *vv'*, in front of the glass plate. The tube *ce* is cemented into one branch of a piece of copper, *edf*, whose other branches communicate, one, *d*, with a manometric apparatus with a stop-cock *r* (Fig. 59, *bis*), and the other, *f*, with an air-pump, if necessary. The tubes *hi*, *kl* are first completely filled with mercury, the air being expelled through *fg*; the tube *fg* is then closed with the blow-pipe, and a portion of the mercury being allowed to flow out through the stop-cock *r*, the pressure of the

air in the branch *ncc* is so far diminished as ultimately to become nearly equal to the pressure of the vapour in *am*, when the mercury in the two branches of the syphon-shaped tube falls nearly to the same level. The force of the vapour is then measured by the atmospheric pressure, diminished by the column $a\beta$ in the manometer and the column *mn* in the tube *ab*, both these columns, whose temperatures are known, being reduced to their heights at 0° . The temperature of the vapour is ascertained as in the preceding experiment.

(iv.) None of the foregoing methods answer for temperatures above 60° or 70° C. At higher degrees the water in the vessel *vv* separates so promptly into strata of different temperatures, as to require constant agitation to prevent this result from taking place. For temperatures above 100° C., moreover, those methods become impracticable from other causes. For higher degrees, therefore, M. Regnault had recourse to the well-known method employed by Mr. Dalton and other physicists subsequently, of ascertaining the temperature of the vapour of water boiling under determined pressures.

In order to obtain results of the degree of accuracy which this method is capable of giving, it is necessary to boil the water in a vessel communicating freely with a space of tolerable capacity, in which we can dilate or condense air at will, and by this means form an artificial atmosphere, which exerts a determined pressure on the surface of the heated liquid. We thus obtain a temperature of ebullition as perfectly stationary as that of water boiling in free air, and we can maintain this temperature stationary as long as we will. The apparatus employed for this purpose by M. Regnault is represented in Fig. 60.

It consists of a retort of red copper, *A*, closed with a cover. This cover carries four iron tubes, closed below; of these, two descend to the bottom of the retort, the others only reach half way down. These tubes, which are 7^{mm} in internal diameter, and about 1^{mm} thick, are surrounded by a case of very thin copper attached to the cover, and having apertures, *o, o, o*, in its upper part. They are filled with mercury to within a few centimetres of their upper edge, and hold four mercurial thermome-

ters, whose bulbs descend to the bottom of the tubes. From the arrangement of the tubes it appears that two of the thermometers are plunged in vapour, and two in water (Fig. 61).

The neck of the retort is connected with a tube, tt' , about one metre in length, surrounded by a copper cylinder, through which flows a constant current of cold water, supplied by a reservoir P . This tube communicates with a copper balloon, of about twenty-four litres in capacity, contained in a vessel, vv' , full of water at the temperature of the surrounding medium. To the balloon is attached a pipe with two branches, one of which is cemented to the tube egh , of the apparatus represented in Fig. 56, when the experiments are made at pressures inferior to that of the atmosphere, or with the tube pq of the apparatus in Fig. 62, for greater pressures. The second branch is connected by means of a lead tube, tt' , with an exhausting or condensing air-pump.

For pressures inferior to the atmospheric, the air in the balloon having been rarefied, the water in the retort is heated until ebullition commences. The vapour, according as it forms, is condensed in the refrigerator tt' , and falls back into the retort. The pressure is measured by the difference of heights of the mercury in the tubes ha and o . It may be remarked, that the column in the barometric tube ha , connected with the balloon, is never absolutely stationary; the amplitude of its oscillations, however, when the fire is properly regulated, is very small, not exceeding one-tenth of a millimetre. The mercury in the barometer o , on the other hand, remains perfectly stationary. The difference of the heights of these columns is observed by means of a kathetometer, and an assistant at the same instant notes the height of the thermometers. Several observations were made, at intervals of eight or ten minutes, under the same pressure, and it was thus easy to perceive the perfect constancy of the temperatures indicated by the thermometer for the same pressure, and to show that the least change in the latter was followed by a corresponding change in the former.

The height of the kathetometers not exceeding one metre, when greater differences of level in the apparatus, Fig. 62, required to be measured, it was necessary to employ two of them. In order to ascertain if the divisions of the scales of the two in-

struments were identical, M. Regnault read off the divisions of one, in lengths of centimetres, by means of the other, and such was the accuracy of their graduation, that he in no case encountered a difference of more than one-twentieth of a millimetre. "To attain such a degree of precision in the measures, it is evident that the instruments must be constructed with the greatest accuracy; the telescopes must not have too great a focal length ($0^m.30$), and the levels, in particular, must possess extreme sensibility. Those in the kathetometers constructed by M. Gambey indicated inclinations of one second. The verniers gave directly one-fiftieth of a millimetre, and easily admitted of the estimation of one-hundredth."*

"The thermometers employed in the experiments made at pressures inferior to the atmospheric ranged from 0° to 100° C.; they had from six to eight divisions in 1° C.; it was consequently easy to read with certainty the one-sixtieth of a degree. Those employed for higher pressures had a range from 0° to 240° , and 1° C. contained 2.5 or 3 divisions of their scale. All these instruments were graduated and verified by ourselves with the greatest care."†

In estimating the temperature from the indications of the thermometers, a correction required to be made for the portion of mercury in the stem unassimilated in temperature. The temperature of this portion was ascertained by means of a sensitive mercurial thermometer, suspended between the four stems. It may be remarked that the temperature indicated by the thermometer in the liquid was always higher, at low temperatures, than that indicated by those in the vapour; the difference in some cases amounted to $0^{\circ}.7$. As the pressure approached the atmospheric, this difference became less, and at high temperatures was quite insensible.

(v.) The method‡ by which M. Regnault investigated the force of vapours at high temperatures was similar to that last described, and the apparatus which he employed for this purpose differed

* Ann. de Chimie et de Phys., 3^{me} Serie, tome xi. p. 311.

† Ibid., p. 313.

‡ Memoires de l'Institut, tome xxi. p. 538.

from the preceding merely in the size and strength of the parts. The boiler was made of copper about 5^{mm} thick, and had a total capacity of seventy litres. The reservoir B, forming the artificial atmosphere, was also of copper 13^{mm} thick, and contained about 280 litres. The manometer destined to measure the pressure of the artificial atmosphere, and thus of the vapour in the boiler, consisted of a tube, or rather a series of tubes placed vertically over one another, and attached to the walls of the building in the vicinity of which the experiments were performed, and continued along a strong pole or mast firmly secured to the top of the wall. This system of tubes was open above, and contained the column of mercury which measured the tension in B. Its total height was twenty-four metres, and it was accordingly able to measure a pressure of thirty atmospheres.

The experiments were conducted as follows. The water in the boiler having been raised nearly to its boiling point, the air in the reservoir B was compressed until it had reached the pressure under which it was desired to make the experiment. A column of mercury of the corresponding height was next forced up the manometer from below by means of a force-pump, and connexion was then made between the manometer and the reservoir. Meanwhile the temperature of the water in A was rising, and continued to increase until it reached its boiling point under the pressure to which it was exposed. It was then kept in a state of ebullition for at least half an hour, and no observation was made until it was ascertained that the mercurial thermometers connected with the boiler were perfectly stationary. The temperatures indicated by these thermometers were then observed, and also the indications of an air thermometer, which, to insure greater accuracy, was employed in addition to the former. At the same time the difference of level of the mercury in the branches of the manometer was noted. M. Regnault's experiments with this apparatus extended to a pressure of about twenty-eight atmospheres, corresponding to a temperature of 230°.56, measured on the air thermometer. For the results of the preceding experiments see sect. vi.

125. *Experiments of Sr. Avagadro and M. Regnault on the Tension of Vapour of Mercury.*—Signor Avagadro has published

in the Memoirs of the Academy of Turin* a paper on the elastic force of the vapour of mercury. He has employed in his investigations the principle that the elastic force of a mixture of air and vapour is equal to the sum of the forces of each, considered separately, and that, accordingly, if we ascertain by observation the force of such a mixture at any temperature, and subtract from it the force of the air, calculated from the known laws of its expansion, we obtain the elastic force of the vapour alone at the observed temperature. His apparatus consisted of a glass syphon, whose shorter branch was closed and terminated in a ball, containing about one-third of its volume of air carefully dried. The remainder of the ball, as well as the bend of the syphon, was filled with freshly boiled mercury, which at the ordinary temperature stood at about the same level in both branches. This apparatus was immersed in a bath of fixed oil, whose temperature was raised by means of a charcoal fire placed underneath. Attached to the open branch of the syphon was a scale divided into millimetres, which enabled the observer to determine directly the rise of the mercury in this branch, and hence, by calculating approximately the corresponding depression in the ball from the relative areas of the ball and tube, whose diameters were 15^{mm} and 3^{mm}.75, respectively, to ascertain approximately the difference of level, and therefore the pressure and increased volume of the mixed air and vapour at any temperature.

The following Table contains the corrected results of his experiments.

TABLE of the Tension of the Vapour of Mercury from 230° C. to 290° C.

Temperature..	Pressure.	Temperature.	Pressure.
230°	58 ^{mm} .01	270°	165 ^{mm} .22
240	80 .02	280	207 .59
250	105 .88	290	252 .51
260	133 .62		

* Tome xxxvi. p. 215 (1833).

M. Regnault* also made some experiments on the vapour of mercury at temperatures inferior to the preceding. The following Table gives his results.

SERIES I.		SERIES II.		SERIES III.	
Temperature.	Pressure.	Temperature.	Pressure.	Temperature.	Pressure.
0°.00	0 ^{mm} .000	0°.00	0 ^{mm} .000	100°.6	0 ^{mm} .56
23°.57	0°.068	25°.39	0°.034	146°.3	3°.46
38°.01	0°.098	49°.15	0°.087	177°.9	10°.72
100°.60	0°.555	72°.74	0°.183	200°.5	22°.01
		100°.11	0°.407		

"Series I. and II. differ notably," remarks M. Regnault, "when we compare the *relative* values which they give for the elastic forces of mercurial vapour; still the *absolute* differences which we observe between these forces are really very small, and of the order of the errors of observation. The preceding experiments are, however, sufficient to show that the tension of the vapour of mercury at 100° is about 0^{mm}.5, and that at 50° it scarcely reaches 0^{mm}.1." "In Series III. the observations evidently became erroneous on approaching the temperature of 200°, owing to a partial distillation of the mercury. The results, accordingly, can only be regarded as approximations."

SECT. V.—EXPERIMENTAL RESEARCHES ON THE DENSITIES OF VAPOURS.

126. *Theoretic Determination of Density of Vapours from M. Gay-Lussac's Law of Volumes.*—When in the form of gases and vapours bodies reach, as it were, the very limit of material existence, we find them under the dominion of a force, the tendency of which is to constrain them to follow in their compression the law of Boyle and Mariotte, and in their expansion by heat that of Dalton and Gay-Lussac. This latter physicist has, moreover,

* *Memoires de l'Institut*, tome **xxi.** p. 502.

pointed out that under the same circumstances the law of chemical combination assumes a remarkable simplicity of form, for he has shown that gases and vapours not only combine according to definite proportions by weight or volume, as solids and liquids do, but that the volumes in which they combine always bear to one another a ratio expressed by some simple whole number; and further that the volume of the compound bears an equally simple ratio to those of the component gases.

This discovery of M. Gay-Lussac's of what is commonly called the *law of volumes* has been since so amply confirmed by subsequent experimenters, as to be regarded one of the best established laws in physics. The following examples, taken from Turner's Chemistry,* will serve as illustrations of it.

Volumes of Elements.	Volumes of resulting Compounds.
100 nitrogen + 300 hydrogen yield	200 ammonia.
50 oxygen + 100 hydrogen "	100 water.
50 oxygen + 100 nitrogen "	100 protoxide of nitrogen.
100 sulphur + 600 hydrogen "	600 hydrosulphuric acid.
100 sulphur + 600 oxygen "	600 sulphurous acid.
100 chlorine + 100 hydrogen "	200 hydrochloric acid.
100 iodine + 100 hydrogen "	200 hydriodic acid.
100 bromine + 100 hydrogen "	200 hydrobromic acid.
100 cyanogen + 100 hydrogen "	200 hydrocyanic acid.
100 oxygen + 100 nitrogen "	200 binoxide of nitrogen.

When we know the densities and volumes of the elements of a compound, and the volume of the latter, we can readily determine its density, from the principle that the weight of the compound is equal to the sum of the weights of the components. Thus let $n, n', n'', \&c., d, d', d'', \&c.$ represent the volumes and densities of the elements of a compound; N, D the volume and density of the compound itself; the weights of the elements will be $nd, n'd', n''d'', \&c.$, and the weight of the compound ND ; and consequently we have $nd + n'd' + n''d'', \&c. = ND$; and therefore,

$$D = \frac{nd + n'd' + n''d'', \&c.}{N}$$

* Turner's Chemistry, eighth edition, p. 161.

Now in the case of gases and vapours the quantities n , n' , n'' , &c., N , may all be expressed by simple whole numbers 1, 2, 3; hence, if we know the volumes and densities of the elements of a compound, and have an *approximate* experimental value for the density of the latter, we can determine its *exact* value, since we know in this case the exact value of N , which by the preceding law must be the *whole* number which gives a value for D nearest to the experimental one.

Thus we know by analysis that two volumes of hydrogen + one of oxygen form water, and that the density of the former gas, as referred to air at 0° and 760^{mm}, is 0.06926, and that of the latter 1.10563, while the density of aqueous vapour, as determined by direct experiment, is about 0.623. In this case, therefore, we have, approximately,

$$\frac{2 \times 0.06926 + 1.10563}{N} = 0.623;$$

and accordingly N must equal 2, and the exact value of D , the density of aqueous vapour, is 0.62207. The density here spoken of is, as will have suggested itself to the student, the *absolute density* of a vapour, that is (110), its density, as referred to air at the same temperature and pressure.

In the same way, knowing the volume and density of a binary compound and of one of its components, we can determine the exact density of the other component from its approximate value determined by experiment.

From what has been said of the unequal expansion of gases by heat, when the pressures to which they are submitted are at all considerable, it follows that, under these circumstances, their volumes will not preserve a constant ratio when they undergo equal changes of temperature. Hence we may expect to find in the law of volumes the same characteristic of a "limit law," which marks the other laws of gases which we have noticed.

127. *M. Gay-Lussac's Method of determining by Experiment the Density of Vapours.*—The first method of determining experimentally the density of vapours is due to M. Gay-Lussac. It consists in ascertaining the volume occupied by a known weight of liquid when converted into vapour, at a certain temperature

and pressure. The apparatus which he employed for this purpose was constructed as follows. A graduated glass jar (Fig. 65), filled with freshly boiled mercury, was inverted in a metal cistern, *MN*, containing a quantity of the same liquid, and placed over a furnace, *f*. The upper edge of this cistern was ground plane, to admit of its being accurately levelled, and supported a frame, *RS*, to which was attached at right angles a graduated rule, *PQ*. The lower point of this rule was brought to coincide with the surface of the mercury in the cistern, and a telescope of short focus moved along the upper part, by means of which the level of the mercury in the jar could be observed, and thus its height above that in the cistern measured. The jar *AB* was surrounded by a cylindrical vessel, *CC*, containing water or some other fluid, whose point of ebullition was generally higher than that of the liquid under consideration.

The manner of using this apparatus was as follows. A small bubble of thin glass, *m*, filled with a known weight of the liquid whose vapour was to be examined, and sealed at both ends, was introduced into the jar *AB*. In consequence of its superior lightness it rose through the mercury, and floated on its upper surface. The furnace being then lighted, the apparatus became gradually heated, and the glass bubble being burst by the expansion of the fluid which it contained, the vapour which was immediately formed depressed by its tension the level of the mercury in the jar. When the fluid in the vessel *CC* was raised to its boiling point, the elastic force of the vapour in *AB* was measured; if this was found inferior to the maximum force of vapour at the existing temperature, it was certain that all the fluid had been converted into vapour; but if the force was equal to the maximum, there was no evidence that this had taken place. In this latter case, therefore, it was necessary to repeat the experiment with a bubble containing a smaller quantity of fluid. When it was found that all the fluid introduced into the jar had been vaporized, the volume which it occupied was observed, and also its elastic force and temperature, and from these data, and its known weight, its absolute density was easily computed.

For let *v* represent the volume at 0° of one division of the jar, *k* the coefficient of cubical expansion of its material for 1° C.,

T the temperature of the apparatus as given by the thermometer in cc, and n the number of divisions of the jar occupied by the vapour; then the volume of vapour equals $nv(1 + kT)$, and w being its weight, the weight of the unit of volume of vapour, under the circumstances of the experiment, equals

$$\frac{w}{nv(1 + kT)}.$$

If H represents the height of the barometer at the time of the experiment, and h the difference of the heights of the mercury in the jar and in the cistern, these heights being reduced to their equivalents at 0° , the elastic force of the vapour equals $H - h$. Now if W represent the weight of the unit of volume of dry air at 0° and 760^{mm} , its weight at T° and $H - h$ is equal to

$$W \frac{H - h}{760} \cdot \frac{1}{1 + \alpha T},$$

and consequently the absolute density of the vapour, that is (110), the ratio of the weight of a given volume of the vapour to the weight of the same volume of dry air at the same temperature and elastic force, is given by the expression,

$$D = \frac{w}{nvW} \cdot \frac{760}{H - h} \cdot \frac{1 + \alpha T}{1 + kT}.$$

In an experiment made by M. Gay-Lussac on the density of aqueous vapour, the following results were obtained.

The weight of the glass vessel empty was $0^{\text{gr}}.791$

Its weight filled with water was . . . 1.391

Therefore w equalled . . . $0^{\text{gr}}.600$

The value of v was $0^{\text{lit}}.00499316$, n equalled 220, $T = 100^\circ$, and assuming $k = 0.0000262$, $nv(1 + kT) = 1^{\text{lit}}.10137$, and therefore $w \div nv(1 + kT)$, the weight of 1 litre of the vapour, = $0^{\text{gr}}.5473$.

Again, the height of the barometer at 15° was $755^{\text{mm}}.5$, consequently $H = 753^{\text{mm}}.4$; the height of the mercury in the jar over that in the cistern was 52^{mm} at 100° , and therefore $h = 51^{\text{mm}}.0$; and thus the elastic force of the vapour = $702^{\text{mm}}.4$.

Assuming W , the weight of the litre of dry air at 0° and 760^{mm} , after MM. Biot and Arago, to be equal to $1^{\text{gr}}.29954$, and the value of α to be 0.00375 ; we have,

$$W \frac{H-h}{760} \cdot \frac{1}{1+\alpha T} = 0^{\text{gr}}.8730;$$

consequently $D = 0.627$.

Moreover, as the weight of 1^{lit} of aqueous vapour at 100° and $702^{\text{mm}}.4$ equalled $0^{\text{gr}}.5473$, assuming Mariotte's law to hold true, its weight at 100° and 760^{mm} would be $0^{\text{gr}}.5925$; and the weight of 1^{lit} of water at its maximum density being 1000 grammes, it follows, from the preceding experiment, that the density of aqueous vapour at 100° and 760^{mm} , as referred to water at 4° , $= 0.0005925$, and consequently the volume occupied by 1^{gr} of vapour at 100° and 760^{mm} is to that occupied by 1^{gr} of water at 4° , as $1688:1$, or, in round numbers, as $1700:1$.

128. *Method of M. Despretz.*—It is obviously essential to the accuracy of the preceding method that the whole of the liquid should be converted into vapour, and as there is no means of ascertaining whether this condition has been complied with, except by so regulating the experiment that the elastic force at the time of observation is inferior to that due to the temperature, it follows that this method is inapplicable to the determination of the density of vapour, when this latter possesses the maximum value due to the temperature. M. Gay-Lussac, therefore, was obliged to compute the latter, namely, the maximum density, or the density corresponding to the maximum elastic force, from the density at an inferior force, by means of Mariotte's law.

In order to remedy this defect, and at the same time to examine whether Mariotte's law is really applicable to vapours, M. Despretz employed the following apparatus for the determination of their densities.*

This apparatus consisted of a barometer tube, AB (Fig. 66), of wider bore than usual, closed with a stop-cock, r . The tube being filled with mercury and inverted in a cistern, a small quantity of the liquid whose vapour was to be examined was introduced into

* *Annales de Chimie et de Physique*, tome **xxi**. p. 143.

the vacuum. A glass balloon, *E*, also furnished with a stop-cock, *r'*, and exhausted of air, was then screwed on the top of the barometer tube, and, the stop-cocks being opened, the balloon was filled with vapour. The stop-cocks were then closed, and the balloon being removed its weight was determined, and, the weight of the empty balloon and its capacity having been previously ascertained, the weight and volume of vapour were known. If there was liquid in excess, the elastic force of the vapour and its density were at their maximum for the temperature of the surrounding medium; but if there was a deficiency of liquid in proportion to the volume of the barometric chamber and balloon, the density was inferior to the maximum. The elastic force was given by the height of the mercurial column in the tube, compared with that of a barometer, *CD*, immersed in the same cistern. The temperature was always that of the air at the time of the experiment.

From the weights of vapour which filled the balloon at different temperatures and pressures, M. Despretz calculated the weights of a constant volume at a constant temperature, under the different observed pressures; and on comparing these weights and pressures he arrived at the conclusion, that in the liquids which he examined, namely, sulphuret of carbon, ether, and water, "the densities of vapours reduced by calculation to a fixed temperature are proportional to the corresponding elastic forces," that is, that Mariotte's law is applicable to vapours, as it is to permanent gases.

The following Table exhibits a view of his results:

Vapours.	t° .	w .	w' .	f .	$\frac{w'}{f}$
Sulphuret of carbon,	15°.87	8 ^{gr} .187	8 ^{gr} .655	0 ^m .1991	43.47
Ditto,	14 .78	5 .194	5 .486	0 .1272	43.13
Ditto,	15 .26	3 .285	3 .470	0 .0797	43.53
Ditto,	16 .62	3 .053	3 .229	0 .0739	43.69
Ditto,	15 .34	2 .773	2 .932	0 .0686	42.74
Ether,	12 .04	4 .967	5 .230	0 .1270	41.18
Ditto,	11 .43	3 .197	3 .364	0 .0820	41.02
Water,	19 .31	0 .134	0 .142	0 .0137	10.36
Ditto,	17 .44	0 .102	0 .108	0 .0100	10.80

In the preceding table t is the temperature of the vapour, w the weight of vapour filling the balloon at that temperature, w' the weight of vapour at 0° , calculated from Gay-Lussac's law of expansion, which would fill the balloon at 15° , when its capacity equalled $9^{\text{lit.}}.3746$, and finally f the observed elastic force.* The last column gives the ratio $\frac{w'}{f}$, which should be constant for the *same* vapour, on the supposition that Mariotte's law applies to it, and in the case of *different* vapours is proportional to their absolute densities.

129. *Method of M. Dumas.*—Neither of the preceding methods is applicable to vapours which act chemically upon mercury, or require for their formation a temperature much higher than that of boiling water. M. Despretz's method, indeed, is only applicable at temperatures which do not differ much from that of the surrounding medium. For vapours, accordingly, which fall under either of the preceding classes, we must employ the method of M. Dumas,† which is of very extensive application, and susceptible of considerable accuracy. This method consists in ascertaining the weight of vapour filling a balloon of known capacity, at a temperature superior by 20° or 30° to the boiling point of the liquid, and under the atmospheric pressure. M. Dumas' method of operating was as follows:

A balloon of glass (Fig. 67), whose neck was drawn out to a capillary termination, was filled with perfectly dry air, and weighed, the temperature and barometric pressure at the time being noted. A small quantity of liquid was then introduced into the balloon, which was placed in the bath, by means of which its temperature was to be raised. This bath varied in M. Dumas' experiments according to the temperature required. If this temperature did not exceed 150° C., he employed diluted sulphuric acid, contained in a cylinder of glass enveloping the balloon, and placed in a cistern of mercury, as in M. Gay-Lussac's method. If the temperature was required to reach 200° , he made use of concentrated sulphuric acid, contained

* In making this calculation M. Despretz assumed the coefficient of cubical expansion of glass to be 0.000 026 8, after MM. Lavoisier and Laplace; and that of vapour,

0.003 75, according to M. Gay-Lussac.

† *Annales de Chimie et de Physique* (2^{me} Serie), tome xxiii. p. 342 (1826).

in a glass vessel set in a sand bath, to guard against accidents which might arise from fracture of the vessel holding the acid. And for temperatures superior to 200° he used a bath of D'Arcet's fusible metal.

For these different forms of bath M. Mitscherlich has since substituted, with great advantage, one of chloride of zinc.

When the temperature of the balloon was raised to within a few degrees of the boiling point of the liquid which it contained, the vapour of the latter began to issue from the orifice, and displace the air, and as the temperature increased the vapour was projected in a strong jet, and with a loud, whistling noise, which subsided when the liquid was all vaporised, and thus gave evidence that the balloon was then filled merely with vapour. The temperature of the bath was then considerably raised, and kept stationary for some time, at the expiration of which the end of the capillary tube was sealed with a blow-pipe, the temperature and barometric pressure being noted. Before sealing the vessel it was necessary to observe if any condensed liquid was lodged in the tube, and if so to vaporise it by passing a live coal along it. The balloon was now removed from the bath, carefully dried, and allowed to acquire the temperature of the surrounding medium, when it was again weighed, observing the atmospheric temperature and pressure.

It now only remained to ascertain the volume of air which remained along with the vapour in the balloon, and also the volume of the latter. These volumes M. Dumas obtained by opening the capillary termination of the tube under water, the latter then rose up into the balloon and condensed the vapour, while the air remained in a bubble above the water at the top of the balloon. This air was then collected and measured, its temperature and pressure being observed.*

The balloon was next weighed quite full of water, and all the data were then ascertained which were requisite for the calculation, i. of the volume of the balloon; ii. of the volume of air remaining with the vapour; and iii. of the density of the latter.

* Or the temperature and pressure of the bubble of air in the balloon being observed, it might have been permitted to escape, and the balloon weighed, first with the quantity of water then remaining in it, and afterwards when full.

130. *Formulae of Calculation applicable to M. Dumas' Method.*

—M. Dumas' method of calculation was as follows. Let w be the weight of the balloon filled with dry air, w' its weight full of the mixture of vapour and air, and W its weight full of water, at the temperature t of the surrounding medium. Then the approximate volume of the balloon, expressed by the weight of water which it contained, was equal to $(W - w) + (W - w)\delta$, if δ represent the ratio of the densities of air and water under the circumstances of the experiment. For w was in fact the weight of the glass vessel + the weight of the dry air it contained – the weight of air displaced by the balloon at the time of the experiment; and assuming as a first approximation that the weights of these two volumes of air were equal, we have $w =$ the weight of the balloon. Further W represented the weight of the balloon + the weight (W') of the water it contained – the weight of the volume of air it displaced, and if this volume is expressed by the weight W' of water which would occupy it, the weight of air occupying the same volume will be $W'\delta$, for the weights of equal volumes of different masses are as their densities; therefore

$$W = w + W' - W'\delta, \text{ or } W' = (W - w) + W'\delta.$$

Now as δ is very small, we may put for W' , which multiplies it, the rudely approximate value, $W - w$ (obtained by neglecting the quantity $W'\delta$ in the preceding equation), which gives finally

$$W' = (W - w) + (W - w)\delta,$$

as above.*

Having thus ascertained the volume of the balloon, M. Dumas next calculated from it the weight w'' of the air it contained, which in fact was equal to $W'\delta$; this being known, he was able to determine the weight of the mixture of vapour and air. For the weight (w) of the balloon full of the mixture equalled the weight of the glass (w) + the weight of the mixture

* Or since $W' = (W - w) + W'\delta$, we have which, if we neglect δ^2 , becomes $W' = W' (1 - \delta) = W - w$; therefore $W' = (W - w) (1 + \delta) = (W - w) + (W - w)\delta$, as in the text.
 $(W - w) \div (1 - \delta) = (W - w) (1 - \delta)^{-1}$,

(W_1) – the weight of air displaced by the balloon (w''); therefore

$$w' = w + W_1 - w'',$$

and

$$W_1 = w' + w'' - w.$$

The volume of this mixture was then corrected for temperature, pressure, and dilatation of glass, and reduced to 0° C., and $\text{cm}^3.76$. The volume of the air remaining in the mixture was also reduced to its value under the same conditions, and corrected, moreover, for the presence of aqueous vapour; it was then subtracted from the volume of the mixture, the same operation was performed on its weight, and there remained, finally, the corrected values of the weight and volume of pure vapour.

Let V_0 represent the volume of the balloon at 0° .

This volume is thus found. W' , the weight of water filling the balloon at the temperature t , having been ascertained as above, expressed in grammes, we obtain immediately the weight (W''), which would fill it at 4° , the temperature corresponding to the maximum density of water. For, taking this latter density as unity, and representing by Δ' the density of water at t° , as given by Despretz' or Halstrom's tables, pp. 73, 74, we have $W'' : W' ::$

$1 : \Delta'$, and therefore $W'' = \frac{W'}{\Delta'}$. Now as 1000 grammes of distilled water at 4° occupy one litre, the volume of the balloon at t° , or $V_t : 1^{\text{lit}} :: \frac{W'}{\Delta'} : 1000$, and therefore $V_t = \frac{W'}{1000\Delta'}$; and since,

if k represents the coefficient of expansion of glass for 1° C., $V_0 = \frac{V_t}{1 + kt}$, we have, finally,

$$V_0 = \frac{W'}{1000\Delta'} \cdot \frac{1}{1 + kt}.$$

Let T represent the temperature of the bath when the balloon is sealed, and H the corrected height of the barometer at the same time; v the volume of air remaining in the balloon, collected over water, and saturated with aqueous vapour at the temperature t , and under the pressure H' ; and f the elastic force of aqueous vapour at t° , so that $H' - f$ is the elastic force of the air alone.

Further, let w' , as before, represent the weight of the balloon

full of the mixture of vapour and air, taken when the temperature of the surrounding medium is t'' and the height of the barometer H'' ; and w the weight of the balloon itself.

Then the volume of the balloon at the time of sealing is $V_0(1 + kT)$; this is also the volume of the mixture of vapour and air at T° and H ; at the temperature 0° and under the pressure 760^{mm} it would be

$$\frac{V(1 + kT)}{1 + \alpha T} \cdot \frac{H}{760}.$$

The volume of air contained in this mixture is equal to v at the temperature t' and under the pressure $H' - f$; at 0° and 760^{mm} it would be

$$\frac{v}{1 + \alpha t'} \cdot \frac{H' - f}{760}.$$

Consequently the *volume* of vapour alone at 0° and 760^{mm} is

$$\frac{V_0(1 + kT)}{1 + \alpha T} \cdot \frac{H}{760} - \frac{v}{1 + \alpha t'} \cdot \frac{H' - f}{760},$$

and the weight γ of the same volume of dry air, under the same circumstances of temperature and pressure, is

$$\gamma = \left\{ \frac{V_0(1 + kT)}{(1 + \alpha T)} \cdot \frac{H}{760} - \frac{v}{1 + \alpha t'} \cdot \frac{H' - f}{760} \right\} 1.293,$$

the volumes V and v being expressed in parts of a litre.

It now remains to determine the *weight* of the vapour alone.

The weight of the mixture of vapour and air is, as we have seen, $w' - w$ + the weight of air displaced by the balloon at the time of weighing. The volume of the balloon being then $V_0(1 + kt'')$, the weight of the air displaced by it is

$$V(1 + kt'') (1.293) \frac{1}{1 + \alpha t''} \cdot \frac{H''}{760};$$

consequently the weight of the mixture of vapour and air is

$$(w' - w) + (1.293) \frac{V_0(1 + kt'')}{1 + \alpha t''} \cdot \frac{H''}{760}.$$

The volume of remaining air at t' and $(H' - f)$ is v , consequently its weight is

$$(1.293) \frac{v}{1 + \alpha t'} \cdot \frac{H' - f}{760};$$

therefore the weight β of vapour alone is

$$\beta = (w' - w) + (1.293) \left\{ \frac{V_0(1 + k t'')}{1 + \alpha t''} \cdot \frac{H''}{760} - \frac{v}{1 + \alpha t'} \cdot \frac{H' - f}{760} \right\}.$$

And the absolute density, D , of the vapour is given by the expression

$$D = \frac{\beta}{\gamma}.$$

These formulæ may easily be reduced to English measures, in which the weights are expressed in grains and the volumes in cubic inches. For let W'' be the weight of water at 62° F., which would fill the balloon, we have $W'' = \frac{W'}{\Delta'}$ Δ'' , if Δ'' represents the density of water at 62° ; and as (p. 104) 252.722 grains of distilled water at 62° occupy one cubic inch, we have

$$V_t = \frac{W'}{252.722} \cdot \frac{\Delta''}{\Delta'},$$

and

$$V_{32^\circ} = \frac{W'}{252.722} \cdot \frac{\Delta''}{\Delta'} \cdot \frac{1}{1 + k'(t - 32)},$$

k' being the coefficient of expansion of glass for 1° Fahr.

Reducing the volumes of vapour and air to 32° Fahr. and 30° , the expression for γ becomes

$$\gamma = \left\{ \frac{V_{32^\circ} \{1 + k'(T - 32^\circ)\}}{1 + \alpha'(T - 32^\circ)} \cdot \frac{H}{30} - \frac{v}{1 + \alpha'(t - 32)} \cdot \frac{H' - f}{30} \right\} (0.32776),$$

since the weight of one cubic inch of air at 32° and 30° is equal to 0.878.32776,* and that for β

$$\beta = (w' - w) + (0.32776) \left\{ \frac{V_{32} \{1 + k'(t'' - 32)\}}{1 + \alpha'(t'' - 32)} \cdot \frac{H''}{30} - \frac{v}{1 + \alpha'(t - 32)} \cdot \frac{H' - f}{30} \right\}.$$

* For, according to Dr. Prout, 100 cubic inches at 60° and 30° weigh 31875.0117.

We give as examples the results obtained by M. Dumas in the case of the vapour of iodine, and of protochloride of arsenic.

Iodine.

107^{gr}.532, balloon full of dry air at 24° and 0^m.757, . . . = *w*.
 110 .025, balloon full of vapour and air at 185° and 0^m.757, = *w'*.
 664 .550, balloon full of water at 22°, = *W*.
 0^{lit}.066, air remaining in vapour, measured over water
 at 22°, and under the pressure 0^m.757.

Hence weight of litre of vapour of iodine = 118^{gr}.323, and
 absolute density = 8.716.

Protochloride of Arsenic.

97^{gr}.432, balloon full of dry air at 25° and 0^m.758, . . . = *w*.
 99 .420, balloon full of vapour (pure) at 175° and 0^m.758, = *w'*.
 638 .740, balloon full of water at 23°, = *W*.
 Hence weight of litre of vapour = 88^{gr}.1852, and absolute
 density = 6.3006.

The composition of this vapour* being 1 volume vapour of arsenic + 6 volumes of chlorine = 4 volumes of vapour, and the densities of vapour of arsenic and of chlorine being 10.37 and 2.44 respectively, the density of the vapour of the protochloride, according to the law of volumes, is given by the expression,

$$\frac{10.37 + 6 \times 2.44}{4} = 6.251.$$

4

131. *M. Regnault's Method of determining the Density of aqueous Vapour, I. in vacuo, (a) at the Temperature of boiling Water, and under feeble Pressures.*—We have seen that, on the supposition of the applicability of Mariotte and Gay-Lussac's laws, the density of a vapour, referred to any fixed standard, may be calculated from its absolute density, and further, that, on the same supposition, this latter quantity has a constant value for the same vapour. And as it is of the utmost importance in hygrometrical researches to know to what extent those laws may be held to apply, without sensible error, in the case of aqueous vapour,

* Regnault, Cours de Chimie, tome i. p. 309.

M. Regnault has made two series of experiments with this object. In the *first* he has investigated the density of aqueous vapour *in vacuo*, (α) at the temperature of boiling water and under feeble pressures, not exceeding half an atmosphere, and also (β) at temperatures extending from that of the surrounding medium to about 40° C. above it, and under different pressures. In the *second* he has examined the density of vapour in *air*, at its maximum for the temperature between the limits of 0° and 25° C.

In the conduct of these experiments M. Regnault has introduced several important modifications and improvements of previous methods, which we purpose briefly to bring before the notice of the student in the following pages.

(α). To ascertain the density of aqueous vapour *in vacuo*, at the temperature of boiling water, and under feeble pressures, M. Regnault made use of the following apparatus.

A (Fig. 68) represents a glass balloon about ten litres in capacity, to the neck of which is attached* a brass mounting furnished with a cock, r . Into this balloon a small quantity of water is introduced, and connexion is made with an air-pump, a tube containing coarsely powdered pumice stone and concentrated sulphuric acid being interposed to prevent the aqueous vapour penetrating into the pump. The vacuum is maintained for a considerable time, and the vapour of water which is perpetually formed in the balloon, eventually expels all the air from it, when the cock r is closed. The balloon is then placed in a vessel, BCDE, of galvanized sheet iron, so that the cock r is opposite the

* The method of connecting this mounting with the neck of the balloon is represented in Fig. 69. The mounting consists of two pieces, $abcd$, $efgh$, which, on being screwed together, compress between them a piece of hemp packing lying in the annular space oo' . This packing is saturated with a cement formed of equal parts of minium and ceruse, which are rubbed together with linseed oil, so as to form a thick paste. When the packing is compressed, a portion of the cement is driven from it into the space between the brass mounting

and the neck of the balloon; to prevent any passing into the balloon itself, the upper edge, mn , is carefully ground to fit the mounting. This cement quickly hardens, especially if the balloon is repeatedly heated in the vapour of boiling water, and forms a perfectly close joint not only at low but also at high temperatures, and possesses the additional advantage of not being liable to crack in consequence of sudden changes of temperature.—*Annales de Chimie et de Physique* (3^{me} Serie), tome xiv. p. 216.

[The main body of the page contains several lines of text that are extremely faint and illegible due to the quality of the scan. The text appears to be organized into paragraphs, but the specific words and sentences cannot be discerned.]

Rognault has made two series of experiments with this object. *First* he has investigated the density of aqueous vapour (a) at the temperature of boiling water and under feeble pressures, not exceeding half an atmosphere, and also (β) at temperatures extending from that of the surrounding medium to 100° C. above it, and under different pressures. In the *second* he has examined the density of vapour in air, at its maximum, the temperature between the limits of 0° and 25° C. In the conduct of these experiments M. Rognault has introduced several important modifications and improvements of procedure, which we purpose briefly to bring before the student in the following pages.

To ascertain the density of aqueous vapour in *vacuo*, at
the temperature of boiling water, and under feeble pressure,
I made use of the following apparatus.

Fig. 23 represents a glass balloon about ten lines in diameter, the neck of which is attached to a brass mounting by a cork, *r*. Into this balloon a small quantity of water is introduced, and connexion is made with an air-pump, a tube of finely powdered pumice stone and concentrated sulphuric acid being interposed to prevent the escape of vapour into the pump. The vacuum is made and for some time, and the vapour of water which is temporarily condensed in the balloon, eventually expels all the air from it, when the stopcock is closed. The balloon is then placed in a vessel of water, so that the cork *r* is opposite to the

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tubular opening *t*. This vessel contains a quantity of water, two decimetres deep, and is heated over a furnace. The extremity of the tube *bc* is connected with the brass tube *cd*, which is itself soldered to a flexible lead tube, *de*, constituting part of the mounting *N* of a large jar *F*. This jar is maintained at the temperature of the surrounding medium, and a small lead tube, *t*, puts it in communication with an air-pump, and the barometric manometer represented in Fig. 70.*

A partial vacuum having been made in the jar *F*, the cock *r* is opened when the water is boiling briskly in the vessel *BCDE*. The vapour passes over, and is distilled in *F*. At the expiration of about an hour the force of the vapour is observed by means of the manometer, and the cock *r* is closed.

The tube *bc* is now detached from *cd*, and carefully dried, and the balloon removed from *BCDE* and weighed, after having been allowed to remain for twenty-four hours suspended from the scale of the balance.

After its weight has been ascertained the balloon is again placed in the vessel *BCDE*, and in connexion with the jar *F*. The water is brought up to the boiling point, and a very advanced vacuum made in *F*. The cock *r* being opened, the greater part of the vapour is condensed in the jar, and there only remains a quantity which equilibrates the pressure in *F*. This latter force is carefully measured after the equilibrium is established, and the cock *r* is then closed. The balloon is again weighed as before.

If these weights are determined by the ordinary methods, allowance should be made for any change which may have taken

* This manometer consists of a perfect barometer, *AB*, and a barometer tube, *CD*, open above, both secured to the same board, and standing in the same cistern. When this instrument is not in use, mercury is allowed to flow out of the cistern through the cock *r*, until the level falls below the top of the partition *mn*, by which means all communication is cut off between the two tubes. This is found necessary in order to prevent any sudden oscillations in the open tube affecting the vacuum in *AB*, as they are apt

to do when frequently repeated, by forcing minute quantities of air through the mercurial column. When in use the mercury is brought to the level of the lower point of the screw *v*, and the heights of the columns in the two tubes above the upper point of the screw being observed by a kathetometer, these heights, increased by the height of the screw, which has been previously ascertained, give the true heights of the columns in the two tubes.

place, during the interval between the weighings, in the weight of the volume of air displaced by the balloon, and also in the small amount of moisture which is always condensed on the surface of glass vessels, apparently by some kind of molecular action. This latter quantity, which is, no doubt, in ordinary circumstances, very minute, we have no means of estimating with accuracy; the former might be calculated if it depended merely on changes of temperature and pressure, and on variations of the hygrometric state of the atmosphere, but there appears reason to believe that, where the observations are separated by an interval of some hours, the weight of a given volume of atmospheric air is also affected, though to a very slight extent, by changes which take place in its constitution.

In order to obviate, then, the necessity of those corrections, and the consequent liability to error, M. Regnault followed, in these experiments, the same method of weighing which he had adopted in his experiments on the densities of gases. The peculiarity of this method consists in equilibrating the balloon, not with weights, as in the usual way, but with another balloon full of air, hermetically sealed, constructed of the same kind of glass as the former, and having exactly the same external capacity. Any alteration produced, then, in the weight of the gas contained in the balloon A, was *exactly* represented by the counterpoise necessary to restore equilibrium,—neglecting as quite insensible the weight of the air displaced by the counterpoise itself,—for the volume of air displaced by the balloon, and the moisture condensed on its surface, were accurately equilibrated at any instant by the similar quantities in the case of the other balloon, and the weight of the balloon itself, and of its mounting, of course remained unaltered.

This second balloon, B, was prepared as follows. Having filled the balloon A with water, M. Regnault weighed it first in water and then in air, in this way he ascertained the weight of water which it displaced. He then selected another balloon of the same kind of glass, which displaced as nearly as possible the same quantity of water, and accordingly had nearly the same external capacity. To the neck of this balloon he cemented a

brass cap, and if the displacement of this cap, added to that of the balloon itself, was still inferior to the displacement of A, he selected a piece of glass tube, sealed at both ends, whose displacement equalled the difference. This he succeeded in doing without difficulty after a few trials.

If the balloon B, with its mounting, was lighter than A, M. Regnault introduced a little mercury into it before sealing, so that the addition of a small weight was sufficient to render the equilibrium perfect.

When taking the weights the balloons were attached by hooks to the scales of an accurate balance, so that they floated in exactly the same stratum of air. They were, moreover, enclosed in a case which protected them from local currents, and also from any effects which might be produced by the presence of the experimenter.

As a proof of the accuracy of this method, M. Regnault mentions that two balloons, suspended as described above, remained in perfect equilibrium for upwards of fifteen days, although in the interval the temperature had changed from 0° to 17° C., and the pressure from 741^{mm} to 771^{mm} .

To return to our experiment, let w be the weight of the counterpoise necessary to restore equilibrium after the partial exhaustion of the balloon A.

This is evidently the weight of the quantity of vapour which would fill the balloon at the temperature τ of boiling water, and under the pressure h , equal to the difference of the pressures observed in the two cases.

Let W be the weight of dry air which fills the balloon at 0° and 760^{mm} . This weight was determined by direct experiment. The weight of air at T° and h , filling the balloon at T° , is evidently

$$W \frac{h}{760} \cdot \frac{1}{1 + \alpha T} (1 + kT),$$

where α is the coefficient of expansion of dry air, and k that of the glass balloon for 1° C. Therefore the absolute density of aqueous vapour under the circumstances of the experiment, being

equal to the former of those weights (w) divided by the latter, is represented by the expression,

$$\frac{w}{W} \cdot \frac{1 + \alpha T}{1 + kT} \cdot \frac{760}{h}.$$

The following Table contains the results of four experiments made in this way:

Number of Experiment.	W .	w .	T .	h .	D .
1	12 ^{gr} .9937	2 ^{gr} .959	99°.91	378 ^{mm} .72	.62311
2	2 .802	99 .14	357 .51	.62377
3	1 .261	99 .63	161 .32	.62292
4	2 .696	99 .78	345 .28	.62229

The near agreement of the numbers in the last column with one another and with the theoretic density, shows that Mariotte's and Gay-Lussac's laws are applicable within the limits of the preceding experiments; for, as was remarked (112), the absolute density of a vapour is not constant, except on the supposition of the applicability of those laws.

On applying the same method, however, to the determination of the density of aqueous vapour at the temperature of boiling water, and under pressures which approach more nearly to 760^{mm}, values were obtained for D , which are sensibly larger than the preceding, proving, as might have been expected from the analogy of permanent gases, that the density increases more rapidly than the elastic force, when the vapour is near its point of maximum density for the temperature, and consequently near its point of liquefaction.

The preceding method is inapplicable, except at the temperature of boiling water; it fails also in giving accurate results at very feeble pressures, as the least error in the weights produces very sensible ones in the numerical value for the density. In order to obtain, therefore, correct values for the density at temperatures approaching more and more nearly to that of saturation, M. Regnault had recourse to the following method.

132. *Method of determining the Density of aqueous vapour in vacuo (β) within a limited Range of Temperature on either Side of that of the surrounding Medium, and at Pressures gradually diminishing from the Maximum.*—(β). The capacity of a large balloon of glass is accurately determined by weighing it when full of distilled water at a known temperature. Into this balloon is introduced a small glass bubble, hermetically sealed, and full of water, whose weight is carefully ascertained. The balloon is connected with an air-pump and manometer, as in the last method, and both it and the manometer are enclosed in vessels of water, in the manner represented in Fig. 38. and the usual precautions are taken to insure uniformity of temperature through the apparatus. The interior of the balloon is next carefully dried, and, after as perfect a vacuum as possible has been made in it, the pressure of the air remaining is observed. The glass bubble is then broken by means of some live coals, and the temperature of the water surrounding the balloon raised above the point of saturation of the vapour. This will be known to be the case when the elastic force, as given by the manometer, diminished by that due to the remaining air, is inferior to the maximum force of aqueous vapour at the existing temperature.

If V , expressed in cubic centimetres, represents the volume at 0° of the balloon and part of the manometer tube occupied by the vapour in these experiments, this volume at t equals $V(1 + kt)$, k being the coefficient of glass for 1°C. , and w being the weight of water introduced into the balloon, which we will suppose at the temperature t to be all converted into vapour, we have for the weight of 1^{cc} of the vapour at t° and the observed pressure f , the expression

$$\frac{w}{V(1 + kt)}.$$

The weight of 1^{cc} of dry air at 0° and 760 M. Regnault assumed, after MM. Biot and Arago, to be $0.87.0012995$, consequently its weight at t and f equals

$$0.87.0012995 \frac{1}{1 + at} \cdot \frac{f}{760};$$

and the absolute density of the vapour, in terms of the data of the preceding experiment, is given by the expression,

$$D = \frac{w}{V(0.0012995)} \cdot \frac{1 + at}{1 + kt} \cdot \frac{760}{f}$$

The following Table gives a view of the results of some experiments made by this method, in which

$$V = 9612^{\text{cc}}.4, \text{ and } w = 0.67.308.$$

No. of Experiment.	t .	f .	F_t .	$\frac{f}{F_t}$.	D .
1	15°.06	12 ^{mm} .81	12 ^{mm} .75	1.000
2	23°.74	21°.97	21°.85	1.000
3	25°.94	25°.05	24°.91	1.000
4	30°.82	32°.14	32°.14	1.000	0.646 93
5	31°.23	32°.66	33°.86	0.964	0.638 49
6	31°.54	33°.24	34°.46	0.964	0.627 86
7	32°.37	33°.49	38°.47	0.870	0.624 99
8	37°.05	34°.19	46°.82	0.733	0.621 40
9	41°.51	34°.65	59°.51	0.582	0.621 95
10	41°.88	34°.61	60°.68	0.570	0.623 33
11	45°.78	35°.22	74°.33	0.474	0.620 03
12	48°.38	35°.48	84°.84	0.418	0.620 46
13	55°.41	36°.23	119°.84	0.302	0.620 78

F_t denotes the maximum tension of vapour due to the temperature, taken from M. Regnault's tables; $\frac{f}{F_t}$, the ratio of the elastic force observed to the maximum force at the temperature, is generally called the *fraction of saturation*.

In experiments 1, 2, 3, liquid was in excess, and accordingly the elastic force had its maximum value; in the remaining experiments the value of this force was inferior to the maximum, the liquid, therefore, was all converted into vapour, and the data served for the calculation of the density.

The experiments from 8 to 13 give for this density numbers sensibly equal to one another and to the theoretic density, thus establishing within their limits the applicability of Mariotte and Gay-Lussac's laws. But the experiments from 4 to 7, which were made at temperatures near that corresponding to the saturation

of the space, give for the density values much greater than the preceding, and constantly increasing with the fraction of saturation.

“From this we may conclude,” says M. Regnault, “that *the density of aqueous vapour in vacuo and under feeble pressures may be calculated according to Mariotte’s law, provided that the fraction of saturation does not exceed 0.8*; but that this density is notably greater when we approximate more nearly to the state of saturation.

“This latter circumstance may be owing to one or both of two causes; either aqueous vapour really suffers an anomalous condensation on approaching its point of saturation, or a portion of the vapour remains condensed on the surface of the glass, and does not assume the aeriform state until the mass of vapour is at some distance from the point of saturation. The daily experience of our laboratories proves the hygroscopic attraction of glass; this substance holds water condensed on its surface, even when it has remained a long time in air far from its point of saturation, so that we cannot doubt but that the hygroscopic affinity of glass has some influence on the phenomenon in question, though it is difficult to decide whether it is its sole cause. If we were to determine the density of aqueous vapour near its point of saturation, in balloons of different materials, or in glass balloons covered with different kinds of varnish, or in balloons of this material of very different forms, presenting, accordingly, very different proportions between their surface and capacity, we might, perhaps, arrive at an approximate estimate of the influence exerted in the present case by the nature of the surface. But it would be difficult to get rid of the superficial condensation completely and with certainty.”

133. II. *M. Regnault’s Method of determining the Density of aqueous Vapour in Air, in a State of Saturation.*—M. Regnault describes as follows his method of determining the density of aqueous vapour in air, at its maximum for various temperatures.

“I have determined the density of aqueous vapour in a state of saturation in air, by weighing the quantity of moisture which a known volume of air, when saturated, contains at different tem-

peratures. For this purpose I have employed the method of M. Brunner, which consists in filling with water a vessel of known capacity, and then, after putting the upper part of this vessel, which is called an aspirator, in connexion with tubes containing desiccating substances, in allowing the water to flow out at a uniform rate through an orifice below. The water thus withdrawn from the aspirator is replaced by an equal volume of air, which has been deprived of all its moisture on its passage through the desiccating tubes. These tubes are weighed before the aspiration has commenced, and after the water has all flowed out of the aspirator, and the difference of the weights represents the weight of water contained in a volume of air equal to the capacity of the aspirator.

"The aspirator which I employed consists of a cylindrical vessel (Fig. 71) of galvanized sheet iron, terminated by conical ends. The upper part contains two tubular openings, the one, *a*, central, in which is hermetically secured a tube, *tt'*, which acts as a Mariotte's tube in rendering the flow of water uniform; through the second aperture, *b*, is passed a thermometer, *T*, whose reservoir occupies the middle of the vessel. The lower part has a single tubular aperture, with a graduated cock, *R*; this cock has a discharge pipe, one decimetre in length, which remains filled with water after the discharge is completed, and prevents the entrance of air by the inferior aperture when the aspirator is emptied.

"The Mariotte's tube has a cock, *r*, which serves to stop the aspiration of air, and a U-shaped tube filled with *sulphuric pumice*, which is constantly attached to the apparatus; the object of this tube is to prevent the vapour of the water in the aspirator reaching the desiccating tubes *B* and *C*.

"To absorb the moisture of the air I only employ two U-shaped tubes, *B* and *C*, 0^m.18 in height, and filled with sulphuric pumice in large pieces, for if reduced to fine powder it would present too great a resistance to the flow of the air, and consequently the air in the aspirator would not possess the same elastic force as the exterior air."

After having described some experiments by which he satisfied himself that these tubes were sufficient to absorb all the moisture contained in the air drawn into the aspirator, M. Reg-

nault states his reasons for desiring to render this part of the apparatus as small as possible. These reasons are founded on the uncertainty attached to the weights of vessels of glass of considerable dimensions, determined under different atmospheric conditions, arising from the circumstances to which we alluded in a previous paragraph. He then proceeds to describe the part of the apparatus designed to saturate the air with moisture before its aspiration.

“A vessel of tin, *MN*, of twenty-five litres in capacity, closed above, is placed in a large dish full of water; this vessel has three tubular apertures. The upper aperture *e* admits a very sensitive thermometer, whose reservoir occupies the centre of the vessel; in the aperture *f* is fitted the extremity of the first tube *c*, so that this tube may derive the air from the middle of *MN*; and finally, by means of the aperture *g*, the vessel is put in communication with the balloon *o*, filled with wet sponge, which the air is obliged to traverse before entering the apparatus. To insure still further the saturation of the air, a cylinder of wove wire was placed in the interior of the tin vessel; this cylinder was covered internally and externally with moist linen cloth, which dipped into the water covering the bottom of the dish. A little opening, *o*, in the side of this cylinder, allowed the air to be drawn from the middle of the vessel, and in the neighbourhood of the reservoir of the thermometer.

“This apparatus was arranged in a room whose temperature varied very little, and no experiment was commenced until some time after it was put together. The rate of aspiration of the air within certain limits made no difference in the results of the experiment. Thus in one case the water was drawn off in forty-five minutes, in another in three hours, but the weight of water collected in the tubes *B*, *c* was exactly the same in both experiments.

“Under ordinary circumstances the aspirator was emptied in $1^{\text{h}} 15^{\text{m}}$ or $1^{\text{h}} 30^{\text{m}}$. Every five minutes the thermometer in the tin vessel was read from a distance by means of a small telescope, and the mean of the observed temperatures was taken as the temperature of the saturated air. These temperatures never varied above one or two tenths of a degree. When the water ceased to flow a few minutes were allowed to elapse, to permit the air

in the aspirator to acquire exactly the pressure of the external air; the cock *r* was then closed, and the thermometer *T*, as well as the barometer, noted.

“To obtain the quantity of water in air saturated at 0° , I made use of the following arrangement. A tin tube (Fig. 72), 0^m.55 long and 0^m.10 in diameter, has in its axis a tube, *ab*, 0^m.02 in diameter. This tube is open at its two ends; a lateral opening, *cd*, forms the communication between the tube *ab* and the drying tube *c*. The upper end of the tube *ab* is closed. The outer vessel is filled with pounded ice, which escapes when melted through the cock *r*.

“When the aspirator is in operation, the exterior air is drawn through the ice, which reduces it to 0° ; it enters the tube *ab* through the inferior orifice, and from thence passes into the desiccating tubes through the orifice *cd*.

“Now let

t represent the mean temperature of the saturated air;

f the maximum elastic force of aqueous vapour at that temperature;

t the temperature of the aspirator at the end of the experiment;

f' the elastic force at the temperature *t*;

H the barometric height reduced to 0° at the end of the experiment;

a the coefficient of dilatation of air for 1° C.;

k that of sheet iron, assumed to be 0.000 036 6;

V the volume of the aspirator at 0° .

“The volume of the aspirator at *t* will be $V(1 + kt')$; this is the volume which the air drawn through the tubes had in the aspirator at the conclusion of the experiment, but it was then saturated with vapour of water, whose elastic force was *f'*, therefore the air alone had a pressure equal only to $H - f'$, whereas when in the tin vessel its pressure was $H - f$; consequently its volume in the latter case would have been

$$V(1 + kt') \frac{H - f'}{H - f},$$

had its temperature been the same as in the aspirator; but as its temperature in MN was t , its volume there was really

$$V(1 + kt') \frac{H - f'}{H - f} \cdot \frac{1 + at}{1 + at'}.$$

“ If we represent by w the weight of a cubic centimetre of air at 0° and 760^{mm}, and by δ the density of aqueous vapour taken relatively to that of air,—that is, its absolute density,—then, supposing that aqueous vapour in a state of saturation in air follows the same law of dilatation and pressure as air, that is, that Gay-Lussac’s and Mariotte’s laws are applicable to it under those circumstances, we shall have for the weight of aqueous vapour contained in the preceding volume of air, the expression

$$V(1 + kt') \frac{H - f'}{H - f} \cdot \frac{1 + at}{1 + at'} \cdot w\delta \frac{1}{1 + at} \frac{f}{760},$$

or

$$V(1 + kt') \frac{H - f'}{H - f} \frac{1}{1 + at'} \cdot w\delta \frac{f}{760}.$$

“ Equating this expression to the weights determined by experiment, we should have a series of equations to determine δ , by means of which we might assure ourselves whether this value is constant for all temperatures.

“ I have preferred calculating by means of this formula the weight of vapour which should be found in the preceding volume of air, supposing $\delta = 0.622$, the theoretic density, and comparing this weight with that derived from direct experiment. The result of this comparison is, that all the numbers obtained by calculation are a little greater than those found by experiment, and this, sensibly, by the same fraction of the total weight. This fraction is very small, and amounts to about one-hundredth.

“ From this we may conclude that *the density of aqueous vapour in air, in a state of saturation, and under feeble pressures, may be calculated from Mariotte’s law; and that the ratio of the weight of a volume of this vapour to that of the same volume of air, under the same circumstances of temperature and pressure, is a little less than the theoretic density of aqueous vapour.*

"It is true," adds M. Regnault, "that we may explain in another way this difference between the observed and calculated weights of vapour. We may admit that the density of aqueous vapour, under the preceding circumstances, is the same as that which we have found *in vacuo*, namely, 0.622, but that the values of the elastic force, f , employed in the calculation, and which were taken from my table of the elastic forces *in vacuo*, are a little too large; and this would accord with the result of my experiments* on the elastic force of aqueous vapour in *air*, in a state of saturation."

The experiments referred to were made by means of an apparatus similar to that described in (54). The balloon in this case was filled with perfectly dry gas, and connected with the manometer. The elastic force of the gas at various temperatures was determined by a series of experiments, and a quantity of water contained in a glass bubble previously introduced into the balloon, was disengaged in the usual way. The apparatus was then raised to various temperatures, and the elastic force of the mixture of gas and vapour observed. The elastic force of the gas being subtracted, the remainder gave the force of the vapour, and this was uniformly found to be less than that derived from M. Regnault's tables, which, however, agreed exactly with his experiments on vapour *in vacuo*. The difference varied from 0^{mm}.1 to 0^{mm}.7, and appeared to have no relation to the temperature. M. Regnault states that the water introduced into the balloon had not been boiled, and was consequently saturated with air; but it is difficult to resist the impression that the difference between the elastic force of the mixture and the sum of the forces due to the gas and vapour separately, was owing to the absorption of a definite portion of the gas by the water, at the commencement of the experiment. The gases operated on were atmospheric air and nitrogen, and the results were the same in both cases.

134. *Researches on the Application of the Law of Volumes to the Determination of the Density of Vapours.*—On comparing the densities of compound vapours, as determined by Gay-Lussac's

* Ann. de Chimie et de Physique (3^{me} Serie), tome xiv. p. 134.

law of volumes, with their densities derived from direct experiment, M. Dumas was led to the conclusion, that in the case of the hydracids the volume of the vapour is always represented by 4, so that one atom of the acid furnishes four volumes of vapour. He noticed* a striking deviation from this law, however, in the case of acetic acid, for the density of the vapour of this body, derived from its atomic constitution, on the preceding hypothesis, is only 2.08, while its value, as determined by direct experiment, at 20° or 30° above its boiling point, is 2.7 or 2.8. The difference of these numbers being nearly equal to the density of aqueous vapour, M. Dumas suggested as a probable explanation of the anomaly, that the vapour obtained by the ordinary method consists of a mixture of equal volumes of pure acetic acid vapour and aqueous vapour.

This conjecture appeared to M. Bineau† to be rendered more probable by some investigations of his own as to the combinations of water with the hydracids; on being submitted, however, to the test of more direct experiments, it was not sustained by facts, and accordingly M. Bineau subsequently‡ came to the conclusion, that acetic as well as formic and sulphuric acids do really deviate from the preceding law, and furnish for each atom of the acid only *three* instead of *four* volumes of vapour.

Thus the atomic constitution of acetic acid being $C_4H_4O_4$, we obtain

8 volumes vapour of carbon, . . .	3.368
8 volumes hydrogen,	0.552
4 volumes oxygen,	4.424
	<hr/>
Density of vapour	$= \frac{8.344}{3} = 2.78,$

which agrees very closely with the experimental density.

So also for formic acid, whose constitution is represented by $C_2H_2O_4$, we have

* *Traité de Chimie*, tome v. p. 146.

† *Comptes Rendus*, tome xv. p. 777 (1842).

‡ *Ibid.*, tome xix. p. 767 (1844).

4 volumes vapour of carbon, . . .	= 1.684
4 volumes hydrogen,	0.276
4 volumes oxygen,	4.424
	<hr/>
Density of vapour,	$\frac{6.384}{3} = 2.12,$

which, like the preceding, is in close accordance with experiment; and similarly for the vapour of sulphuric acid.

Simultaneously, however, with the publication of these views, M. Cahours* announced his discovery of the fact, that the density of acetic acid vapour varies materially with the temperature at which it is taken, and that the preceding anomaly disappears completely at a temperature 100° or 110° above its boiling point. Pursuing his investigations, he found that the density of this vapour, which at 125° equals 3.20, decreases to 2.08, the theoretic density, at 250°, and retains this value up to the highest temperature, 338°, at which he observed it. Similarly butyric acid vapour has a density equal to 3.68 at 177°, which becomes 3.07, in conformity with the general law, at 261°, and remains constant at this value up to 330°.

M. Cahours ascertained that in the case of water, the greater number of compound ethers, ethylic, amylic, and methylic alcohol, and a large proportion of the volatile oils, especially the hydrocarburetted oils, the density of the vapour at 30° or 40° above the boiling point agrees very well with its theoretic density, determined on the supposition that its volume is represented by *two* or *four*.

But, on the other hand, in the case of the following acids derived from the alcohols, viz. the acetic, butyric, and valeric, and also of the essences of aniseed and fennel, in order to agree with the theoretic on the same supposition as to volume, the density of the vapour must be taken at temperatures considerably above the boiling point. "It is curious to see," adds M. Cahours, "that these bodies, which present such striking analogies in other respects, exhibit the same peculiarities in their molecular arrangement."

This subject has been since more fully examined by M.

* Comptes Rendus, tome xix. p. 771 (1844).

Bineau,* in reference to the vapours of acetic, formic, and sulphuric acid. For low temperatures he employed a method analogous to that of M. Despretz; for those ranging from 99° to 126° he had recourse to the method of M. Gay-Lussac, surrounding the graduated jar with saline solutions of different degrees of strength, and consequently boiling at different temperatures, while for the highest points he used the method of M. Dumas.

The following Table contains the results of his experiments in the case of acetic acid.

SERIES I.			SERIES II.			SERIES III.			SERIES IV.			SERIES V.		
t° .	f .	δ .	t° .	f .	δ .	t° .	f .	δ .	t° .	f .	δ .	t° .	f .	δ .
12.0	2.44	3.80	11.5	3.76	3.88	12.0	5.23	3.92	20.0	8.55	3.88	20.5	10.03	3.95
19.0	2.60	3.66	19.0	4.00	3.75	20.0	5.56	3.77	22.0	8.64	3.85	28.0	10.03	3.75
22.0	2.70	3.56	21.0	4.06	3.72	24.0	5.75	3.70	35.0	11.19	3.64
...	30.0	6.03	3.60	36.5	11.32	3.62

The first column in each series gives the temperature of the vapour, the second its elastic force, expressed in millimetres, and the third its corresponding absolute density. The change of the latter quantity with the temperature, even within the limits of the preceding experiments, is very striking. M. Bineau remarks that the results of the preceding series of experiments are more deserving of confidence in proportion as the corresponding elastic forces are greater, since any error in the measurement of these forces by means of the kathetometer, which might easily amount to some hundredths of a millimetre, will have less influence on the final results according as the forces themselves are greater.

The maximum tension of acetic acid vapour at 15° is about $7^{\text{mm}}.7$; at 22° , $14^{\text{mm}}.5$; and at 23° , 32^{mm} .

Where the elastic force and temperature of a vapour change together, any alteration in its *absolute density* may result from its following a different law from air, either in its compression or in its expansion by heat, or in both; but where the pressure remains constant, or nearly so, any considerable change in its density, as

* Annales de Chimie et de Physique (3^{me} Serie), tome xviii. p. 226.

referred to that of air under the same circumstances of temperature and pressure, must be referred to the difference of its dilatability by heat. To understand how its expansion for a given change of temperature may be compared with that of air in terms of the absolute densities at the limiting temperatures, conceive two equal volumes, v , of vapour and of air under the same pressure, f , and at the same temperature, t , and let their densities under those circumstances, referred to a fixed standard, be d and a . Suppose now, the pressure remaining unchanged, that the temperature of both is raised to t' , and that the volume of the vapour in consequence becomes v' , and that of the air v'' , their densities now being d' and a' . Further, let the absolute density of the vapour in the first instance be δ , and in the latter δ' .

Then if we represent by β the coefficient of expansion of the vapour for the change of temperature $t' - t$, as referred to the volume at the lower temperature, we have (22) $v' = v(1 + \beta)$, and if β' represent the analogous coefficient for air, $v'' = v(1 + \beta')$. Further, as the density of a body referred to a *fixed* standard varies inversely as the volume occupied by a given weight, we have

$$\frac{d'}{d} = \frac{v}{v'} = \frac{v}{v(1 + \beta)} = \frac{1}{1 + \beta} \quad \text{and} \quad \frac{a'}{a} = \frac{1}{1 + \beta'};$$

therefore

$$\frac{a'}{a} \cdot \frac{d}{d'} = \frac{1 + \beta}{1 + \beta'}.$$

But further,

$$\frac{d}{a} = \delta, \quad \text{and} \quad \frac{d'}{a'} = \delta';$$

therefore

$$\frac{d}{d'} \cdot \frac{a}{a'} = \frac{\delta}{\delta'},$$

and consequently

$$\frac{\delta}{\delta'} = \frac{1 + \beta}{1 + \beta'};$$

from which we obtain

$$\beta = \frac{\delta}{\delta'} (1 + \beta') - 1.$$

It is to be remembered that β' here represents the coefficient of expansion of air for $(t' - t)^\circ$, referred to the volume at t° as the

standard. We obtain the relation between this quantity and the coefficient for 1° referred to the volume at 0° , viz. 0.00366, from the formula in page 47, by substituting β' for $\kappa\tau$, $t' - t$ for τ , $a = 0.00366$ for k , and t for t' , which gives

$$\beta' = \frac{a(t' - t)}{1 + at}.$$

Or we may obtain the value of $(1 + \beta')$ directly thus:

$$v'' = v(1 + \beta');$$

but

$$v'' = v \frac{1 + at'}{1 + at},$$

where $a = 0.00366$; therefore

$$1 + \beta' = \frac{1 + at'}{1 + at};$$

and

$$\beta' = \frac{a(t' - t)}{1 + at}.$$

Substituting this expression in the value for β , we have

$$\beta = \frac{\delta}{\delta'} \cdot \frac{1 + at'}{1 + at} - 1.$$

Applying these formulæ to the results of the first and second experiments in Series V., we find $\beta = .0802$, and $\beta' = .0255$, proving that the increments of equal volumes at $20^\circ.5$ of acetic acid vapour and air, for a change of temperature equal to $7^\circ.5$, are in the ratio 3.1 : 1. At low temperatures, therefore, this vapour is very far indeed from following Gay-Lussac's law of equal expansion.

Comparing the correlative pressures and densities at the same temperatures in the above experiments, we find:

$t^\circ = 20^\circ.$		$t^\circ = 30^\circ.$	
$f.$	$\delta.$	$f.$	$\delta.$
4 ^{mm.} .0	3.74	6 ^{mm.} .0	3.60
5 .6	3.77	10 .7	3.73
8 .5	3.88
10 .0	3.96

The density, therefore, increases much more rapidly than it

should do according to Mariotte's law, and that at pressures much inferior to those of saturation.

At temperatures superior to 250° , however, it appears that both these laws, namely, Gay-Lussac's and Mariotte's, apply to this vapour, since M. Cahours has shown that at those temperatures the absolute density is a constant quantity.*

The results of M. Bineau's experiments on formic acid are contained in the following Tables:

A.

SERIES I.			SERIES II.			SERIES III.			SERIES IV.		
t° .	f .	δ .	t° .	f .	δ .	t° .	f .	δ .	t° .	f .	δ .
$15^{\circ}.5$	$2^{\text{mm}}.61$	2.86	$11^{\circ}.0$	7.26	3.02	$10^{\circ}.5$	14.69	3.23	$18^{\circ}.5$	23.53	3.23
20.0	2.72	2.80	15.0	7.60	2.93	12.5	15.20	3.14	22.0	25.17	3.05
$31^{\circ}.5$	3.04	2.60	20.0	7.99	2.85	16.0	15.97	3.13	29.0	27.40	2.83
...	$30^{\circ}.5$	8.83	2.69	20.0	16.67	2.94	$34^{\circ}.5$	28.94	2.77
...	24.5	17.39	2.86
...	30.0	18.28	2.76

B.

t° .	f .	δ .	t° .	f .	δ .	t° .	f .	δ .
$99^{\circ}.5$	690^{mm}	2.52	$101^{\circ}.0$	693^{mm}	2.44	$115^{\circ}.5$	649^{mm}	2.20
99.5	684	2.49	101.0	650	2.41	$115^{\circ}.5$	640	2.16
99.5	676	2.46	$105^{\circ}.0$	691	2.35	$117^{\circ}.5$	688	2.13
99.5	662	2.44	$105^{\circ}.0$	650	2.33	$118^{\circ}.0$	650	2.13
99.5	641	2.42	$105^{\circ}.0$	630	2.32	$124^{\circ}.5$	670	2.06
99.5	619	2.41	$108^{\circ}.0$	687	2.31	$124^{\circ}.5$	640	2.04
99.5	602	2.40	$111^{\circ}.5$	690	2.25	$125^{\circ}.5$	687	2.05
99.5	557	2.34	$111^{\circ}.5$	690	2.22	$125^{\circ}.5$	645	2.03

C.

t .	f .	δ .
$184^{\circ}.0$	750^{mm}	1.68
216.0	690	1.61

* If, however, as is probable, M. Cahours operated at those high temperatures under a pressure nearly constant, the constancy of the absolute density only proves the applicability of Gay-Lussac's law.

The experiments whose results are contained in Table A were made by means of an apparatus similar to that of M. Despretz, those in B with M. Gay-Lussac's, and those in C with the apparatus of M. Dumas.

The maximum tension of formic acid vapour at 13° is about 19^{mm} ; at 15° , 20^{mm} .5; and at 32° , 53^{mm} .5.

The values of β and β' , determined from the data in the first line of the first column, and the mean of the last two in the second column of Table B, are .1664 and .0322; proving that the dilatation of formic acid vapour, under the circumstances referred to, is five times greater than that of air.

Comparing the correlative pressures and densities at the same temperatures in Table A, we find the following results:

$t = 15^{\circ}$.		$t = 20^{\circ}$.		$t = 25^{\circ}$.		$t = 30^{\circ}$.	
f .	δ .	f .	δ .	f .	δ .	f .	δ .
2^{mm} .6	2.87	2^{mm} .7	2.80	2^{mm} .9	2.71	3^{mm} .1	2.61
7 .6	2.93	8 .0	2.85	8 .4	2.77	8 .8	2.70
15 .8	3.06	16 .7	2.94	17 .5	2.85	18 .3	2.76
.....	24 .2	3.15	26 .2	2.94	27 .8	2.81

Which prove the inapplicability of Mariotte's law to this vapour, even at pressures amounting only to one-half of the maximum.

For sulphuric acid M. Bineau obtained the following results:

t .	f .	δ .
332°	690^{mm}	2.50
345	708	2.24
365	745	2.12
416	735	1.69
498	725	1.68

"Theory assigns to the vapour of hydrated sulphuric acid the density 1.64, on the hypothesis of a combination of water and anhydrous acid without condensation. This value agrees very

well with the result of experiment at temperatures superior to 400° ."

The great densities of all the preceding vapours at low temperatures, so far superior to those which an extensive analogy would lead us to assign to them, and their enormous dilatability by heat, joined to the fact that this dilatability subsequently diminishes, contrary to all analogy, and ultimately becomes equal to that of air, appear to warrant the conclusion, that, even at temperatures extending far beyond their boiling points, their liquids are very incompletely vaporized. Their great dilatability would be explained, on this supposition, by the fact, that every addition of heat should tend to complete the imperfect vaporization, and develop a fresh quantity of vapour in the apparently already vaporized mass.

But when the temperature is attained at which the vaporization becomes complete, this process can no longer go on, and the dilatability should then become equal, or nearly so, to that of air and the other gases, as we find in fact it does.

It cannot be denied that it is difficult to conceive how this imperfect state of vaporization can coexist with the high degree of rarefaction observed in some of the preceding experiments, in which, nevertheless, the anomalies of which we have been speaking, in both density and dilatation, were as striking as under higher pressures. This difficulty does not, however, appear so great as to prevent our entertaining the hypothesis in question, for there is reason to believe that, even under the feeblest pressures, vapours undergo anomalous condensations throughout their mass, when near their point of saturation.

Some light might, probably, be thrown on this interesting subject by an examination of the latent heat of the vapours in question at different temperatures.

M. Bineau* appears inclined to the hypothesis, that these vapours admit of two distinct forms of molecular arrangement, one corresponding to the theoretic value of the density, the other in which the molecules have double the mass, and accordingly the

* Ann. de Chim. et de Phys. (3^{me} Serie), tome xviii. p. 242.

density double the value. He further supposes that at extreme temperatures one or other of these forms of grouping predominates, but that at all intermediate temperatures they are mixed. These extreme densities in the case of formic acid he conceives to be 1.59 and 3.18, and he accounts for the existence of densities superior to the latter, by referring them to the same cause which produces those anomalous condensations which occur, as we have mentioned, in the case of all vapours, near their point of saturation.

SECT. VI.—ON THE GRAPHIC CONSTRUCTION OF EXPERIMENTS, AND FORMULÆ OF INTERPOLATION.

135. *Determination of the most probable Values of observed Quantities, from the actual Results of Experiment.*—Having thus described the principal methods which have been employed for determining, by experiment, the elastic force and density of vapours at certain temperatures, it remains briefly to explain the manner in which the most probable values of those quantities are derived from the actual results of experiment, and also the method of calculating their values at different temperatures at which no observations may have been made.

For, as regards the first point, experience proves that errors are unavoidable even in the case of the simplest observations, and where all possible care has been taken to avoid everything which might interfere with the accuracy of the result. Thus in the direct observation of quantities depending on a single variable, as, for instance, of the temperature of a fluid, indicated by the point at which the mercury stands in the stem of a thermometer immersed in the liquid, or of the elastic force of a vapour, where this is given directly by the position of the mercury in a barometer tube, we find that a succession of observations of the same quantity, under circumstances as nearly the same as possible, always gives results differing somewhat from each other. It is accordingly a matter of importance to ascertain how the true value of the quantity under observation may be determined from the actual results of experiment, or, if absolute truth is unattainable

ble, to ascertain how the most probable value of this quantity, derivable from a given series of observations, may be deduced from them, and also to learn the probable limits of error.

In applying the calculus of probabilities to the solution of this question, it is assumed that all errors which result from defective methods, inaccurate instruments, and peculiarities of the observer, have either been wholly obviated, or reduced within such limits that they will counteract one another when different methods of observation are adopted, or when the same methods are used by different observers. Besides these errors, however, there exists another class, which, whether we attribute them to imperfections of the senses, or some other causes peculiar to each particular set of observations, unknown to us, and perhaps undiscoverable, are found by experience to be limited in their magnitude, and to be such as sometimes to cause the observed quantity to err by excess, and sometimes by defect, while the tendency of errors of the former class, on the contrary, is always in one and the same direction. Hence it is easy to see, in a general way, how a repetition of experiments, affected chiefly by errors of the latter class, enables us to obtain a more correct result. For if there were no *constant* errors, and if the observations were sufficiently numerous to embrace all possible combinations of the causes producing *irregular* errors, as those of the second class may be called, and if, moreover, these causes produced in all cases equal positive and negative errors, then the arithmetic mean of all the observed values would obviously be the true one, or indeed, more simply, the arithmetic mean of the *extreme* observed values. This supposition, however, of absolute exemption from *constant* errors, and of the existence of so numerous a series of experiments as to fulfil the preceding conditions with respect to *irregular* errors, is in fact never realized, and accordingly absolute truth is unattainable by any single method of observation. It is necessary, therefore, to multiply methods as well as observations, and to apply the calculus of probabilities to deduce from their results the most probable value of the observed quantity. This calculus furnishes the following propositions, which we subjoin without proof, for the information of the experimental student, referring him for their demonstration to a memoir by I. F. Encke, published in

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the second volume of Taylor's Scientific Memoirs. These propositions are applicable to the results of all those experiments in which we observe *directly* a quantity depending on a *single* variable, in which class are comprehended most of the experiments described in the present treatise. The practical rules for the calculation of the probable value of the observed quantity, so far as it can be determined from a given series of experiments, follow directly from the propositions themselves.

136. *Rules for determining the most probable Value of a single observed Quantity, deduced from the Method of least Squares.*

PROPOSITION I.—In a series of *equally good, direct* observations of a quantity depending on a *single* variable, that hypothesis with respect to the unknown quantity is most probable, which assigns it such a value that the sum of the squares of the remaining errors is a minimum.

Call the unknown quantity x , and suppose we have a series of m observations assigning the values $n, n', n'', \&c.$; by the preceding proposition the most probable value of x is that which renders the sum

$$(x - n)^2 + (x - n')^2 + (x - n'')^2, \&c.,$$

a minimum. This condition, by the ordinary rules of the differential calculus, gives

$$x = \frac{n + n' + n'' + \&c.}{m};$$

and accordingly,

(a). The most probable value of the unknown quantity is the arithmetic mean of the observed values.

Definition.—The *mean error* is that error which, if it alone were assumed in all the observations indifferently, would give the same sum of the squares of the errors as that which actually exists. It is denoted by the symbol ϵ_2 .

The value of the *mean error* is given by the equation

$$\epsilon_2 = \sqrt{\left(\frac{(n^2) - \frac{(n)^2}{m}}{m - 1} \right)},$$

in which (n^2) represents $n^2 + n'^2 + \&c.$, and $(n)^2 = (n + n' + \&c.)^2$.

Definition.—The *probable error*, r , of a particular class of observations, is that error below which there are as many errors less than itself as there are larger ones above it, so that there are as many cases in which the errors are less than r , as there are cases in which the errors are greater.

The *probable error* is connected with the mean error by the equation

$$r = 0.674489 \varepsilon_1;$$

this is the most probable value of r , but it may lie between the limits

$$\frac{r}{1 + \frac{0.476936}{\sqrt{m}}} \text{ and } \frac{r}{1 - \frac{0.476936}{\sqrt{m}}};$$

or $q. p.$

$$r \left(1 - \frac{0.476936}{\sqrt{m}} \right) \text{ and } r \left(1 + \frac{0.476936}{\sqrt{(m)}} \right).*$$

(3). The probable error of the arithmetic mean itself is

$$\frac{r}{\sqrt{m}}.$$

If, therefore, we represent the arithmetic mean of a set of observations by a , there is an equal chance that the true value of the unknown quantity lies between

$$a + \frac{r}{\sqrt{m}} \text{ and } a - \frac{r}{\sqrt{m}}.$$

PROPOSITION II.—If there be several sets of observations giving the probable values $a, a', a'', \&c.$, with the probable errors $r, r', r'', \&c.$, then

* If we denote by ε_1 the quotient obtained by dividing the sum of all the errors, without regard to their signs, by $(m-1)$, we obtain a less accurate value of r by means of the equation,

$$r = 0.845347 \varepsilon_1,$$

whose limits are

$$r \left(1 \pm \frac{0.509584}{\sqrt{m}} \right).$$

This value, as appears from the magnitude of the limits, is less accurate than that given in the text, but, owing to the simpler nature of the quantity ε_1 , it is much easier of calculation.

(a). The most probable value, A , derived from all these sets, is

$$A = \frac{\frac{a}{r^2} + \frac{a'}{r'^2} + \frac{a''}{r''^2} + \&c.}{\frac{1}{r^2} + \frac{1}{r'^2} + \frac{1}{r''^2}}.$$

(β). And the probable error, R , is

$$R = \frac{1}{\sqrt{\left(\frac{1}{r^2} + \frac{1}{r'^2} + \frac{1}{r''^2} + \&c.\right)}}.$$

PROPOSITION III.—If X represents a known function, f , of certain independent magnitudes, $x, x', x'', \&c.$, of which we know the probable values, $a, a', a'', \&c.$, and the probable errors, $r, r', r'', \&c.$, then

(a). The most probable value of X , which we will call A , is

$$A = f(a, a', a'', \&c.)$$

(β). And its probable error, R , is

$$R = \sqrt{\left\{ \left(\frac{dA}{da}\right)^2 r^2 + \left(\frac{dA}{da'}\right)^2 r'^2 + \&c. \right\}}$$

This proposition is rigorously true for linear functions, but only approximately so for higher ones.

PROPOSITION IV.—If X is expressed in terms of different sets of magnitudes, $a, a', a'', \&c., a', a', a'', \&c.$, which give, as in the last proposition, the probable values, $A, A, A, \&c.$, and the probable errors, $R, R, R, \&c.$, then, by Prop. II.,

(a). The probable value, A , derived from all these, is

$$A = \frac{\frac{A}{R^2} + \frac{A}{R'^2} + \frac{A}{R''^2} + \&c.}{\frac{1}{R^2} + \frac{1}{R'^2} + \frac{1}{R''^2} + \&c.}.$$

(β). And the probable error, R ,

$$R = \frac{1}{\sqrt{\left(\frac{1}{R^2} + \frac{1}{R'^2} + \frac{1}{R''^2} + \&c.\right)}}.$$

As an example of the application of the rules deducible from the first two propositions, we select the following sets of experiments by M. Regnault on the elastic force of aqueous vapour at 0° .

Number of Experiment.	Series <i>i</i> <i>f</i> =	Series <i>l</i> <i>f</i> =	Series <i>h</i> <i>f</i> =
1	4.54	4.66	4.54
2	4.54	4.67	4.54
3	4.52	4.64	4.54
4	4.54	4.62	4.58
5	4.52	4.64	4.58
6	4.54	4.66	4.57
7	4.52	4.67	4.58
8	4.50	4.66	
9	4.50	4.66	
10	4.54		

The value of *a* derived from Series *i* is $a = 4.526$, with the probable error $r = 0.0105$; from Series *l*, $a' = 4.653$, and $r' = 0.0105$; and from Series *h*, $a'' = 4.561$, and $r'' = 0.0127$. Hence (Prop. II.)

$$A = \frac{\frac{a}{r^2} + \frac{a'}{r'^2} + \frac{a''}{r''^2}}{\frac{1}{r^2} + \frac{1}{r'^2} + \frac{1}{r''^2}} = 4.582,$$

and

$$R = \frac{1}{\sqrt{\left(\frac{1}{r^2} + \frac{1}{r'^2} + \frac{1}{r''^2}\right)}} = .0064;$$

accordingly the most probable value of the elastic force at 0° , as far as it can be determined from the preceding series of experiments, is 4.582, and there is an equal chance of its possessing any value between 4.584 and 4.5756.

137. *Method of correcting the observed Values of a Series of Quantities dependent on a single Variable.*—The propositions contained in the last paragraph furnish us with the only known means of correcting the results of individual observations in the case of independent quantities; but if we have an extensive series of obser-

vations of two quantities, so *dependent* on each other that a determinate value of the one is always connected with a given value of the other, as in the case of elastic forces and temperatures, we possess in this case another means of correcting the results of experiments with respect at least to one of the observed quantities. This means is derived from the principle, that whenever, in the system of physical nature, we find two quantities connected with one another in the manner described, so that one is, in mathematical language, a *function* of the other considered as an independent variable, then if we vary the latter constantly in one direction, the former will vary constantly in one direction also. And accordingly, if the different values of the quantity which is chosen to represent the independent variable, are represented by abscissæ, and those of the other by the corresponding ordinates, the line on which the extremities of all those ordinates will be found, will either be a right line, or more frequently a curve of uniform progress, free alike from sinuosities and abrupt changes of direction. The only apparent exceptions to this rule occur in cases where, at a particular point or points, new forces are called into play; thus, as a general rule, we have seen that the molecules of all bodies approximate on decrease of temperature, but in the case of water and fusible metal, after this approximation has been carried to a certain extent, and has reached a certain limit, the forces which determine crystalline or polar arrangement appear to be brought into operation, either suddenly at different points, or gradually through the entire mass, and the decrease of temperature, if still continued, is accompanied by a remotion instead of an approximation of the molecules; and in a similar manner change of state is accompanied by an *abrupt* change of volume.

Unless in such singular cases, however, the curve passing through the extremities of the ordinates which represent the *true* values of the quantity under consideration, is a curve of uniform progress; and accordingly if we find that the extremities of ordinates representing the corrected or *mean* values of observations, lie some above and some below a curve of this description, this law both enables us to detect the existence of errors in the quantity represented by the ordinates, which might otherwise have remained undiscovered, and also furnishes us with a means of

correcting them. For if, having marked off on the axis of the abscissæ the values of the independent variable, we erect perpendicular ordinates representing the corresponding *mean* values of the other observed quantities, and then describe a curve which shall pass through one or two points determined with particular accuracy, and among, and as near to, the rest as possible, the ordinates of this curve may be regarded as representing the *true* values of the observed quantity, with more fidelity than the mean values derived from experiment. And if the observations have been uniformly distributed between the extreme limits, and have been sufficiently numerous, and if on tracing the curve we find that it passes between the extremities of the *mean* ordinates, above some and below others, we may feel confident that it exhibits, with a very close approximation to the truth, the relation between the quantity under investigation and the variable on which it depends.

For an error in any *mean* value arises from the number of observations from which it was derived not having been sufficiently great to include the fair proportion of positive and negative errors in the *observed* values, so that a *mean* will be in excess or defect according as positive or negative errors abound in the observations from which it was deduced; but from what has been said as to the nature of these errors it is evidently equally probable that any *mean* will deviate from the *true* value by excess as by defect, and consequently, if the observations have been conducted so as to admit only errors of the class alluded to, the curve of the *true* values should pass in the manner described among the *mean* values.

138. *Methods of Interpolation; by graphic Construction and Formulæ.*—The curve thus described serves not only to correct the mean values of observation, but also to determine the value of the quantity under investigation corresponding to any *assigned* value of the variable, and lying between any two values derived from experiment, since we have evidently merely to erect an ordinate at the extremity of a given abscissa, and by direct measurement ascertain the value of the quantity sought. The process of *interpolation*, however, is generally effected by finding the equation of the curve which coincides most nearly with the curve traced as above, and from this equation calculating the

required quantity. The form of this equation is in most cases arbitrarily chosen, in some instances, however, it is determined by theoretical considerations, and in others by some relation more or less clearly marked between the values of the quantity sought and the variable on which it depends. Such equations as these, inasmuch as they cannot be regarded as representing the physical connexion between the quantities in question, but merely as exhibiting the progress of the curves which represent their relation between the extreme limits of experiment, cannot be relied upon much beyond those limits, and from this restriction in their use they are frequently called *formulae of interpolation*, as they are also called *empirical formulæ*, from the tentative manner in which they are formed.

Having selected the particular form of equation which, on trial, is found to represent with most accuracy the general results of observation, the values of its constant coefficients may either be obtained by assuming it to pass through the requisite number of points on the graphic curve, or their most probable value may be determined from the results of experiments directly by means of the propositions in (136).

139. *Methods of calculating the Constants in Formulae of Interpolation.*—For suppose the equation is of the form

$$y = a (10)^{\frac{bx}{1+cx}},$$

in which y represents the elastic force, and x the corresponding temperature; then if we have from the graphic curve three values, y', y'', y''' , corresponding to x', x'', x''' , we obtain a, b, c from the three equations,

$$y' = a (10)^{\frac{bx'}{1+cx'}}, \quad y'' = a (10)^{\frac{bx''}{1+cx''}}, \quad y''' = a (10)^{\frac{bx'''}{1+cx'''}}.$$

It may be remarked that one point on the graphic curve of elastic forces is always given by the graduation of the thermometer employed in the experiments, namely, the extremity of the ordinate representing the pressure corresponding to the upper fixed point on the scale of temperatures; thus on the centigrade scale a force of 760^{mm} corresponds to 100°. A second point on the curve is

generally obtained by the determination of the force at 0° , as this temperature is capable of being produced with certainty and maintained invariable for any length of time. If, therefore, we select these two fixed points on the curve to give the values y' , x' , and y'' , x'' , we need only derive the other y'' , x'' , from the trace of the curve itself.

Or, as we have said, we may obtain the values of the coefficients a , b , c directly from the experiments, by the aid of the rules in (136). For, let y' be the elastic force corresponding to $x' = 0$; then we have $y' = a$, and therefore in the general equation,

$$y = y' (10)^{\frac{bx}{1+cx}},$$

leaving only b and c to be determined. Now let y''' represent the force 760^{mm} , corresponding to $x''' = 100^\circ$, and y'' the force at any intermediate point, x'' , then, substituting these values in the preceding equation, and solving for b and c , we obtain

$$b = \log \frac{y'''}{y'} \cdot \frac{\left(\frac{1}{x''} - \frac{1}{x'''}\right) \log \frac{y''}{y'}}{\log \frac{y'''}{y'} - \log \frac{y''}{y'}};$$

$$c = \frac{\frac{1}{x''} \log \frac{y''}{y'} - \frac{1}{x'''} \log \frac{y'''}{y'}}{\log \frac{y'''}{y'} - \log \frac{y''}{y'}}.$$

In these equations y' , y''' , and x''' , are supposed to be known with certainty; the only uncertain quantities are y'' , x'' . Suppose then that we select from the series of experiments m corresponding elastic forces and temperatures, whose probable values are a_1 , a'_1 ; a_2 , a'_2 ; . . . a_m , a'_m ; and their probable errors r_1 , r'_1 ; r_2 , r'_2 ; . . . r_m , r'_m ; substituting a_1 , a'_1 ; a_2 , a'_2 , &c. for y'' , x'' , we obtain, by Proposition III., m probable values for b and c , with their corresponding probable errors, and from these, by Proposition IV., we finally obtain the most probable values of those quantities, as far as they are determinable from the m selected observations.

140. *M. Regnault's Method of graphic Construction.*—The gra-

phic representation of the results of experiment is generally effected by laying down those results on a sheet of paper, divided by a series of engraved lines into a number of small squares. This paper, however, as it is generally met with in commerce, is never very accurately divided, partly owing to a want of care in tracing the divisions on the plate from which it is printed, and partly to the fact that, previous to taking the impressions, it is necessary to moisten the paper, which then contracts unequally in different directions on drying.

M. Regnault, accordingly, laid down his curves directly on a plate of copper, divided by himself with the greatest care. This plate was afterwards retouched by the graver, and the plates accompanying his memoirs were printed from it. On the copper plate M. Regnault first traced two lines strictly at right angles, adjacent to two edges of the plate. These lines were eight decimetres in length, and were each divided by an accurate machine into 100 parts, so that the value of each division was 8^{mm} . By this means the whole plate was divided into a number of small squares, the further subdivision of which was effected by means of a micrometric apparatus, which enabled him to appreciate the one-thousandth part of each of their sides.

In the construction of the curve of elastic forces of aqueous vapour, M. Regnault represented the temperatures by the abscissæ, and the forces by the vertical ordinates. "It was impossible," he remarks, "to represent these forces on the same scale through the whole range of the experiments. For at low temperatures a difference of 1° C. produces a variation in the elastic force of only a few millimetres, while at higher temperatures the same difference produces a change of many decimetres in the force. If, therefore, we adopt for the elastic forces a unit sufficiently large to render sensible the small variations exhibited by experiments at low temperatures, these forces would be represented, at high temperatures, by lengths so considerable that it would be impossible to introduce them into the same sheet, and the arcs of the curve would approximate so nearly to a right line, that their curvature would be imperceptible. If, on the other hand, we adopt a scale sufficiently small to represent con-

veniently the forces at high temperatures, the accidental variations of experiments at low temperatures would completely disappear.

“To avoid these inconveniences I have adopted three different scales to represent the elastic forces of vapour within different ranges of temperature, but these different vertical scales all correspond to the same horizontal scale of temperature, in which each centigrade degree is represented by one division of the axis of abscissæ.”

The first scale corresponds to the range from -33° to $51^{\circ}.6$; in this scale each millimetre of elastic force is represented on the vertical ordinate by one division whose absolute length is 8^{mm} .

The second scale embraces the forces corresponding to temperatures from 0° to 100° . In this each vertical division represents 10^{mm} .

The third extends from 100° to 232° ; each vertical division is equivalent to 100^{mm} ; between 100° and 197° the ordinates represent the elastic forces diminished by 760^{mm} ; and between 197° and 232° , the same forces diminished by 10760^{mm} .

Above 100° the mercurial thermometer does not strictly accord with the air thermometer; it was accordingly necessary to construct two sets of curves at these temperatures; in the one the abscissæ represent the indications of the mercurial, in the other of the air thermometer.

Every point determined by direct experiment is represented on the plate accompanying M. Regnault's memoir, by the centre of a small cross, which is further distinguished by a letter indicating the series of experiments from which it was derived. We may remark, that the correspondence between the mean ordinates determined by M. Regnault's experiments and the ordinates of the mean curve corresponding to the same abscissæ, is as close as might have been expected from the accuracy of his methods and his skill as an observer.

141. *Classification of Formulæ of Interpolation for the elastic Force of aqueous Vapour.*—The greater number of the formulæ which have been proposed to represent the relation between the elastic force of aqueous vapour and its temperature, have been based on Dalton's law formerly referred to (117), namely, that within

the limits of experiment the elastic forces of aqueous vapour are represented by the terms of a geometric series with constantly decreasing ratio, when the corresponding temperatures form an arithmetic series with equal differences.

For convenience of enumeration we may divide these formulæ into the following classes:

In the *first* the force is represented by the general term of a series, in which each successive term is formed by the multiplication of the preceding by the ratio diminished by a quantity depending on the number of the term. This form is a direct and simple algebraic statement of Dalton's law. The general expression is

$$y = y_0 a (a - b) (a - 2b) \dots (a - (x - 2) b),$$

in which y is the general term of the series of forces corresponding to the x^{th} term of the series of temperatures, y_0 the first term of the forces, a the first ratio, b the quantity by which each preceding ratio is constantly diminished.

In the *second* class the force is represented by the general term of a geometric series with constant ratio, modified by the addition of certain terms to the exponent of the ratio, so as to produce the retardation in the progress of the series required by Dalton's law. The general term of a geometric series with constant ratio being $y = y_0 r^x$, in which y , y_0 have the same signification as before, r is the constant ratio, and x the number of the term diminished by 2, the general expression of formulæ of the second class is

$$y = y_0 r^{a+bx+cx^2+dx^3+\&c.}$$

In the *third* class the force is represented by the sum of the general terms of different geometric series; the expression is of the form

$$y = a^{\mu+\lambda x} + a^{\mu'+\lambda'x} + a^{\mu''+\lambda''x} + \&c.$$

In the *fourth* class the retardation of the geometric series is effected by dividing the exponent of the ratio by a linear function of the same exponent; the general term has the form

$$y = y_0 a^{\frac{x}{1+\beta x^2+\gamma x^3+\&c.}}$$

In the *fifth* the forces are supposed to form the terms of an ordinary geometric series, when the temperatures constitute a similar series, but with a different ratio. The general expression in this case would be $y = Ax^m$, which, further generalized, becomes

$$y = y_0 (1 + bx)^m.$$

In the *sixth* the general term is formed by a combination of the forms in the second and third class; its general expression is

$$y = e^{a + Aat + B\beta t + C\gamma t + \&c.}$$

142. I. *Formulae whose general Expression is* $y = y_0 a (a - b) (a - 2b) \dots \{a - (x - 2)b\}$.

(i.) Mr. Dalton* employed a formula of this class, in which the temperature was not counted by degrees on any of the scales usually adopted, but by intervals of 5° Reaumur, or 11.25° Fahr., counted from 32° F. Let F represent the elastic force in English inches of mercury, corresponding to any temperature expressed by T of these intervals; f the force at 32° F.; then for temperatures above 32° ,

$$F = fa (a - b) (a - 2b) \dots \{a - (T - 1)b\},$$

and below 32° ,

$$F = \frac{f}{a' (a' + b) (a' + 2b) \dots \{a' + (T - 1)b\}};$$

in these expressions $f = 0.2$ inch,

$$a = 1.4872, \quad b = 0.01567, \quad a' = a + b = 1.5029.$$

(ii.) Mr. Ure† adopted a similar formula, differing from Mr. Dalton's merely in the value of the arbitrary degrees by which he expressed the temperature, and in the point from which he started. His degrees consisted of intervals of 10° F., and starting from 210° , the first ratio, $a = 1.23$, and the constant decrement $b = 0.01$.

* Manchester Memoirs, vol. v. Part ii.

† Philosophical Transactions, 1818, Part ii., p. 350.

Above 210° F. the expression, therefore, is,

$$F = fa(a-b)(a-2b) \dots \{a - (T-1)b\};$$

and below 210° ,

$$F = \frac{f}{a(a+b)(a+2b) \dots \{a + (T-1)b\}},$$

in which f , the force at 210° , = $28^{\text{in.}}9$, $a = 1.23$, $b = 0.01$.

To preserve the law of continuity in this formula, as was remarked before, the first divisor giving the force at 209° should be 1.24 instead of 1.23, and accordingly the values of a and b should be modified so as to exhibit this continuity in the formula.

Formulae of this class are very badly adapted for the purposes of interpolation. To obtain any term it is necessary to calculate all those preceding it; moreover, they cannot be immediately applied to fractional parts of the arbitrary units of temperature, for which they are constructed, and consequently will not give the forces corresponding to assigned degrees on any of the ordinary scales.

143. II. *Formulae whose general Expression is $y = y_0r^{ax+bx^2+cx^3+\&c.}$*

(i.) M. Laplace* first proposed a formula of this class, in which the exponent consisted of two terms. His formula is

$$F = f(10)^{pt - qt^2},$$

in which t , the temperature, is expressed in centigrade degrees counted from 100° , the boiling point of water under the pressure $f = 0^{\text{m.}}76$. These degrees are positive above 100° , negative below. F is expressed in metres of mercury. M. Laplace calculated the values of p and q from Mr. Dalton's experiments, and found $p = 0.015457$, $q = 0.0000625826$. Reduced to logarithms this expression becomes

$$\text{Log } F = \log 0^{\text{m.}}76 + 0.015457 t - 0.0000625826 t^2.$$

If we express the temperature by centigrade degrees counted from 0° , this formula becomes

$$\text{Log } F = \log 0^{\text{m.}}005123 + 0.0279712 t - 0.00006258526 t^2,$$

* *Mécanique Céleste*, tome iv. p. 273 (1805).

in which $0^{\text{m}}.005\ 123$ represents the force of aqueous vapour at 0° C.

(ii.) M. Biot* proposed a formula of this class, with three terms in the exponent. He calculated the values of the constants from Mr. Dalton's experiments. Expressing the force in English inches, and the temperature in Fahrenheit degrees, counted from 212° , positive descending, and negative ascending, his formula is

$$\text{Log } F = \log 30^{\text{in}} - 0.008\ 541\ 219\ 72\ t - 0.000\ 020\ 810\ 91\ t^2 \\ + 0.000\ 000\ 005\ 80\ t^3.$$

Referred to French measures this formula becomes

$$\text{Log } F = \log 0^{\text{m}}.76 - 0.015\ 372\ 787\ 57\ t - 0.000\ 067\ 319\ 95\ t^2 \\ + 0.000\ 000\ 033\ 74\ t^3,$$

where F is expressed in metres, and t in centigrade degrees, counted from 100° C., positive below this point, and negative above.

On calculating the values of the constant coefficients in M. Biot's formula, from Mr. Ure's observations, Mr. Ivory† obtained the following expression,

$$\text{Log } F = \log 30^{\text{in}} + 0.008\ 746\ 6\ t - 0.000\ 015\ 178\ t^2 \\ + 0.000\ 000\ 024\ 825\ t^3,$$

in which F is expressed in English inches, and t in Fahrenheit degrees, counted from 212° , positively ascending, negatively descending. If we desire to make the degrees positive descending, as in M. Biot's formulæ, it is only necessary to change the signs of t and t^3 .

This expression may be put under the form

$$\frac{\text{Log } \frac{F}{30}}{t} = a - bt + ct_2.$$

* *Traité de Physique*, tome i. p. 273. Colonel Boileau, Superintendent of the Hon. East India Company's Magnetic Observatory at Simla, has remarked, that in consequence of some error in M. Biot's calculation, the values of the coefficients, as

given above, are incorrect, and should be as follows: $a = -.008\ 541\ 170\ 60$, $b = -.000\ 020\ 812\ 37$, $c = +0.000\ 000\ 005\ 805$.

† *Philosophical Magazine* (New Series), vol. i. p. 1 (1827).

$\frac{F}{30}$ represents the force expressed in atmospheres of thirty inches, and the first member of the equation is the ratio of the logarithm of this force to the temperature. If this ratio were constant the elastic force would increase in geometric progression when the temperature increased in arithmetic; the terms affected by t and t^2 represent the deviation from this law.

144. III. *Formulæ whose general Expression is* $y = a^{\mu + \lambda x} + a^{\mu' + \lambda' x} + a^{\mu'' + \lambda'' x} + \&c.$

M. Prony* applied formulæ of this kind, with two, three, and four terms, to represent the experiments of M. Betancourt. As the results of these experiments, however, were not very accurate, we do not consider it necessary to give the numerical values of the constants as determined by M. Prony.

145. IV. *Formulæ whose general Expression is*

$$y = y_0 a^{\frac{x}{1 + \beta x + \gamma x^2 + \&c.}}$$

(i.) Professor August,† of Berlin, proposed a formula of this class, with two terms in the denominator of the exponent. It was, accordingly, of the form

$$F = f a^{\frac{t}{1 + \beta t}}.$$

In this formula there are three constants to be determined, f , a , and β ; f is evidently the elastic force corresponding to a temperature = 0° ; this being known, we only require two additional data to determine a and β ; one of these is given by the construction of the thermometric scale which we adopt, on which a certain arbitrary temperature is assigned to the vapour of water, when it has a certain pressure; the other may be derived from experiment. M. August, however, determined it from the supposition that vapour loses its elastic force at the absolute zero, which, as we have seen (62), has been fixed at $-266\frac{2}{3}$. On this supposition we have $F = 0$, when $t = -266\frac{2}{3}$, which gives $1 - 266\frac{2}{3}\beta$

* Nouvelle Architecture Hydraulique, § 1522.

† Poggendorff's Annalen, vol. xiii. p. 122 (1828).

= 0, and $\beta = 1 \div 266\frac{2}{3} = 3 \div 800$. Adopting the centigrade scale, we have $F = 0^m.76$, when $t = 100^\circ$. Substituting these values for F and t , and for f the value $0^m.005\ 057\ 8$, determined by M. Gay-Lussac, and for β the number $\frac{3}{800}$ as above, we have, to determine a , the equation

$$0^m.76 = 0^m.005\ 057\ 6\ a^{\frac{800}{11}},$$

whence

$$a = \left(\frac{0.76}{0.005\ 057\ 6} \right)^{\frac{11}{800}} = (150.260)^{\frac{11}{800}};$$

substituting this in the general equation, we have

$$F = 0^m.005\ 057\ 6 (150.263)^{\frac{11t}{800+3t}}.$$

Reduced to the logarithmic form, this expression becomes

$$\text{Log } F = \log (0.005\ 057\ 6) + \frac{11t}{800+3t} \log (150.263),$$

or

$$\text{Log } F = -2.296\ 0555 + \frac{23.945\ 371\ t}{800+3t};$$

and solving for t ,

$$t = \frac{800}{3} \cdot \frac{2.296\ 0555 + \log F}{5.685\ 735\ 0 - \log F}.$$

This formula represents the force of aqueous vapour with considerable accuracy, although we have reason to believe that the principle from which the value of β was derived is erroneous. This is owing to the circumstance, that the point at which aqueous vapour loses its elastic force, although not that assigned, is probably very low, and that the coefficient β is not very influential in the formula.

(ii.) M. Roche* has proposed a similar formula, whose constants, with the exception of one, he determined from certain

* Ferussac's Bulletin, tome xiv. (1830).

theoretic considerations, which are neither very precise nor accurate. The remaining constant he obtained by comparing his formula with the provisional table compiled by the Academy of Paris in 1823, prior to their experiments in 1829.

M. Roche's formula is

$$F = 10^{\frac{11nt}{11+0.03 \cdot t}},$$

in which F is expressed in atmospheres, and t in centigrade degrees, counted from 100° . To n he assigned the value 0.0155 ; the Commissioners subsequently found that the value 0.0149 agreed better with their experiments, and M. Roche finally adopted 0.0152 , which gives

$$F = 10^{\frac{0.167t}{11+0.03t}}.$$

The accordance of this formula with the results of experiment can only be explained, as in the preceding case, by the fact that the more important coefficients are determined, directly or indirectly, from experiment, while those derived from theoretical considerations are less influential.

(iii.) Professor Magnus* has also adopted a formula of this class, all the constants in which he has determined from his own experiments, applying, as in (139), the method of least squares. His formula is

$$F = 4.525 (10)^{\frac{7.4475t}{234.69+t}},$$

where F is expressed in millimetres and t in centigrade degrees, counted from 0° .

146. V. *Formulae whose general Expression is $y = y_0 (1 + bx)^m$.*

If, instead of supposing the elastic forces of vapour to constitute a geometric series with a decreasing ratio, when the temperatures form an arithmetic series, we assume the forces and temperatures to form the corresponding terms of two ordinary geometric series, with different constant ratios, we obtain for the force an expression of the form $F = At^m$: for the general term

* Taylor's Scientific Memoirs, vol. iv. p. 234.

in the series of forces is $F = ar^n$, and the corresponding term in the series of temperatures is $t = a'r^n$. Eliminating n we have

$$F = a \left(\frac{t}{a'} \right)^m = At^m,$$

where

$$m = \log \frac{r}{r'} \text{ and } A = \frac{a}{a'^m}.$$

(i.) Mr. Tregaskis* proposed a formula of this kind. He supposed $r = 2$ and $r' = 1.2$; hence $m = 3.8$, and counting the temperature on the centigrade scale, starting from 0° , and taking the force corresponding to 100° as unity, we have

$$F = 100^{-3.8} t^{3.8}.$$

This formula is not found to accord well with experiment. We have referred to it for the purpose of connecting the preceding formulæ with those which follow, and which may, in fact, be considered generalizations of it.

(ii.) Dr. Young† appears to have been the first to propose a formula of this latter class, whose general expression, as we have remarked, is $F = f(1 + bt)^m$. In this formula, as in all of the class, f represents the force at the point from which the temperature is reckoned. Counting from 32° F. , and using English measures, Dr. Young's formula‡ was

$$F = 0.1781 (1 + 0.006 \cdot t)^7.$$

(iii.) Tredgold§ adopted the following formula of this class,

$$F = \left(\frac{t + 100}{177} \right)^6,$$

in English measures, which, being reduced to the *metric* system, and the centigrade scale, becomes

$$F = \left(\frac{75 + t}{85} \right)^6;$$

* Edinburgh Journal of Science, No. 19.

† Natural Philosophy, vol. ii. p. 400.

‡ For low temperatures Dr. Young proposed the formula

$F = .18 + .007 t + .00019 t^2,$

t being counted as above.

§ Treatise on the Steam Engine (1838), p. 57.

in this F is expressed in centimetres of mercury, and t is counted from 0° .

All formulæ of this class admit of being simplified by taking as the unit the force at the assumed 0° , since f in that case becomes $= 1$, and the general form is $F = (1 + bt)^m$.

Thus in the preceding formula of Tredgold's, if we count the temperature from 100° , in units of 100° Cent., and take as the unit of force an atmosphere of seventy-six centimetres, the expression becomes

$$F = (1 + 0.5714 t)^6.$$

(iv.) M. Coriolis* proposed the formula

$$F = \left(\frac{1 + 0.1878 t}{2.878} \right)^{5.355},$$

in which the force is expressed in atmospheres, and the temperature in centigrade degrees, counted from 0° . If t be counted from 100° , this formula becomes

$$F = (1 + 0.006525 t)^{5.355},$$

and if t is counted in units of 100° ,

$$F = (1 + 0.6525 t)^{5.355}.$$

(v.) Mr. Southern† adopted a formula of this kind under the logarithmic form,

$$\text{Log } F = 5.13 \log (t + 51.3) - 10.94123$$

where t is expressed in Fahrenheit degrees, and F in English inches.

This formula is equivalent to

$$F = \left(\frac{1 + 0.01949 \cdot t}{2.647} \right)^{5.13},$$

which is of the same form as that proposed by M. Coriolis.

* *Mécanique des Corps solides* (1844),
p. 190.

† Robison's *Mechanical Philosophy*,
vol. ii. p. 172.

(vi.) The French Commissioners* also adopted a formula of this class,

$$F = (1 + 0.7153 \cdot t),$$

in which the force is expressed in atmospheres of $0^m.76$, and t is counted from 100° in units of 100 degrees.

They have remarked, however, that for the lower part of the scale Mr. Tredgold's formula agrees better with the results of experiments, and accordingly they have employed the two formulæ in the calculation of their tables,—Mr. Tredgold's, as far as four atmospheres, and their own beyond that pressure.

If, instead of assuming arbitrarily the exponent in the preceding formulæ, as has generally been done, we determine both it and the coefficient of the temperature from two extreme observations, as, for instance, from the forces at 0° and 224° , we obtain, as St. Avogadro† has shown, the expression

$$F = (1 + 0.5706 \cdot t)^{5.94},$$

which may be expected to accord better with the whole range of observations than either the Commissioners' or Mr. Tredgold's.

All formulæ of the class $F = (1 + bt)^m$ represent the force as becoming equal to cypher when $t = -\frac{1}{b}$; if we count the temperatures from this point we have

$$F = ct^m,$$

where $c = b^m$.

Applying this modification to the last formula, we obtain

$$F = 0.036 t^{5.94},$$

where t is counted from $-75^\circ.25$ C. in units of 100 degrees.

(vii.) The American Commissioners‡ found the results of their experiments represented by the formula

$$F = (1 + .00333 t)^6,$$

in which F is expressed in atmospheres and t in Fahrenheit degrees, counted from 212° .§

* Annales de Chimie et de Physique, tome xliii. p. 107 (1830).

† Fisica de' Corpi ponderabili, tomo iv. p. 325.

‡ Journal of the Franklin Institute, vol. xxii. p. 296 (1836).

§ It is unnecessary to do more than allude to a formula of this class, proposed by

147. VI. *Formulae whose general Expression is* $y = e^{a + Aa^x + B\beta^x + \&c.}$

—M. Biot has given in the *Connaissance des Temps* for 1844 a table of the elastic force of aqueous vapour, calculated by means of a formula of this class. His formula is

$$\text{Log } F = a - ba^t - c\beta^t.$$

He has based the determination of the five constants on experiments of M. Gay-Lussac's between -20° and 100° , and on those of MM. Dulong and Arago between 100° and 220° . Their values are

$$\begin{aligned} a &= 5.961\ 313\ 302\ 559 \\ \text{Log } b &= 1.823\ 406\ 881\ 930 \\ a &= -0.013\ 097\ 342\ 951 \\ \text{Log } c &= 0.741\ 109\ 518\ 370 \\ \beta &= -0.002\ 125\ 105\ 843 \end{aligned}$$

In this formula t is supposed to be counted from -20° on an *ideal* air thermometer, whose indications are derived from those of the mercurial thermometer by the relations assigned by MM. Dulong and Petit.† This thermometer is supposed to mark 100° at a temperature corresponding to that of aqueous vapour, whose elastic force equilibrates the weight of a column of mercury equal to 760^{mm} at 0° , under the influence of gravity at Paris.

148. *M. Regnault's Formulae of Interpolation.*—M. Regnault has employed three formulæ of interpolation adapted to different parts of the thermometric scale.

(i.) Between -32 and 0° he has employed the formula

$$F = a + ba^\tau, \quad (\text{E})$$

in which τ is reckoned from -32 , so that $\tau = t + 32^\circ$. The con-

Mr. Alexander in the *Philosophical Magazine* for January and February, 1849. The reasoning by which the author attempts to deduce the constants from theoretical considerations is a mass of mathematical and physical blunders, and the formula itself fails to exhibit with accuracy the results of the most trustworthy experiments at high temperatures.

† Additions à la *Connaissance des Temps*, p. 8 (1844). In reducing the indications of the mercurial thermometer to those of the ideal air thermometer, M. Biot has assumed that the indications of different mercurial thermometers are always strictly comparable, which, however, M. Regnault has shown not to be the case.

stants were determined from the three following data given by the graphic curve,

$t = -32$	$\tau = 0$	$F = 0^{\text{mm}}.32$
$t = -16$	$\tau = 16$	$F = 1.29$
$t = 0$	$\tau = 32$	$F = 4.60$

hence

$$\begin{aligned}\text{Log } b &= \bar{1}.602\,472\,4 & \text{Log } a &= 003\,398\,0 \\ a &= 0.080\,38\end{aligned}$$

(ii.) From 0° to 100° he adopted the formula

$$\text{Log } F = a + b\alpha' - c\beta', \quad (\text{D})$$

similar to M. Biot's in (147). In this t is counted from 0° , and the constants were determined from the following data of the graphic curve:

$t = 0$	$F = 4^{\text{mm}}.60$
$t = 25$	$F = 23.55$
$t = 50$	$F = 91.98$
$t = 75$	$F = 288.50$
$t = 100$	$F = 760.00$

Hence

$$\begin{aligned}\text{Log } a &= 0.006\,865\,036 \\ \text{Log } \beta &= \bar{1}.996\,724\,9 \\ \text{Log } b &= 2.134\,033\,9 \\ \text{Log } c &= 0.611\,648\,5 \\ a &= + 4.738\,438\,0\end{aligned}$$

(iii.) From 100° to 230° , the formula which he used was

$$\text{Log } F' = a - b\alpha' - c\beta', \quad (\text{H})$$

in which $\tau = t + 20$, t being the centigrade temperature counted from 0° . This formula might also have been employed to calculate the forces through the whole range from -20° to 230° , as it represents them with considerable fidelity, except that between -20° and 40° it gives values a little less than the true ones.

The data for the calculation of the constants were,

$t = -20$	$F' = 0^{\text{mm}}.91$
$t = 40$	$F' = 54.91$
$t = 100$	$F' = 760.00$

$$\begin{array}{ll} t = -160 & F = 4651^{\text{mm}}.60 \\ t = 220 & F = 17390 .0 \end{array}$$

and hence

$$\begin{aligned} \text{Log } a &= \bar{1}.994\ 049\ 292 \\ \text{Log } \beta &= \bar{1}.998\ 343\ 862 \\ \text{Log } b &= 0.139\ 774\ 3 \\ \text{Log } c &= 0.692\ 435\ 1 \\ a &= 6.264\ 034\ 8 \end{aligned}$$

In these three formulæ the temperatures are supposed to be counted on the air thermometer, which for the first two (E), (D), may be supposed to coincide with the mercurial.

149. *Formulæ for the elastic Force of the Vapours of various Liquids.*—Sr. Avogadro* has found that the elastic force of the vapour of mercury is represented with considerable accuracy, between the limits of his experiments, by the formula

$$\text{Log } e = -0.646\ 37\ t + 0.075\ 956\ t^2 - 0.184\ 52\ t^3,$$

in which e represents the elastic force expressed in atmospheres of 760^{mm}, and t the temperature as given by his mercurial thermometer, in units of 100° C., counted from 360°, the boiling point of mercury, positively towards 0°.

The same author has calculated the following formulæ for the elastic forces of the vapours of alcohol, spirits of turpentine, and ether, from the experiments of M. Despretz:†

For alcohol,

$$\text{Log } F = \log 0^{\text{m}}.76 + 0.018\ 023\ t - 0.000\ 146\ 34\ t^2;$$

For spirits of turpentine,

$$\text{Log } F = \log 0^{\text{m}}.76 + 0.011\ 608\ t - 0.000\ 091\ 083\ t^2;$$

For ether,

$$\text{Log } F = \log 0^{\text{m}}.76 + 0.014\ 848\ 3\ t - 0.000\ 014\ 595\ t^2.$$

In these formulæ F is the elastic force in metres, t the temperature in centigrade degrees, counted from the respective boiling points of the liquids, positively, ascending.

Comparing this latter formula with that for the elastic force of aqueous vapour similarly expressed, namely (p. 235),

* Avogadro, *Fisica*, tomo iv. p. 420.

† Ibid., p. 416.

$$\text{Log } F = \log 0^{\text{m}}.76 + 0.015\,372\,8\,t - 0.000\,067\,32\,t^2,$$

we see the approximation towards Dalton's law of equal pressures at temperatures equidistant from the boiling point, exhibited in the case of water and ether, as the coefficients of the first power of the temperature are nearly equal in those two formulæ.

As we have mentioned (138) that formulæ of interpolation can only be regarded as exhibiting the curve of values of the unknown quantity *within* the limits of experiment, we have not thought it necessary to discuss the form of the curves represented by the various formulæ in the preceding pages, *beyond* those limits, or to notice the singular points which they present. Any person desirous of pursuing this subject will find it fully discussed by Signor Avogadro in his valuable work on the physics of ponderable bodies.*

150. *Explanation of Tables I., II., III. (i.) Method of expressing the Temperature in these Tables.*—We now proceed to explain the construction and arrangement of the following tables.

The first three tables give the elastic force of aqueous vapour within different limits of temperature, calculated from M. Regnault's experiments.

In these tables the temperature is supposed to be determined by means of a standard air thermometer, corrected for the expansion of its envelope, and graduated according to Fahrenheit's scale, the point marked 212° on which corresponds to the temperature of the vapour of water boiling under a pressure equal to that which a column of mercury thirty inches high at 32° F. would produce under the equator and at the level of the sea.

(a.) *Material of standard Thermometer.*—We have selected as the standard an *air thermometer*, because, as has been previously remarked, it is the only one whose indications can be relied upon at high temperatures. This instrument may be constructed either on the principle of the expansion of air by heat under a constant pressure, or on that of the increase of its elastic force owing to the same cause, under a constant volume. Of these two principles, M. Regnault recommends the adoption of the latter. An air thermometer, constructed according to this method, will

* Avogadro, Fisica, tomo iv. pp. 266–355, 405–432.

resemble in its arrangement the form of apparatus represented in Figs. 36, 37, and described in page 84. Where the thermometer is designed to measure very high temperatures, M. Regnault recommends that it be filled, in the first instance, with highly rarefied air, so that, when heated, its elastic force shall not much exceed that of the surrounding medium. In this way we obviate all danger of alteration in the form and volume of the reservoir, arising from excess of internal pressure at high temperatures, when the material of the envelope might be partially softened by heat.

The value of the coefficient of dilatation of air for 1° of our standard thermometer is 0.002 036. This is the quantity which should be employed in calculating the temperature from the data of the instrument. If we adopt in its construction the principle of the expansion of air under a constant pressure, the value of the coefficient would be a little greater, namely, 0.002 038 7.*

Where extreme accuracy is required, the amount of the correction to be applied for the expansion of the envelope should be determined by previous experiments in each particular case. In most cases, however, it will be sufficient to derive this correction from Table VI., which gives its amount for flint glass, and for such specimens of ordinary or crown glass as do not contain a sensible quantity of lead.

Although the best adapted for a standard, however, the air thermometer is not at all fit for general use. Every observation made with this instrument is in fact an experiment. For general use, accordingly, we must have recourse to the mercurial thermometer, graduated, however, not independently, but by comparison with a standard instrument. Between 32° and 212° , indeed, the indications of a mercurial thermometer, graduated independently, may be considered as identical with those of one filled with air; but beyond 212° , as M. Regnault has shown, the peculiar nature of the glass forming the envelope of the mercurial thermometer affects its indications in such a manner as to render a

* These values are obtained from the coefficients corresponding to 1° of M. Regnault's thermometers, scil. 0.003 665 and 0.003 67, by multiplying them by the fac-

tor 0.555 497, which, as we shall see in p. 250, expresses the ratio of a degree on our thermometer to one on his.

particular correction necessary for each instrument. Accordingly, the indications of a mercurial thermometer should be corrected by direct comparison with an air thermometer in all cases in which extreme accuracy is sought to be obtained; in others it may be sufficient to apply a correction depending on the general nature of the glass of which the envelope is formed, the amount of which is given for four different kinds of glass in the seventh of the following tables.

(b.) *Method of Graduation.*—It is with considerable reluctance that we have employed in these tables Fahrenheit's scale, the only advantage possessed by which, namely, the convenient magnitude of its unit, is far more than counterbalanced by the inconvenient position of its zero, and the arbitrary number of divisions introduced between the fixed points. British men of science, however, appear so unwilling to adopt, even in scientific treatises, a system different from that universally prevalent among practical men, that we have felt we should only diminish the utility of the following tables by deviating from established usage in this respect.

(c.) *Method of determining upper fixed Point.*—As the temperature of the vapour of boiling water varies with the pressure to which it is submitted, and as the weight of a mercurial column of a given height, by which this pressure is usually measured, varies with the force of gravity, at different latitudes, and different heights above the mean level, it follows that in order to render the upper fixed point in the graduation of the thermometer absolutely determined, we should specify the latitude and elevation at which a mercurial column of given height is supposed to represent the normal pressure. We have selected for this purpose the latitude of 0° , and the level of the sea, and accordingly the normal pressure adopted for the graduation of our standard thermometer is that which a column of mercury of thirty inches high, at the temperature of melting ice, would have at the level of the sea, under the equator.

To find the height of the corresponding column at any latitude and at the same level, it is only necessary to multiply 30^{i} by the ratio of the force of gravity at the equator to its force at the given latitude. In this way we can readily find the barometric pressure

at which a thermometer should be graduated at any latitude, so as to agree with our standard. As it is impossible, however, without the help of apparatus similar to that represented in Fig. 60, to command the requisite pressure, the following method of determining the point on a given mercurial thermometer, corresponding to 212° of the standard, will be found at once most accurate and most convenient.

(d.) *Method of finding the Point on a given mercurial Thermometer corresponding to 212° of the Standard.*—The stem of the thermometer being divided into portions of equal length, marked on the tube itself, and the point corresponding to 32° carefully determined, the instrument is plunged in the vapour of boiling water, in a vessel similar to that represented in Fig. 34, so that the whole of the mercurial column is brought to the temperature of the vapour. The division on the tube at which the mercury stands is then noted. The height of the barometric column and its temperature are next observed, and this height is reduced to the equivalent at 32° by means of the fifth table.

If the place of observation is at any considerable height above the level of the sea, the length of the barometric column at 32° must be reduced to its equivalent at that level, by multiplying it by the ratio of the forces of gravity at those two positions. This ratio is given by the expression*

$$g = g' \left(1 + \frac{5h}{4r} \right),$$

in which g is the force of gravity at the sea-level, g' the force at the height h , and r the mean radius of the earth. Accordingly if l represents the length of the barometric column at the height h above the mean radius, the length of the column of equal weight at the sea-level will be

$$= l \frac{g'}{g} = \frac{l}{\left(1 + \frac{5h}{4r} \right)}, \text{ or } q. p. = l \left(1 - \frac{5h}{4r} \right).$$

This length is next to be reduced to the length of a column of equal pressure at the equator, by multiplying it by the ratio

* Poisson, *Mecanique*, tome i. p. 459.

of the force of gravity at the place of observation to its force at the equator. This ratio is given by the formula

$$G' = G (1 + n \sin^2 \lambda),$$

in which G' represents the force at the latitude λ , G the force at the equator, and

$$n = 0.0053132; \log n = \bar{3}.7253561640.*$$

So that if l be the length of the barometric column, reduced to 32° , at the place of observation, and L the length of the column of equal pressure at the sea-level at the equator, we have

$$L = l \left(1 - \frac{5h}{4r} \right) (1 + n \sin^2 \lambda).$$

Having thus found the pressure, in inches of mercury at the equator, under which steam is formed at the time of observation, a reference to Table III. gives the corresponding temperature as marked by our standard thermometer. This temperature, therefore, is that corresponding to the point noted on the thermometer under comparison, and hence the value of each division on its stem, and the point on it corresponding to 212° on the standard, are readily found.

ii. *Method of expressing elastic Forces.*—The elastic forces of aqueous vapour are represented in the following tables by the lengths of mercurial columns of equal pressure, both at the equator and also at the latitude of Dublin ($53^\circ 21' N.$) From the former, namely, the lengths of mercurial columns which would equilibrate the elastic force of the vapour at the equator, the corresponding lengths at any latitude may readily be obtained by dividing the values given in the tables by the quantity $(1 + 0.0053132 \sin^2 \lambda)$. For Great Britain the values contained

* This value of n is determined, by means of Clairaut's theorem, from the value of the earth's ellipticity, scil. $1 \div 299.7$, deduced by Bessel from a comparison of the most recent arc-measurements. The value of the mean radius, according to the same authority, is 20888022.077 English feet, or 6366554.09 metres.—*Taylor's Scientific*

Memoirs, vol. ii. p. 387. The value of n , deduced from Bowditch's formula for the length of the pendulum vibrating seconds at various latitudes, does not differ much from the preceding; it is $n = 0.0053329$.—*Bowditch's Laplace*, vol. ii. p. 284.

or within the chamber shall be graduated at a rate corresponding to that of the Standard.—The thermometer being filled and portions of equal weight of the liquid, and the point corresponding to the zero of the instrument is placed in the vessel, in a vessel similar to that represented in the sketch of the mercurial column is brought to the surface of the liquid. The division on the tube at which the liquid stands is noted. The height of the barometer is then observed, and this height is reduced to its equivalent at sea-level by means of the fifth table.

If the place of observation is at any considerable distance from the level of the sea, the length of the barometer must be reduced to its equivalent at that level by the ratio of the forces of gravity at the two places. This ratio is given by the expression*

$$g = g' \left(1 + \frac{2h}{R} \right),$$

in which g is the force of gravity at the sea-level, h the height of the place of observation above the sea-level, and R the mean radius of the earth. If l represents the length of the barometer at the place of observation, the length of the barometer at the sea-level will be

$$= l \frac{g'}{g} = \frac{l}{\left(1 + \frac{2h}{R} \right)}, \text{ or } g \cdot g' = 1 \left(1 + \frac{2h}{R} \right).$$

This height is now to be reduced to the level of the sea by multiplying by the ratio of the forces of gravity at the two places, by multiplying

* See also, *Physical Geography*, p. 100.

to 1,

ues assigned by
(E), having been

in the column for Dublin will be found sufficiently accurate, as its latitude is not far from the mean latitude of these islands.

With respect to the degree of accuracy with which the pressures are sought to be expressed, it is obvious that it is unnecessary to affect a greater accuracy than our means of measuring temperatures will enable us to realize. Thus if the smallest change of temperature which a thermometer can estimate is the one-tenth of a degree Fahr., and if the change of pressure corresponding to this change of temperature at any part of the scale is 0.001 , it is clearly useless to carry the values of the pressures beyond three decimal places at that part of the scale; and as the change of pressure for 1° , under those circumstances, will be 0.01 , it follows that we may omit the fourth decimal as soon as the difference of the pressures expressed with four decimals equals 100 ; and similarly, if the smallest appreciable change of temperature is one-twentieth of a degree, we may drop the last decimal where the difference for 1° equals 200 ; in general, if $1 \div n^{\text{th}}$ of a degree is the smallest quantity capable of being estimated by the thermometer, we may omit the last decimal as soon as the difference for 1° equals $10n$ of the units expressed by figures in that place. We have supposed one-fiftieth of a degree Fahr. to be the smallest change of temperature capable of being determined with certainty,* and have, accordingly, in the following tables retained the last decimal figure until the change for 1° exceeded 500 .

(iii.) *Method of Calculation.*—It now remains to explain how M. Regnault's formulæ were adapted to the calculation of the following tables. In the first place it was necessary to determine the point on his thermometers corresponding to 212° of our standard, that is, the temperature as marked by them, corresponding to a pressure equal to that of thirty inches at the equator. Now thirty inches equal $761^{\text{mm}}.9862$, and a column of this length at the equator is equivalent to one of $759^{\text{mm}}.69827$ at the level of the sea at Paris, and to one of $759^{\text{mm}}.7072$, at the observatory, which is about sixty metres above that level. On reference to

* It may be remarked that M. Regnault states that one of his thermometers was capable of indicating with accuracy a change

of $1 \div 2000$ th of a degree Cent., or about $1 \div 100$ th of a degree Fahr.

M. Regnault's tables, we find that a pressure of $759^{\text{mm}}.707$ corresponds to a temperature of $99^{\circ}.9895$, which is, accordingly, the point corresponding to 212° of our standard, and hence 1° of the latter equals $0^{\circ}.555\,497$ Cent., and $t^{\circ}\text{C.} = .555\,497\,T^{\circ}\text{Fahr.}$

In Regnault's formulæ the elastic forces are expressed by the length in millimetres (F) of the equilibrating column of mercury, at the latitude of Paris ($48^{\circ}50'14''$) and at the height of sixty metres above the level of the sea. The equivalent length in English inches (i) at the level of the sea, and under the equator, is given by the expression

$$i = F(0.039\,370\,79) \frac{1 + n \sin^2 48^{\circ}50'14''}{1 + \frac{5h}{4r}}$$

since $1^{\text{mm}} = 0.039\,370\,79$.

Substituting the values for n , h , and r , we get

$$i = F(0.039\,488\,9),$$

and $\log i = \log F + 2.596\,474\,838\,2$.

Hence formula (D), namely,

$$\text{Log } F = a + b\alpha^t - c\beta^t,$$

becomes

$$\text{Log } i = 2.596\,474\,838\,2 + a + b\alpha^{.555\,497\,T} - c\beta^{.555\,497\,T},$$

or

$$\text{Log } i = a_{\prime\prime} + b_{\prime\prime}\alpha_{\prime\prime}^T - c_{\prime\prime}\beta_{\prime\prime}^T,$$

where

$$a_{\prime\prime} = a + 2.596\,474\,838\,2$$

$$\text{Log } a_{\prime\prime} = .555\,497 \log a,$$

$$\text{Log } \beta_{\prime\prime} = .555\,497 \log \beta;$$

and T denotes the number of degrees Fahr., corresponding to t , and counted from the same zero.

To express the force in inches i , at Dublin we have

$$i = \frac{i}{1 + n \sin^2 53^{\circ}.21} = \frac{1}{1.003\,420\,003};$$

hence

$$\text{Log } i = \log i - 0.001\,482\,752\,7.$$

M. Regnault remarks that the numerical values assigned by him to the constants in the formulæ (D), (H), (E), having been

calculated by means of logarithmic tables extending only to seven places of decimals, are not as accurate as if they had been computed by the help of more extended tables. Previously, therefore, to adapting those formulæ to English measures, we calculated the values of their constants from Vlacq's tables, in which the logarithms are given to ten places of decimals. The following are the resulting values of the constants:*

In Formula (D).

$$\begin{aligned} a &= 4.739\,389\,852\,7 \\ \text{Log } a &= 0.006\,864\,921\,1 \\ \text{Log } \beta &= 1.996\,725\,549\,5 \\ \text{Log } b &= 2.131\,947\,611\,0 \\ \text{Log } c &= 0.611\,742\,663\,0 \end{aligned}$$

In Formula (H).

$$\begin{aligned} a &= 6.263\,509\,686\,5 \\ \text{Log } a &= 1.998\,343\,377\,8 \\ \text{Log } \beta &= 1.994\,048\,173\,7 \\ \text{Log } b &= 0.692\,450\,419\,2 \\ \text{Log } c &= 0.139\,553\,958\,4 \end{aligned}$$

In Formula (E).

$$\begin{aligned} a &= -0.082\,094 \\ \text{Log } b &= 1.604\,327\,6 \\ \text{Log } a &= 0.033\,316\,014 \\ b &= 0.402\,094 \\ a &= 3.412\,371\,1 \end{aligned}$$

These values reproduce the data from which the constants are derived, with a much nearer approach to accuracy than those given by M. Regnault.

We are now prepared to adapt M. Regnault's formulæ to English measures and our standard thermometer.

Formula (E), which ranges from -32°C. to 0°C. , becomes

$$i = a_{\prime\prime} + b_{\prime\prime} a_{\prime\prime}^{-T}, \quad (\text{E}_{\prime\prime})$$

in which

$$\begin{aligned} a_{\prime\prime} &= -0.003\,241\,802 \\ \text{Log } a_{\prime\prime} &= 0.018\,506\,945\,829 \\ \text{Log } b_{\prime\prime} &= 1.266\,914\,897 \end{aligned}$$

* As some typographical errors have crept into the equations given by M. Regnault (Mem. de l'Inst., tome xxi. p. 596), for the determination of the constants α , and β , in the formula

$$\text{Log } F = a + b\alpha^t + c\beta^t,$$

we subjoin the correct expressions. α and β are the roots of the quadratic $z^2 - Mz +$

$N = 0$, in which

$$\begin{aligned} M &= \frac{\Delta y_1 \Delta y_2 - \Delta y_0 \Delta y_3}{(\Delta y_1)^2 - \Delta y_0 \Delta y_2}, \\ N &= \frac{(\Delta y_2)^2 - \Delta y_1 \Delta y_3}{(\Delta y_1)^2 - \Delta y_0 \Delta y_2}, \end{aligned}$$

and

$$\text{Log } \alpha = \frac{1}{n} \log \alpha, \quad \text{Log } \beta = \frac{1}{n} \log \beta.$$

In this formula T is counted from 32° F., positively, downwards; it ranges from $T = 0$ to $T = 64$, or from 32° F. to -32 F.

Formula (D), which applies between 0° and 100 C., becomes

$$\text{Log } i = a_{\text{,,}} + b_{\text{,,}} a_{\text{,,}}^T - c_{\text{,,}} \beta_{\text{,,}}^T, \quad (D_{\text{,,}})$$

where

$$\begin{aligned} a_{\text{,,}} &= 3.335\,864\,690\,9 \\ \text{Log } a_{\text{,,}} &= 0.003\,813\,443\,076 \\ \text{Log } \beta_{\text{,,}} &= 1.998\,181\,052\,570 \\ \text{Log } b_{\text{,,}} &= 2.131\,947\,611\,000 \\ \text{Log } c_{\text{,,}} &= 0.611\,742\,663\,000 \end{aligned}$$

In this formula, which answers between the limits 32° and 212° F., T denotes the number of Fahr. degrees, counted from 32° positively, upwards.

Formula (H) becomes

$$\text{Log } i = a_{\text{,,}} - b_{\text{,,}} a_{\text{,,}}^T - c_{\text{,,}} \beta_{\text{,,}}^T, \quad (H')$$

in which

$$\begin{aligned} a_{\text{,,}} &= 4.859\,984\,524\,7 \\ \text{Log } a_{\text{,,}} &= 1.999\,079\,751\,3 \\ \text{Log } \beta_{\text{,,}} &= 1.996\,693\,778\,3 \\ \text{Log } b_{\text{,,}} &= 0.659\,317\,975\,2 \\ \text{Log } c_{\text{,,}} &= 0.020\,517\,432\,4 \end{aligned}$$

T represents degrees Fahr., counted from 32° F., and this formula has been employed between the limits 212° and 432° .

A break will be observed in the continuity of the first and second tables at the pressure corresponding to 32° ; this is owing to the circumstance that the curve represented by formula $(D_{\text{,,}})$ cuts the curve represented by $(E_{\text{,,}})$ at a sensible though very small angle.

In the second and third tables the pressures corresponding to whole degrees were calculated from the formulæ; the values for the intermediate fractional parts were obtained from the formula of interpolation,*

$$u' = u + \frac{h'}{h} \Delta u + \frac{h' (h' - h)}{h \cdot 2h} \Delta^2 u + \frac{h' (h' - h) (h' - 2h)}{h \cdot 2h \cdot 3h} \Delta^3 u,$$

in which $h = 1$, $h' = 0.1, 0.2, 0.3$, &c., and Δu , $\Delta^2 u$, $\Delta^3 u$, are the first, second, and third differences.

151. *Description of remaining Tables.*—In Table IV. we have

* Lacroix, Calcul. Diff. et Integr., tome iii. p. 24.

given the values of $(1 + n \sin^2 \lambda)$, and also of its reciprocal for every five degrees of latitude from the equator to the pole, by means of which the length of a column of mercury at the equator equivalent to a given column at any latitude, and *vice versa*, may be readily found with a degree of accuracy sufficient for most cases.

In Table V. we have given the values of the absolute dilatation of mercury, and of its mean coefficient for 1° F. for every twenty degrees of our standard thermometer. This table has been calculated from the formula in p. 70, which, being adapted to our thermometer, becomes

$$\delta_T = a_{\text{m}} T + b_{\text{m}} T^2,$$

where T is the temperature in degrees Fahr., counted from 32° , and

$$\text{Log } a_{\text{m}} = 5.997\,550\,7$$

$$a_{\text{m}} = 0.000\,099\,437\,6$$

$$\text{Log } b_{\text{m}} = 9.891\,307\,5$$

$$b_{\text{m}} = 0.000\,000\,007\,785\,88$$

This table is useful for reducing the length of the mercurial column at any temperature to its equivalent at 32° F. For this purpose it is necessary to substitute in the expression

$$H_t = H_{32} \{1 + \delta (T - 32)\},$$

the value of δ , as given in the third column of the table, for the temperature nearest to T . If extreme accuracy is required, the value corresponding to any temperature T is easily obtained by interpolation, on the supposition that the rate of increase of the mean coefficient is uniform for each interval of 20° .

In all reductions of the height of the *barometric* column, we may safely take for δ the value corresponding to 60° F. This value, reduced to the form of a vulgar fraction, is $1 \div 10034.5$; substituting this in the expression for H_t , and solving for H_{32} , we obtain

$$H_{32} = H_t \frac{10034.5}{10034.5 + (T - 32)}.$$

Table VI. contains the mean coefficient of dilatation for 1° Fahr., from 32° to 652° F., of flint and crown glass, calculated from the formulæ given by M. Regnault, which, when adapted to our thermometer, are

<i>For Flint Glass.*</i>		<i>For Crown Glass.</i>	
k_T	$= bT + cT^2$	k_T	$= bT + cT^2$
$\log b$	$= \overline{5}.098\ 853\ 6$	$\log b$	$= 5.161\ 874\ 5$
$\log c$	$= \overline{10}.863\ 499\ 3$	$\log c$	$= 9.658\ 513\ 4$

in which k_T is the absolute dilatation from 32° to T , and T is counted from 32° .

Table VII. is taken from M. Regnault's memoir, and exhibits a comparison of the indications of an air thermometer corrected for the expansion of its envelope, and of mercurial thermometers constructed with four different kinds of glass, namely, French *crystal*, a double silicate of potash and oxide of lead, similar to our *flint glass*; *verre ordinaire*, a double silicate of lime and potash, or soda, analogous to our *crown glass*; *verre vert*, a multiple silicate of lime, oxide of iron, alumina, and potash or soda, corresponding to the common *bottle glass*; and lastly, a kind of very infusible glass, of which M. Regnault obtained some specimens from Sweden.

In Table VIII. we have given the absolute densities of various simple and compound gaseous bodies, as determined by direct experiment. It is usual to introduce into tables of this kind a column exhibiting the *theoretic* density of such bodies, deduced from Gay-Lussac's law of volumes, but we have thought it preferable to replace this column by one showing the relation between the volumes of compounds and those of their elements. For by means of this column the student will be able to calculate, either the theoretic density of a compound from the experimental densities of its elements, or the theoretic density of one element from the experimental densities of the compound and the other element, or finally, if the same common element enters into various combinations, to compare the different values resulting for its theoretic density, according as we derive it from one or other of its compounds.

* In M. Regnault's memoir, instead of the formula for the absolute dilatation of flint glass, the formula, apparently, for the apparent dilatation of mercury in that material is given by mistake. The formula in the text is deduced from the tabular values

given in the memoir for k_{100} and k_{300} . It gives values, less than those assigned by M. Regnault, below 212° , which is, however, of little consequence, as it is chiefly for higher temperatures that this table is required.

Table IX. contains the weights, in English grains, of 100 cubic inches of air and four other gases, whose densities have been determined by M. Regnault with extreme accuracy. In the third column is given the weight of this volume under the pressure of thirty inches at the equator; in the fourth the weight of the same volume under the pressure of a column of mercury of the same height at Dublin. The temperature of the gas is supposed to be the same in both cases, namely, that of melting ice.

The weight of the same volume of any of those substances at 32° , and under a pressure of thirty inches of mercury at any latitude, may be obtained by multiplying the number in the third column by $(1 + n \sin^2 \lambda)$, or with sufficient accuracy in most cases, by the value in Table IV., corresponding to the latitude nearest to that of the place of observation.

These weights were calculated as follows:

M. Regnault has ascertained that a litre of dry air at 0° , and under the pressure produced by 760^{mm} of mercury at the Observatory of Paris, weighs $1^{\text{gr}}.293\ 187$. Now those 760^{mm} are equivalent to $762^{\text{mm}}.2796$, at the sea level under the equator, while thirty inches are only equal to $761^{\text{mm}}.9862$; the preceding weight, therefore, must be reduced in the ratio $\frac{761\ 986\ 2}{762\ 279\ 6}$, in order to obtain the weight of the litre of air at 0° C., under a pressure of thirty inches at the equator. This gives the weight equal to $1^{\text{gr}}.292\ 689$.

Now $1^{\text{lit}} = 61.027\ 05$ English inches, and 1 gramme = 15.433 English grains; and hence we obtain, finally, the weight of 100 cubic inches of dry air at the temperature of melting ice, and under the pressure of thirty inches of mercury at 32° F. at the sea level under the equator, equal to $32^{\text{grs}}.690\ 541$.

At Dublin, under the pressure of a column of the same height, the weight equals $32^{\text{grs}}.802\ 342$.

Having thus obtained the weight of 100 cubic inches of dry air, the weights of the same volume of the other substances enumerated in the table are obtained by multiplying the preceding weights by the respective absolute densities.

Table X. exhibits the elastic forces of the vapours of various liquids, according to the best authorities.

TABLE I.

TABLE of the elastic Force of aqueous Vapour from -30° to 432°
Fahr.

Temperature.	Force in Inches of Mercury at 32° , at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dublin (Lat. $53^{\circ} 21'$).	Temperature.	Force in Inches of Mercury at 32° , at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dublin (Lat. $53^{\circ} 21'$).	Temperature.	Force in Inches of Mercury at 32° , at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dublin (Lat. $53^{\circ} 21'$).
-30°	0.0099	0.0099	$+6^{\circ}$	0.0578	0.0576	40°	0.2483	0.2475
29	0.0105	0.0105	7	0.0605	0.0603	41	0.2580	0.2571
28	0.0111	0.0111	8	0.0632	0.0630	42	0.2681	0.2672
27	0.0117	0.0117	9	0.0661	0.0659	43	0.2785	0.2775
26	0.0124	0.0123	10	0.0692	0.0689	44	0.2892	0.2882
25	0.0131	0.0130	11	0.0723	0.0721	45	0.3003	0.2993
24	0.0138	0.0137	12	0.0756	0.0753	46	0.3118	0.3108
23	0.0145	0.0144	13	0.0790	0.0788	47	0.3237	0.3226
22	0.0153	0.0152	14	0.0826	0.0823	48	0.3360	0.3349
21	0.0161	0.0160	15	0.0864	0.0861	49	0.3487	0.3476
20	0.0169	0.0168	16	0.0903	0.0899	50	0.3619	0.3607
19	0.0178	0.0177	17	0.0943	0.0940	51	0.3755	0.3742
18	0.0187	0.0186	18	0.0986	0.0982	52	0.3895	0.3882
17	0.0197	0.0196	19	0.1030	0.1027	53	0.4040	0.4026
16	0.0207	0.0206	20	0.1076	0.1073	54	0.4189	0.4175
15	0.0217	0.0216	21	0.1125	0.1121	55	0.4344	0.4329
14	0.0228	0.0227	22	0.1175	0.1171	56	0.4504	0.4488
13	0.0239	0.0238	23	0.1228	0.1223	57	0.4668	0.4653
12	0.0251	0.0250	24	0.1282	0.1278	58	0.4839	0.4822
11	0.0263	0.0262	25	0.1340	0.1335	59	0.5014	0.4997
10	0.0276	0.0275	26	0.1399	0.1395	60	0.5195	0.5178
9	0.0290	0.0289	27	0.1462	0.1457	61	0.5382	0.5364
8	0.0304	0.0303	28	0.1527	0.1522	62	0.5575	0.5556
7	0.0318	0.0317	29	0.1595	0.1589	63	0.5774	0.5755
6	0.0334	0.0332	30	0.1665	0.1660	64	0.5980	0.5959
5	0.0350	0.0348	31	0.1739	0.1733	65	0.6191	0.6170
4	0.0366	0.0365	32	0.1816	0.1810	66	0.6410	0.6388
3	0.0384	0.0382				67	0.6635	0.6612
2	0.0402	0.0400	32°	0.1816	0.1810	68	0.6867	0.6843
1	0.0421	0.0419	33	0.1890	0.1883	69	0.7106	0.7081
0	0.0440	0.0439	34	0.1966	0.1959	70	0.7352	0.7327
$+1$	0.0461	0.0459	35	0.2045	0.2038	71	0.7606	0.7580
2	0.0482	0.0481	36	0.2127	0.2119	72	0.7868	0.7841
3	0.0505	0.0503	37	0.2211	0.2204	73	0.8137	0.8109
4	0.0528	0.0526	38	0.2299	0.2291	74	0.8415	0.8386
5	0.0553	0.0551	39	0.2389	0.2381	75	0.8701	0.8671

Tempe- rature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').	Tempe- rature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').	Tempe- rature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').
76°	0.8995	0.8964	121°	3.532	3.520	166°	11.126	11.088
77	0.9298	0.9266	122	3.631	3.619	167	11.389	11.350
78	0.9610	0.9577	123	3.733	3.720	168	11.657	11.617
79	0.9931	0.9898	124	3.837	3.824	169	11.930	11.889
80	1.0262	1.0227	125	3.944	3.930	170	12.209	12.167
81	1.0602	1.0566	126	4.053	4.039	171	12.493	12.450
82	1.0952	1.0915	127	4.164	4.150	172	12.783	12.739
83	1.1312	1.1274	128	4.278	4.264	173	13.078	13.033
84	1.1683	1.1643	129	4.395	4.381	174	13.379	13.333
85	1.2064	1.2023	130	4.515	4.500	175	13.686	13.639
86	1.2456	1.2413	131	4.638	4.622	176	13.998	13.951
87	1.2859	1.2815	132	4.763	4.747	177	14.317	14.268
88	1.3273	1.3228	133	4.891	4.874	178	14.642	14.592
89	1.3699	1.3652	134	5.022	5.005	179	14.973	14.922
90	1.4137	1.4088	135	5.156	5.139	180	15.310	15.258
91	1.4587	1.4537	136	5.293	5.275	181	15.653	15.600
92	1.5049	1.4998	137	5.434	5.415	182	16.003	15.949
93	1.5524	1.5471	138	5.577	5.558	183	16.360	16.304
94	1.6012	1.5958	139	5.724	5.704	184	16.723	16.666
95	1.6514	1.6457	140	5.874	5.854	185	17.093	17.034
96	1.7029	1.6971	141	6.027	6.006	186	17.469	17.410
97	1.7558	1.7498	142	6.183	6.162	187	17.853	17.792
98	1.8101	1.8039	143	6.343	6.322	188	18.243	18.181
99	1.8658	1.8595	144	6.507	6.485	189	18.641	18.577
100	1.923	1.917	145	6.674	6.651	190	19.046	18.981
101	1.982	1.975	146	6.845	6.822	191	19.458	19.392
102	2.042	2.035	147	7.019	6.996	192	19.878	19.810
103	2.104	2.097	148	7.198	7.173	193	20.305	20.236
104	2.168	2.160	149	7.380	7.354	194	20.740	20.669
105	2.233	2.225	150	7.566	7.540	195	21.182	21.110
106	2.300	2.292	151	7.756	7.729	196	21.632	21.559
107	2.368	2.360	152	7.949	7.922	197	22.091	22.016
108	2.439	2.430	153	8.147	8.120	198	22.557	22.480
109	2.511	2.502	154	8.349	8.321	199	23.032	22.953
110	2.585	2.576	155	8.556	8.527	200	23.515	23.435
111	2.661	2.652	156	8.767	8.737	201	24.006	23.924
112	2.739	2.729	157	8.982	8.951	202	24.506	24.422
113	2.818	2.809	158	9.201	9.170	203	25.014	24.929
114	2.900	2.890	159	9.425	9.393	204	25.532	25.445
115	2.984	2.974	160	9.654	9.621	205	26.058	25.969
116	3.070	3.059	161	9.887	9.853	206	26.593	26.502
117	3.158	3.147	162	10.125	10.090	207	27.137	27.045
118	3.248	3.237	163	10.368	10.332	208	27.691	27.597
119	3.340	3.329	164	10.616	10.579	209	28.254	28.158
120	3.435	3.423	165	10.868	10.831	210	28.826	28.728

	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub-lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub-lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Mercury 32° at Sea Level at Dub-lin (Lat. 53° 21').
211°	29 ¹ .408	29 ¹ .308	256°	67 ¹ .66	67 ¹ .43	301°	139 ¹ .26	138 ¹ .79
212	30.000	29.898	257	68.84	68.60	302	141.36	140.88
213	30.60	30.50	258	70.02	69.79	303	143.48	142.99
214	31.21	31.11	259	71.23	70.99	304	145.63	145.13
215	31.83	31.73	260	72.45	72.20	305	147.80	147.30
216	32.46	32.35	261	73.69	73.44	306	150.00	149.49
217	33.11	32.99	262	74.94	74.69	307	152.22	151.70
218	33.76	33.64	263	76.22	75.96	308	154.47	153.95
219	34.42	34.30	264	77.51	77.24	309	156.75	156.22
220	35.09	34.98	265	78.81	78.55	310	159.05	158.51
221	35.78	35.66	266	80.14	79.87	311	161.38	160.83
222	36.47	36.35	267	81.48	81.21	312	163.74	163.18
223	37.18	37.05	268	82.85	82.56	313	166.13	165.56
224	37.90	37.77	269	84.23	83.94	314	168.54	167.97
225	38.63	38.50	270	85.63	85.33	315	170.98	170.40
226	39.37	39.23	271	87.04	86.75	316	173.45	172.86
227	40.12	39.98	272	88.48	88.18	317	175.95	175.35
228	40.88	40.74	273	89.94	89.63	318	178.47	177.86
229	41.66	41.52	274	91.41	91.10	319	181.02	180.41
230	42.45	42.30	275	92.91	92.59	320	183.61	182.98
231	43.25	43.10	276	94.42	94.10	321	186.22	185.59
232	44.06	43.91	277	95.96	95.63	322	188.86	188.22
233	44.89	44.73	278	97.51	97.18	323	191.53	190.88
234	45.72	45.57	279	99.09	98.75	324	194.23	193.57
235	46.57	46.42	280	100.68	100.34	325	196.96	196.29
236	47.44	47.28	281	102.30	101.95	326	199.73	199.05
237	48.31	48.15	282	103.93	103.58	327	202.52	201.83
238	49.20	49.04	283	105.59	105.23	328	205.34	204.64
239	50.11	49.94	284	107.27	106.91	329	208.20	207.49
240	51.02	50.85	285	108.97	108.60	330	211.08	210.36
241	51.95	51.78	286	110.69	110.32	331	214.00	213.27
242	52.90	52.72	287	112.44	112.05	332	216.95	216.21
243	53.86	53.67	288	114.20	113.81	333	219.93	219.18
244	54.83	54.64	289	115.99	115.60	334	222.94	222.18
245	55.82	55.63	290	117.80	117.40	335	225.98	225.21
246	56.82	56.62	291	119.64	119.23	336	229.06	228.28
247	57.83	57.64	292	121.49	121.08	337	232.17	231.38
248	58.87	58.66	293	123.37	122.95	338	235.32	235.51
249	59.91	59.71	294	125.27	124.85	339	238.49	237.68
250	60.97	60.76	295	127.20	126.77	340	241.70	240.88
251	62.05	61.84	296	129.15	128.71	341	244.95	244.12
252	63.14	62.92	297	131.12	130.68	342	248.23	247.38
253	64.25	64.03	298	133.12	132.67	343	251.54	250.69
254	65.37	65.15	299	135.14	134.68	344	254.89	254.02
255	66.51	66.28	300	137.19	136.72	345	258.27	257.39

Tempe- rature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').	Tempe- rature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').	Tempe- rature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in In-ches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').
346°	261 ⁱ .69	260 ⁱ .80	375°	377 ⁱ .26	375 ⁱ .98	404°	528 ⁱ .93	527 ⁱ .12
347	265.15	264.24	376	381.86	380.55	405	534.88	533.06
348	268.64	267.72	377	386.49	385.17	406	540.88	539.04
349	272.16	271.23	378	391.17	389.84	407	546.94	545.07
350	275.72	274.78	379	395.89	394.54	408	553.04	551.16
351	279.32	278.37	380	400.66	399.29	409	559.20	557.30
352	282.96	281.99	381	405.47	404.09	410	565.41	563.48
353	286.63	285.65	382	410.32	408.92	411	571.67	569.73
354	290.34	289.35	383	415.22	413.81	412	577.99	576.02
355	294.08	293.08	384	420.17	418.73	413	584.36	582.36
356	297.87	296.85	385	425.16	423.71	414	590.78	588.76
357	301.69	300.66	386	430.19	428.73	415	597.25	595.22
358	305.55	304.51	387	435.27	433.79	416	603.78	601.72
359	309.45	308.39	388	440.40	438.90	417	610.36	608.28
360	313.38	312.32	389	445.57	444.06	418	617.00	614.90
361	317.36	316.28	390	450.79	449.26	419	623.69	621.56
362	321.38	320.28	391	456.06	454.51	420	630.44	628.29
363	325.43	324.32	392	461.38	459.80	421	637.24	635.07
364	329.53	328.40	393	466.74	465.15	422	644.09	641.90
365	333.66	332.52	394	472.15	470.54	423	651.01	648.79
366	337.83	336.68	395	477.60	475.98	424	657.97	655.73
367	342.05	340.88	396	483.11	481.46	425	665.00	662.73
368	346.31	345.13	397	488.66	487.00	426	672.08	669.79
369	350.60	349.41	398	494.27	492.58	427	679.22	676.90
370	354.94	353.73	399	499.92	498.22	428	686.41	684.07
371	359.32	358.10	400	505.62	503.90	429	693.66	691.30
372	363.74	362.50	401	511.37	509.63	430	700.97	698.58
373	368.21	366.95	402	517.17	515.41	431	708.34	705.92
374	372.71	371.44	403	523.02	521.24	432	715.76	713.32

TABLE II.

TABLE of the elastic Force of aqueous Vapour from 0° to 100° F.
for every tenth of a Degree.

Temperature.	Force in Inches of Mercury at 32° , at Sea Level at Equator.	Force in Inches of Merc. at 32° , at Sea Level at Dub- lin (Lat. $53^{\circ} 21'$).	Temperature.	Force in Inches of Mercury at 32° , at Sea Level at Equator.	Force in Inches of Merc. at 32° , at Sea Level at Dub- lin (Lat. $53^{\circ} 21'$).	Temperature.	Force in Inches of Mercury at 32° , at Sea Level at Equator.	Force in Inches of Merc. at 32° , at Sea Level at Dub- lin (Lat. $53^{\circ} 21'$).
0.0	0.0440	0.0439	3.6	0.0519	0.0517	7.2	0.0610	0.0608
.1	0.0442	0.0441	.7	0.0521	0.0519	.3	0.0613	0.0611
.2	0.0444	0.0443	.8	0.0524	0.0522	.4	0.0616	0.0614
.3	0.0446	0.0445	.9	0.0526	0.0524	.5	0.0618	0.0616
.4	0.0449	0.0447	4.0	0.0528	0.0526	.6	0.0621	0.0619
.5	0.0451	0.0449	.1	0.0531	0.0529	.7	0.0624	0.0622
.6	0.0453	0.0451	.2	0.0533	0.0531	.8	0.0627	0.0625
.7	0.0455	0.0453	.3	0.0535	0.0534	.9	0.0630	0.0627
.8	0.0457	0.0455	.4	0.0538	0.0536	8.0	0.0632	0.0630
.9	0.0459	0.0457	.5	0.0540	0.0539	.1	0.0635	0.0633
1.0	0.0461	0.0459	.6	0.0543	0.0541	.2	0.0638	0.0636
.1	0.0463	0.0462	.7	0.0545	0.0543	.3	0.0641	0.0639
.2	0.0465	0.0464	.8	0.0548	0.0546	.4	0.0644	0.0642
.3	0.0467	0.0466	.9	0.0550	0.0548	.5	0.0647	0.0645
.4	0.0469	0.0468	5.0	0.0553	0.0551	.6	0.0650	0.0647
.5	0.0472	0.0470	.1	0.0555	0.0553	.7	0.0653	0.0650
.6	0.0474	0.0472	.2	0.0558	0.0556	.8	0.0656	0.0653
.7	0.0476	0.0474	.3	0.0560	0.0558	.9	0.0658	0.0656
.8	0.0478	0.0476	.4	0.0563	0.0561	9.0	0.0661	0.0659
.9	0.0480	0.0479	.5	0.0565	0.0563	.1	0.0664	0.0662
2.0	0.0482	0.0481	.6	0.0568	0.0566	.2	0.0667	0.0665
.1	0.0485	0.0483	.7	0.0570	0.0568	.3	0.0670	0.0668
.2	0.0487	0.0485	.8	0.0573	0.0571	.4	0.0673	0.0671
.3	0.0489	0.0487	.9	0.0576	0.0574	.5	0.0676	0.0674
.4	0.0491	0.0490	6.0	0.0578	0.0576	.6	0.0679	0.0677
.5	0.0494	0.0492	.1	0.0581	0.0579	.7	0.0682	0.0680
.6	0.0496	0.0494	.2	0.0583	0.0581	.8	0.0685	0.0683
.7	0.0498	0.0496	.3	0.0586	0.0584	.9	0.0689	0.0686
.8	0.0500	0.0499	.4	0.0589	0.0587	10.0	0.0692	0.0689
.9	0.0503	0.0501	.5	0.0591	0.0589	.1	0.0695	0.0692
3.0	0.0505	0.0503	.6	0.0594	0.0592	.2	0.0698	0.0695
.1	0.0507	0.0505	.7	0.0597	0.0595	.3	0.0701	0.0698
.2	0.0509	0.0508	.8	0.0599	0.0597	.4	0.0704	0.0702
.3	0.0512	0.0510	.9	0.0602	0.0600	.5	0.0707	0.0705
.4	0.0514	0.0512	7.0	0.0605	0.0603	.6	0.0710	0.0708
.5	0.0516	0.0515	.1	0.0607	0.0605	.7	0.0714	0.0711

Temperature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').
10°.8	0.0717	0.0714	15°.3	0.0875	0.0872	19°.8	0.1067	0.1063
.9	0.0720	0.0717	.4	0.0879	0.0876	.9	0.1072	0.1068
11°.0	0.0723	0.0721	.5	0.0883	0.0880	20°.0	0.1076	0.1073
.1	0.0726	0.0724	.6	0.0887	0.0884	.1	0.1081	0.1077
.2	0.0730	0.0727	.7	0.0891	0.0888	.2	0.1086	0.1082
.3	0.0733	0.0730	.8	0.0895	0.0892	.3	0.1091	0.1087
.4	0.0736	0.0734	.9	0.0899	0.0896	.4	0.1095	0.1092
.5	0.0739	0.0737	16°.0	0.0903	0.0899	.5	0.1100	0.1096
.6	0.0743	0.0740	.1	0.0907	0.0903	.6	0.1105	0.1101
.7	0.0746	0.0743	.2	0.0911	0.0907	.7	0.1110	0.1106
.8	0.0749	0.0747	.3	0.0915	0.0911	.8	0.1115	0.1111
.9	0.0753	0.0750	.4	0.0919	0.0916	.9	0.1120	0.1116
12°.0	0.0756	0.0753	.5	0.0923	0.0920	21°.0	0.1125	0.1121
.1	0.0759	0.0757	.6	0.0927	0.0924	.1	0.1130	0.1126
.2	0.0763	0.0760	.7	0.0931	0.0928	.2	0.1134	0.1131
.3	0.0766	0.0764	.8	0.0935	0.0932	.3	0.1139	0.1136
.4	0.0770	0.0767	.9	0.0939	0.0936	.4	0.1144	0.1141
.5	0.0773	0.0770	17°.0	0.0943	0.0940	.5	0.1150	0.1146
.6	0.0776	0.0774	.1	0.0947	0.0944	.6	0.1155	0.1151
.7	0.0780	0.0777	.2	0.0952	0.0948	.7	0.1160	0.1156
.8	0.0783	0.0781	.3	0.0956	0.0953	.8	0.1165	0.1161
.9	0.0787	0.0784	.4	0.0960	0.0957	.9	0.1170	0.1166
13°.0	0.0790	0.0788	.5	0.0964	0.0961	22°.0	0.1175	0.1171
.1	0.0794	0.0791	.6	0.0969	0.0965	.1	0.1180	0.1176
.2	0.0797	0.0795	.7	0.0973	0.0969	.2	0.1185	0.1181
.3	0.0801	0.0798	.8	0.0977	0.0974	.3	0.1190	0.1186
.4	0.0805	0.0802	.9	0.0981	0.0978	.4	0.1196	0.1192
.5	0.0808	0.0805	18°.0	0.0986	0.0982	.5	0.1201	0.1197
.6	0.0812	0.0809	.1	0.0990	0.0987	.6	0.1206	0.1202
.7	0.0815	0.0812	.2	0.0994	0.0991	.7	0.1212	0.1207
.8	0.0819	0.0816	.3	0.0999	0.0995	.8	0.1217	0.1213
.9	0.0823	0.0820	.4	0.1003	0.1000	.9	0.1222	0.1218
14°.0	0.0826	0.0823	.5	0.1008	0.1004	23°.0	0.1228	0.1223
.1	0.0830	0.0827	.6	0.1012	0.1009	.1	0.1233	0.1229
.2	0.0834	0.0831	.7	0.1017	0.1013	.2	0.1238	0.1234
.3	0.0837	0.0834	.8	0.1021	0.1018	.3	0.1244	0.1239
.4	0.0841	0.0838	.9	0.1026	0.1022	.4	0.1249	0.1245
.5	0.0845	0.0842	19°.0	0.1030	0.1027	.5	0.1255	0.1250
.6	0.0848	0.0846	.1	0.1035	0.1031	.6	0.1260	0.1256
.7	0.0852	0.0849	.2	0.1039	0.1036	.7	0.1266	0.1261
.8	0.0856	0.0853	.3	0.1044	0.1040	.8	0.1271	0.1267
.9	0.0860	0.0857	.4	0.1048	0.1045	.9	0.1277	0.1272
15°.0	0.0864	0.0861	.5	0.1053	0.1049	24°.0	0.1282	0.1278
.1	0.0867	0.0864	.6	0.1058	0.1054	.1	0.1288	0.1284
.2	0.0871	0.0868	.7	0.1062	0.1059	.2	0.1294	0.1289

Temperature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').
24°.3	0.1299	0.1295	28°.8	0.1581	0.1575	33°.3	0.1912	0.1906
.4	0.1305	0.1300	.9	0.1588	0.1582	.4	0.1920	0.1913
.5	0.1311	0.1306	29°.0	0.1595	0.1589	.5	0.1928	0.1921
.6	0.1317	0.1312	.1	0.1602	0.1596	.6	0.1935	0.1929
.7	0.1322	0.1318	.2	0.1609	0.1603	.7	0.1943	0.1936
.8	0.1328	0.1323	.3	0.1616	0.1610	.8	0.1951	0.1944
.9	0.1334	0.1329	.4	0.1623	0.1617	.9	0.1958	0.1952
25°.0	0.1340	0.1335	.5	0.1630	0.1624	34°.0	0.1966	0.1959
.1	0.1345	0.1341	.6	0.1637	0.1631	.1	0.1974	0.1967
.2	0.1351	0.1347	.7	0.1644	0.1638	.2	0.1982	0.1975
.3	0.1357	0.1353	.8	0.1651	0.1645	.3	0.1989	0.1983
.4	0.1363	0.1359	.9	0.1658	0.1653	.4	0.1997	0.1990
.5	0.1369	0.1364	30°.0	0.1665	0.1660	.5	0.2005	0.1998
.6	0.1375	0.1370	.1	0.1673	0.1667	.6	0.2013	0.2006
.7	0.1381	0.1376	.2	0.1680	0.1674	.7	0.2021	0.2014
.8	0.1387	0.1382	.3	0.1687	0.1682	.8	0.2029	0.2022
.9	0.1393	0.1388	.4	0.1695	0.1689	.9	0.2037	0.2030
26°.0	0.1399	0.1395	.5	0.1702	0.1696	35°.0	0.2045	0.2038
.1	0.1405	0.1401	.6	0.1709	0.1704	.1	0.2053	0.2046
.2	0.1412	0.1407	.7	0.1717	0.1711	.2	0.2061	0.2054
.3	0.1418	0.1413	.8	0.1724	0.1718	.3	0.2069	0.2062
.4	0.1424	0.1419	.9	0.1732	0.1726	.4	0.2077	0.2070
.5	0.1430	0.1425	31°.0	0.1739	0.1733	.5	0.2085	0.2078
.6	0.1436	0.1431	.1	0.1747	0.1741	.6	0.2094	0.2086
.7	0.1443	0.1438	.2	0.1755	0.1749	.7	0.2102	0.2095
.8	0.1449	0.1444	.3	0.1762	0.1756	.8	0.2110	0.2103
.9	0.1455	0.1450	.4	0.1770	0.1764	.9	0.2118	0.2111
27°.0	0.1462	0.1457	.5	0.1777	0.1771	36°.0	0.2127	0.2119
.1	0.1468	0.1463	.6	0.1785	0.1779	.1	0.2135	0.2128
.2	0.1474	0.1469	.7	0.1793	0.1787	.2	0.2143	0.2136
.3	0.1481	0.1476	.8	0.1801	0.1795	.3	0.2152	0.2144
.4	0.1487	0.1482	.9	0.1809	0.1802	.4	0.2160	0.2153
.5	0.1494	0.1489	32°.0	0.1816	0.1810	.5	0.2168	0.2161
.6	0.1500	0.1495	.1	0.1824	0.1817	.6	0.2177	0.2169
.7	0.1507	0.1502	.2	0.1831	0.1825	.7	0.2185	0.2178
.8	0.1513	0.1508	.3	0.1838	0.1832	.8	0.2194	0.2186
.9	0.1520	0.1515	.4	0.1846	0.1839	.9	0.2202	0.2195
28°.0	0.1527	0.1522	.5	0.1853	0.1847	37°.0	0.2211	0.2204
.1	0.1533	0.1528	.6	0.1860	0.1854	.1	0.2220	0.2212
.2	0.1540	0.1535	.7	0.1868	0.1861	.2	0.2228	0.2221
.3	0.1547	0.1542	.8	0.1875	0.1869	.3	0.2237	0.2229
.4	0.1554	0.1548	.9	0.1882	0.1876	.4	0.2246	0.2238
.5	0.1560	0.1555	33°.0	0.1890	0.1883	.5	0.2254	0.2247
.6	0.1567	0.1562	.1	0.1897	0.1891	.6	0.2263	0.2255
.7	0.1574	0.1569	.2	0.1905	0.1898	.7	0.2272	0.2264

Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub-lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub-lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub-lin (Lat. 53° 21').
37°.8	0.2281	0.2273	42°.3	0.2712	0.2702	46°.8	0.3213	0.3202
.9	0.2290	0.2282	.4	0.2722	0.2713	.9	0.3225	0.3214
38°.0	0.2299	0.2291	.5	0.2732	0.2723	47°.0	0.3237	0.3226
.1	0.2308	0.2300	.6	0.2743	0.2733	.1	0.3250	0.3238
.2	0.2317	0.2309	.7	0.2753	0.2744	.2	0.3262	0.3251
.3	0.2326	0.2318	.8	0.2764	0.2754	.3	0.3274	0.3263
.4	0.2335	0.2327	.9	0.2774	0.2765	.4	0.3286	0.3275
.5	0.2344	0.2336	43°.0	0.2785	0.2775	.5	0.3298	0.3287
.6	0.2353	0.2345	.1	0.2795	0.2786	.6	0.3311	0.3299
.7	0.2362	0.2354	.2	0.2806	0.2796	.7	0.3323	0.3312
.8	0.2371	0.2363	.3	0.2817	0.2807	.8	0.3335	0.3324
.9	0.2380	0.2372	.4	0.2827	0.2818	.9	0.3348	0.3336
39°.0	0.2389	0.2381	.5	0.2838	0.2828	48°.0	0.3360	0.3349
.1	0.2399	0.2390	.6	0.2849	0.2839	.1	0.3373	0.3361
.2	0.2408	0.2400	.7	0.2860	0.2850	.2	0.3385	0.3374
.3	0.2417	0.2409	.8	0.2870	0.2861	.3	0.3398	0.3386
.4	0.2426	0.2418	.9	0.2881	0.2871	.4	0.3411	0.3399
.5	0.2436	0.2427	44°.0	0.2892	0.2882	.5	0.3423	0.3412
.6	0.2445	0.2437	.1	0.2903	0.2893	.6	0.3436	0.3424
.7	0.2455	0.2446	.2	0.2914	0.2904	.7	0.3449	0.3437
.8	0.2464	0.2456	.3	0.2925	0.2915	.8	0.3462	0.3450
.9	0.2474	0.2465	.4	0.2936	0.2926	.9	0.3475	0.3463
40°.0	0.2483	0.2475	.5	0.2947	0.2937	49°.0	0.3487	0.3476
.1	0.2493	0.2484	.6	0.2958	0.2948	.1	0.3500	0.3488
.2	0.2502	0.2494	.7	0.2970	0.2960	.2	0.3513	0.3501
.3	0.2512	0.2503	.8	0.2981	0.2971	.3	0.3526	0.3514
.4	0.2522	0.2513	.9	0.2992	0.2982	.4	0.3540	0.3527
.5	0.2531	0.2523	45°.0	0.3003	0.2993	.5	0.3553	0.3541
.6	0.2541	0.2532	.1	0.3015	0.3004	.6	0.3566	0.3554
.7	0.2551	0.2542	.2	0.3026	0.3016	.7	0.3579	0.3567
.8	0.2561	0.2552	.3	0.3038	0.3027	.8	0.3592	0.3580
.9	0.2570	0.2562	.4	0.3049	0.3039	.9	0.3606	0.3593
41°.0	0.2580	0.2571	.5	0.3060	0.3050	50°.0	0.3619	0.3607
.1	0.2590	0.2581	.6	0.3072	0.3061	.1	0.3632	0.3620
.2	0.2600	0.2591	.7	0.3084	0.3073	.2	0.3646	0.3633
.3	0.2610	0.2601	.8	0.3095	0.3085	.3	0.3659	0.3647
.4	0.2620	0.2611	.9	0.3107	0.3096	.4	0.3673	0.3660
.5	0.2630	0.2621	46°.0	0.3118	0.3108	.5	0.3686	0.3674
.6	0.2640	0.2631	.1	0.3130	0.3119	.6	0.3700	0.3687
.7	0.2650	0.2641	.2	0.3142	0.3131	.7	0.3713	0.3701
.8	0.2660	0.2651	.3	0.3154	0.3143	.8	0.3727	0.3714
.9	0.2670	0.2661	.4	0.3166	0.3155	.9	0.3741	0.3728
42°.0	0.2681	0.2672	.5	0.3177	0.3167	51°.0	0.3755	0.3742
.1	0.2691	0.2682	.6	0.3189	0.3178	.1	0.3768	0.3756
.2	0.2701	0.2692	.7	0.3201	0.3190	.2	0.3782	0.3769

Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub-lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub-lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub-lin (Lat. 53° 21').
51°.3	0.3796	0.3783	55°.8	0.4471	0.4456	60°.3	0.5251	0.5233
.4	0.3810	0.3797	.9	0.4487	0.4472	.4	0.5269	0.5251
.5	0.3824	0.3811	56°.0	0.4504	0.4488	.5	0.5288	0.5270
.6	0.3838	0.3825	.1	0.4520	0.4504	.6	0.5307	0.5289
.7	0.3852	0.3839	.2	0.4536	0.4521	.7	0.5326	0.5307
.8	0.3866	0.3853	.3	0.4553	0.4537	.8	0.5344	0.5326
.9	0.3881	0.3867	.4	0.4569	0.4553	.9	0.5363	0.5345
52°.0	0.3895	0.3882	.5	0.4585	0.4570	61°.0	0.5382	0.5364
.1	0.3909	0.3896	.6	0.4602	0.4586	.1	0.5401	0.5383
.2	0.3923	0.3910	.7	0.4618	0.4603	.2	0.5420	0.5402
.3	0.3938	0.3924	.8	0.4635	0.4619	.3	0.5440	0.5421
.4	0.3952	0.3939	.9	0.4652	0.4636	.4	0.5459	0.5440
.5	0.3967	0.3953	57°.0	0.4668	0.4653	.5	0.5478	0.5459
.6	0.3981	0.3968	.1	0.4685	0.4669	.6	0.5497	0.5479
.7	0.3996	0.3982	.2	0.4702	0.4686	.7	0.5517	0.5498
.8	0.4010	0.3997	.3	0.4719	0.4703	.8	0.5536	0.5517
.9	0.4025	0.4011	.4	0.4736	0.4720	.9	0.5556	0.5537
53°.0	0.4040	0.4026	.5	0.4753	0.4737	62°.0	0.5575	0.5556
.1	0.4055	0.4041	.6	0.4770	0.4754	.1	0.5595	0.5576
.2	0.4069	0.4055	.7	0.4787	0.4771	.2	0.5615	0.5595
.3	0.4084	0.4070	.8	0.4804	0.4788	.3	0.5634	0.5615
.4	0.4099	0.4085	.9	0.4821	0.4805	.4	0.5654	0.5635
.5	0.4114	0.4100	58°.0	0.4839	0.4822	.5	0.5674	0.5655
.6	0.4129	0.4115	.1	0.4856	0.4839	.6	0.5694	0.5675
.7	0.4144	0.4130	.2	0.4873	0.4857	.7	0.5714	0.5694
.8	0.4159	0.4145	.3	0.4891	0.4874	.8	0.5734	0.5714
.9	0.4174	0.4160	.4	0.4908	0.4891	.9	0.5754	0.5734
54°.0	0.4189	0.4175	.5	0.4926	0.4909	63°.0	0.5774	0.5755
.1	0.4205	0.4190	.6	0.4943	0.4926	.1	0.5794	0.5775
.2	0.4220	0.4206	.7	0.4961	0.4944	.2	0.5815	0.5795
.3	0.4235	0.4221	.8	0.4979	0.4962	.3	0.5835	0.5815
.4	0.4251	0.4236	.9	0.4996	0.4979	.4	0.5856	0.5836
.5	0.4266	0.4252	59°.0	0.5014	0.4997	.5	0.5876	0.5856
.6	0.4282	0.4267	.1	0.5032	0.5015	.6	0.5897	0.5877
.7	0.4297	0.4282	.2	0.5050	0.5033	.7	0.5917	0.5897
.8	0.4313	0.4298	.3	0.5068	0.5051	.8	0.5938	0.5918
.9	0.4328	0.4314	.4	0.5086	0.5069	.9	0.5959	0.5938
55°.0	0.4344	0.4329	.5	0.5104	0.5087	64°.0	0.5980	0.5959
.1	0.4360	0.4345	.6	0.5122	0.5105	.1	0.6000	0.5980
.2	0.4376	0.4361	.7	0.5140	0.5123	.2	0.6021	0.6001
.3	0.4391	0.4376	.8	0.5159	0.5141	.3	0.6042	0.6022
.4	0.4407	0.4392	.9	0.5177	0.5159	.4	0.6063	0.6043
.5	0.4423	0.4408	60°.0	0.5195	0.5178	.5	0.6085	0.6064
.6	0.4439	0.4424	.1	0.5214	0.5196	.6	0.6106	0.6085
.7	0.4455	0.4440	.2	0.5232	0.5215	.7	0.6127	0.6106

Tempe- rature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').	Tempe- rature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').	Tempe- rature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').
64°.8	0.6148	0.6127	69°.3	0.7179	0.7154	73°.8	0.8359	0.8330
.9	0.6170	0.6149	.4	0.7203	0.7179	.9	0.8387	0.8358
65°.0	0.6191	0.6170	.5	0.7228	0.7203	74°.0	0.8415	0.8386
.1	0.6213	0.6192	.6	0.7253	0.7228	.1	0.8443	0.8414
.2	0.6234	0.6213	.7	0.7277	0.7253	.2	0.8471	0.8442
.3	0.6256	0.6235	.8	0.7302	0.7277	.3	0.8500	0.8471
.4	0.6278	0.6256	.9	0.7327	0.7302	.4	0.8528	0.8499
.5	0.6300	0.6278	70°.0	0.7352	0.7327	.5	0.8557	0.8527
.6	0.6321	0.6300	.1	0.7377	0.7352	.6	0.8585	0.8556
.7	0.6343	0.6322	.2	0.7402	0.7377	.7	0.8614	0.8585
.8	0.6365	0.6344	.3	0.7427	0.7402	.8	0.8643	0.8613
.9	0.6387	0.6366	.4	0.7453	0.7427	.9	0.8672	0.8642
66°.0	0.6410	0.6388	.5	0.7478	0.7453	75°.0	0.8701	0.8671
.1	0.6432	0.6410	.6	0.7504	0.7478	.1	0.8730	0.8700
.2	0.6454	0.6432	.7	0.7529	0.7503	.2	0.8759	0.8729
.3	0.6476	0.6454	.8	0.7555	0.7529	.3	0.8788	0.8758
.4	0.6499	0.6477	.9	0.7580	0.7554	.4	0.8817	0.8787
.5	0.6521	0.6499	71°.0	0.7606	0.7580	.5	0.8847	0.8817
.6	0.6544	0.6521	.1	0.7632	0.7606	.6	0.8876	0.8846
.7	0.6566	0.6544	.2	0.7658	0.7632	.7	0.8906	0.8875
.8	0.6589	0.6567	.3	0.7684	0.7657	.8	0.8935	0.8905
.9	0.6612	0.6589	.4	0.7710	0.7683	.9	0.8965	0.8935
67°.0	0.6635	0.6612	.5	0.7736	0.7709	76°.0	0.8995	0.8964
.1	0.6657	0.6635	.6	0.7762	0.7736	.1	0.9025	0.8994
.2	0.6680	0.6658	.7	0.7788	0.7762	.2	0.9055	0.9024
.3	0.6703	0.6681	.8	0.7815	0.7788	.3	0.9085	0.9054
.4	0.6727	0.6704	.9	0.7841	0.7814	.4	0.9115	0.9084
.5	0.6750	0.6727	72°.0	0.7868	0.7841	.5	0.9145	0.9114
.6	0.6773	0.6750	.1	0.7894	0.7867	.6	0.9176	0.9145
.7	0.6796	0.6773	.2	0.7921	0.7894	.7	0.9206	0.9175
.8	0.6820	0.6796	.3	0.7948	0.7921	.8	0.9237	0.9205
.9	0.6843	0.6820	.4	0.7974	0.7947	.9	0.9267	0.9236
68°.0	0.6867	0.6843	.5	0.8001	0.7974	77°.0	0.9298	0.9266
.1	0.6890	0.6867	.6	0.8028	0.8001	.1	0.9329	0.9297
.2	0.6914	0.6890	.7	0.8055	0.8028	.2	0.9360	0.9328
.3	0.6938	0.6914	.8	0.8083	0.8055	.3	0.9391	0.9359
.4	0.6961	0.6938	.9	0.8110	0.8082	.4	0.9422	0.9390
.5	0.6985	0.6961	73°.0	0.8137	0.8109	.5	0.9453	0.9421
.6	0.7009	0.6985	.1	0.8165	0.8137	.6	0.9484	0.9452
.7	0.7033	0.7009	.2	0.8192	0.8164	.7	0.9516	0.9483
.8	0.7057	0.7033	.3	0.8220	0.8192	.8	0.9547	0.9515
.9	0.7081	0.7057	.4	0.8247	0.8219	.9	0.9579	0.9546
69°.0	0.7106	0.7081	.5	0.8275	0.8247	78°.0	0.9610	0.9577
.1	0.7130	0.7106	.6	0.8303	0.8274	.1	0.9642	0.9609
.2	0.7154	0.7130	.7	0.8331	0.8302	.2	0.9674	0.9641

Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub-lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub-lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub-lin (Lat. 53° 21').
78°.3	0.9706	0.9673	82°.8	1.1239	1.1201	87°.3	1.2982	1.2937
.4	0.9738	0.9704	.9	1.1276	1.1237	.4	1.3023	1.2979
.5	0.9770	0.9736	83°.0	1.1312	1.1274	.5	1.3064	1.3020
.6	0.9802	0.9768	.1	1.1349	1.1310	.6	1.3106	1.3061
.7	0.9834	0.9801	.2	1.1386	1.1347	.7	1.3147	1.3103
.8	0.9866	0.9833	.3	1.1422	1.1383	.8	1.3189	1.3144
.9	0.9899	0.9865	.4	1.1459	1.1420	.9	1.3231	1.3186
79°.0	0.9931	0.9898	.5	1.1496	1.1457	88°.0	1.3273	1.3228
.1	0.9964	0.9930	.6	1.1533	1.1494	.1	1.3315	1.3270
.2	0.9997	0.9963	.7	1.1571	1.1531	.2	1.3357	1.3312
.3	1.0030	0.9995	.8	1.1608	1.1568	.3	1.3399	1.3354
.4	1.0063	1.0028	.9	1.1645	1.1606	.4	1.3442	1.3396
.5	1.0096	1.0061	84°.0	1.1683	1.1643	.5	1.3484	1.3438
.6	1.0129	1.0094	.1	1.1720	1.1680	.6	1.3527	1.3481
.7	1.0162	1.0127	.2	1.1758	1.1718	.7	1.3570	1.3524
.8	1.0195	1.0160	.3	1.1796	1.1756	.8	1.3613	1.3566
.9	1.0229	1.0194	.4	1.1834	1.1794	.9	1.3656	1.3609
80°.0	1.0262	1.0227	.5	1.1872	1.1832	89°.0	1.3699	1.3652
.1	1.0296	1.0260	.6	1.1910	1.1870	.1	1.3742	1.3695
.2	1.0329	1.0294	.7	1.1948	1.1908	.2	1.3785	1.3738
.3	1.0363	1.0328	.8	1.1987	1.1946	.3	1.3829	1.3782
.4	1.0397	1.0361	.9	1.2025	1.1984	.4	1.3872	1.3825
.5	1.0431	1.0395	85°.0	1.2064	1.2023	.5	1.3916	1.3869
.6	1.0465	1.0429	.1	1.2103	1.2061	.6	1.3960	1.3912
.7	1.0499	1.0463	.2	1.2141	1.2100	.7	1.4004	1.3956
.8	1.0533	1.0497	.3	1.2180	1.2139	.8	1.4048	1.4000
.9	1.0568	1.0532	.4	1.2219	1.2178	.9	1.4092	1.4044
81°.0	1.0602	1.0566	.5	1.2258	1.2217	90°.0	1.4137	1.4088
.1	1.0637	1.0601	.6	1.2298	1.2256	.1	1.4181	1.4133
.2	1.0671	1.0635	.7	1.2337	1.2295	.2	1.4226	1.4177
.3	1.0706	1.0670	.8	1.2376	1.2334	.3	1.4270	1.4222
.4	1.0741	1.0704	.9	1.2416	1.2374	.4	1.4315	1.4266
.5	1.0776	1.0739	86°.0	1.2456	1.2413	.5	1.4360	1.4311
.6	1.0811	1.0774	.1	1.2496	1.2453	.6	1.4405	1.4356
.7	1.0846	1.0809	.2	1.2535	1.2493	.7	1.4450	1.4401
.8	1.0881	1.0844	.3	1.2575	1.2533	.8	1.4496	1.4446
.9	1.0917	1.0880	.4	1.2616	1.2573	.9	1.4541	1.4491
82°.0	1.0952	1.0915	.5	1.2656	1.2613	91°.0	1.4586	1.4537
.1	1.0988	1.0950	.6	1.2696	1.2653	.1	1.4632	1.4582
.2	1.1024	1.0986	.7	1.2737	1.2693	.2	1.4678	1.4628
.3	1.1059	1.1022	.8	1.2777	1.2734	.3	1.4724	1.4674
.4	1.1095	1.1057	.9	1.2818	1.2774	.4	1.4770	1.4720
.5	1.1131	1.1093	87°.0	1.2859	1.2815	.5	1.4816	1.4766
.6	1.1167	1.1129	.1	1.2900	1.2856	.6	1.4862	1.4812
.7	1.1203	1.1165	.2	1.2941	1.2896	.7	1.4909	1.4858

Tempe- rature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').	Tempe- rature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').	Tempe- rature.	Force in Inches of Mercury at 32° at Sea Level at Equator.	Force in Inches of Merc. at 32° at Sea Level at Dub- lin (Lat. 53° 21').
91°.8	1.4955	1.4904	94°.6	1.6311	1.6256	97°.4	1.7773	1.7712
.9	1.5002	1.4951	.7	1.6362	1.6306	.5	1.7827	1.7767
92°.0	1.5049	1.4998	.8	1.6412	1.6356	.6	1.7882	1.7821
.1	1.5096	1.5044	.9	1.6463	1.6407	.7	1.7936	1.7875
.2	1.5143	1.5091	95°.0	1.6514	1.6457	.8	1.7991	1.7930
.3	1.5190	1.5138	.1	1.6564	1.6508	.9	1.8046	1.7984
.4	1.5237	1.5185	.2	1.6615	1.6559	98°.0	1.8101	1.8039
.5	1.5285	1.5233	.3	1.6667	1.6610	.1	1.8156	1.8094
.6	1.5332	1.5280	.4	1.6718	1.6661	.2	1.8211	1.8149
.7	1.5380	1.5328	.5	1.6769	1.6712	.3	1.8266	1.8204
.8	1.5428	1.5375	.6	1.6821	1.6764	.4	1.8322	1.8260
.9	1.5476	1.5423	.7	1.6873	1.6815	.5	1.8378	1.8315
93°.0	1.5524	1.5471	.8	1.6924	1.6867	.6	1.8434	1.8371
.1	1.5572	1.5519	.9	1.6976	1.6919	.7	1.8490	1.8427
.2	1.5621	1.5567	96°.0	1.7029	1.6971	.8	1.8546	1.8482
.3	1.5669	1.5616	.1	1.7081	1.7023	.9	1.8602	1.8539
.4	1.5718	1.5664	.2	1.7133	1.7075	99°.0	1.8658	1.8595
.5	1.5766	1.5713	.3	1.7186	1.7127	.1	1.8715	1.8651
.6	1.5815	1.5761	.4	1.7238	1.7180	.2	1.8772	1.8708
.7	1.5864	1.5810	.5	1.7291	1.7232	.3	1.8829	1.8764
.8	1.5913	1.5859	.6	1.7344	1.7285	.4	1.8886	1.8821
.9	1.5963	1.5908	.7	1.7397	1.7338	.5	1.8943	1.8878
94°.0	1.6012	1.5958	.8	1.7451	1.7391	.6	1.9000	1.8935
.1	1.6062	1.6007	.9	1.7504	1.7445	.7	1.9058	1.8993
.2	1.6111	1.6056	97°.0	1.7558	1.7498	.8	1.9115	1.9050
.3	1.6161	1.6106	.1	1.7611	1.7551	.9	1.9173	1.9108
.4	1.6211	1.6156	.2	1.7665	1.7605			
.5	1.6261	1.6206	.3	1.7719	1.7659			

TABLE III.

TABLE of the elastic Force of aqueous Vapour from 185° to 214° F.
for every tenth of a Degree.

Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').
185°.0	17.093	17.034	188°.6	18.481	18.418	192°.2	19.962	19.894
.1	17.130	17.072	.7	18.521	18.458	.3	20.005	19.937
.2	17.167	17.109	.8	18.561	18.498	.4	20.048	19.979
.3	17.205	17.146	.9	18.601	18.537	.5	20.090	20.022
.4	17.242	17.184	189°.0	18.641	18.577	.6	20.133	20.064
.5	17.280	17.221	.1	18.681	18.617	.7	20.176	20.107
.6	17.318	17.259	.2	18.721	18.657	.8	20.219	20.150
.7	17.356	17.296	.3	18.762	18.698	.9	20.262	20.193
.8	17.393	17.334	.4	18.802	18.738	193°.0	20.305	20.236
.9	17.431	17.372	.5	18.842	18.778	.1	20.348	20.279
186°.0	17.469	17.410	.6	18.883	18.819	.2	20.391	20.322
.1	17.507	17.448	.7	18.924	18.859	.3	20.434	20.365
.2	17.545	17.486	.8	18.964	18.900	.4	20.478	20.408
.3	17.584	17.524	.9	19.005	18.940	.5	20.521	20.451
.4	17.622	17.562	190°.0	19.046	18.981	.6	20.565	20.495
.5	17.660	17.600	.1	19.087	19.022	.7	20.608	20.538
.6	17.698	17.638	.2	19.128	19.062	.8	20.652	20.582
.7	17.737	17.677	.3	19.169	19.103	.9	20.696	20.625
.8	17.775	17.715	.4	19.210	19.144	194°.0	20.740	20.669
.9	17.814	17.753	.5	19.251	19.185	.1	20.783	20.713
187°.0	17.853	17.792	.6	19.292	19.226	.2	20.827	20.756
.1	17.891	17.830	.7	19.334	19.268	.3	20.871	20.800
.2	17.930	17.869	.8	19.375	19.309	.4	20.916	20.844
.3	17.969	17.908	.9	19.416	19.350	.5	20.960	20.888
.4	18.008	17.947	191°.0	19.458	19.392	.6	21.004	20.932
.5	18.047	17.986	.1	19.500	19.433	.7	21.049	20.977
.6	18.086	18.025	.2	19.541	19.475	.8	21.093	21.021
.7	18.125	18.064	.3	19.583	19.516	.9	21.137	21.065
.8	18.165	18.103	.4	19.625	19.558	195°.0	21.182	21.110
.9	18.204	18.142	.5	19.667	19.600	.1	21.227	21.154
188°.0	18.243	18.181	.6	19.709	19.642	.2	21.271	21.199
.1	18.283	18.220	.7	19.751	19.684	.3	21.316	21.244
.2	18.322	18.260	.8	19.793	19.726	.4	21.361	21.288
.3	18.362	18.299	.9	19.835	19.768	.5	21.406	21.333
.4	18.401	18.339	192°.0	19.878	19.810	.6	21.451	21.378
.5	18.441	18.378	.1	19.920	19.852	.7	21.496	21.423

Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').	Temperature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').
195°.8	21 ¹ .542	21 ¹ .468	200°.3	23 ¹ .661	23 ¹ .581	204°.8	25 ¹ .952	25 ¹ .863
.9	21 ¹ .587	21 ¹ .513	.4	23 ¹ .710	23 ¹ .629	.9	26 ¹ .005	25 ¹ .916
196°.0	21 ¹ .632	21 ¹ .559	.5	23 ¹ .759	23 ¹ .678	205°.0	26 ¹ .058	25 ¹ .969
.1	21 ¹ .678	21 ¹ .604	.6	23 ¹ .808	23 ¹ .727	.1	26 ¹ .111	26 ¹ .022
.2	21 ¹ .723	21 ¹ .649	.7	23 ¹ .858	23 ¹ .776	.2	26 ¹ .164	26 ¹ .075
.3	21 ¹ .769	21 ¹ .695	.8	23 ¹ .907	23 ¹ .826	.3	26 ¹ .217	26 ¹ .128
.4	21 ¹ .815	21 ¹ .740	.9	23 ¹ .956	23 ¹ .875	.4	26 ¹ .271	26 ¹ .181
.5	21 ¹ .861	21 ¹ .786	201°.0	24 ¹ .006	23 ¹ .924	.5	26 ¹ .324	26 ¹ .234
.6	21 ¹ .907	21 ¹ .832	.1	24 ¹ .056	23 ¹ .974	.6	26 ¹ .378	26 ¹ .288
.7	21 ¹ .953	21 ¹ .878	.2	24 ¹ .105	24 ¹ .023	.7	26 ¹ .431	26 ¹ .341
.8	21 ¹ .998	21 ¹ .923	.3	24 ¹ .155	24 ¹ .073	.8	26 ¹ .485	26 ¹ .395
.9	22 ¹ .045	21 ¹ .969	.4	24 ¹ .205	24 ¹ .122	.9	26 ¹ .539	26 ¹ .449
197°.0	22 ¹ .091	22 ¹ .015	.5	24 ¹ .255	24 ¹ .172	206°.0	26 ¹ .593	26 ¹ .502
.1	22 ¹ .137	22 ¹ .062	.6	24 ¹ .305	24 ¹ .222	.1	26 ¹ .647	26 ¹ .556
.2	22 ¹ .183	22 ¹ .108	.7	24 ¹ .355	24 ¹ .272	.2	26 ¹ .701	26 ¹ .610
.3	22 ¹ .230	22 ¹ .154	.8	24 ¹ .405	24 ¹ .322	.3	26 ¹ .755	26 ¹ .664
.4	22 ¹ .276	22 ¹ .200	.9	24 ¹ .455	24 ¹ .372	.4	26 ¹ .810	26 ¹ .718
.5	22 ¹ .323	22 ¹ .247	202°.0	24 ¹ .506	24 ¹ .422	.5	26 ¹ .864	26 ¹ .772
.6	22 ¹ .370	22 ¹ .293	.1	24 ¹ .556	24 ¹ .473	.6	26 ¹ .918	26 ¹ .827
.7	22 ¹ .416	22 ¹ .340	.2	24 ¹ .607	24 ¹ .523	.7	26 ¹ .973	26 ¹ .881
.8	22 ¹ .463	22 ¹ .387	.3	24 ¹ .657	24 ¹ .573	.8	27 ¹ .028	26 ¹ .936
.9	22 ¹ .510	22 ¹ .433	.4	24 ¹ .708	24 ¹ .624	.9	27 ¹ .082	26 ¹ .990
198°.0	22 ¹ .557	22 ¹ .480	.5	24 ¹ .759	24 ¹ .675	207°.0	27 ¹ .137	27 ¹ .045
.1	22 ¹ .604	22 ¹ .527	.6	24 ¹ .810	24 ¹ .725	.1	27 ¹ .192	27 ¹ .100
.2	22 ¹ .651	22 ¹ .574	.7	24 ¹ .861	24 ¹ .776	.2	27 ¹ .247	27 ¹ .154
.3	22 ¹ .699	22 ¹ .621	.8	24 ¹ .912	24 ¹ .827	.3	27 ¹ .302	27 ¹ .209
.4	22 ¹ .746	22 ¹ .668	.9	24 ¹ .963	24 ¹ .878	.4	27 ¹ .358	27 ¹ .264
.5	22 ¹ .793	22 ¹ .716	203°.0	25 ¹ .014	24 ¹ .929	.5	27 ¹ .413	27 ¹ .319
.6	22 ¹ .841	22 ¹ .763	.1	25 ¹ .066	24 ¹ .980	.6	27 ¹ .468	27 ¹ .374
.7	22 ¹ .889	22 ¹ .811	.2	25 ¹ .117	25 ¹ .031	.7	27 ¹ .524	27 ¹ .430
.8	22 ¹ .936	22 ¹ .858	.3	25 ¹ .169	25 ¹ .083	.8	27 ¹ .579	27 ¹ .485
.9	22 ¹ .984	22 ¹ .906	.4	25 ¹ .220	25 ¹ .134	.9	27 ¹ .635	27 ¹ .541
199°.0	23 ¹ .032	22 ¹ .953	.5	25 ¹ .272	25 ¹ .186	208°.0	27 ¹ .691	27 ¹ .596
.1	23 ¹ .080	23 ¹ .001	.6	25 ¹ .324	25 ¹ .237	.1	27 ¹ .747	27 ¹ .652
.2	23 ¹ .128	23 ¹ .049	.7	25 ¹ .376	25 ¹ .289	.2	27 ¹ .803	27 ¹ .708
.3	23 ¹ .176	23 ¹ .097	.8	25 ¹ .428	25 ¹ .341	.3	27 ¹ .859	27 ¹ .764
.4	23 ¹ .224	23 ¹ .145	.9	25 ¹ .480	25 ¹ .393	.4	27 ¹ .915	27 ¹ .820
.5	23 ¹ .272	23 ¹ .193	204°.0	25 ¹ .532	25 ¹ .445	.5	27 ¹ .971	27 ¹ .876
.6	23 ¹ .320	23 ¹ .241	.1	25 ¹ .584	25 ¹ .497	.6	28 ¹ .027	27 ¹ .932
.7	23 ¹ .369	23 ¹ .289	.2	25 ¹ .636	25 ¹ .549	.7	28 ¹ .084	27 ¹ .988
.8	23 ¹ .417	23 ¹ .338	.3	25 ¹ .689	25 ¹ .601	.8	28 ¹ .140	28 ¹ .044
.9	23 ¹ .466	23 ¹ .386	.4	25 ¹ .741	25 ¹ .653	.9	28 ¹ .197	28 ¹ .101
200°.0	23 ¹ .515	23 ¹ .435	.5	25 ¹ .794	25 ¹ .706	209°.0	28 ¹ .254	28 ¹ .157
.1	23 ¹ .563	23 ¹ .483	.6	25 ¹ .846	25 ¹ .758	.1	28 ¹ .311	28 ¹ .214
.2	23 ¹ .612	23 ¹ .532	.7	25 ¹ .899	25 ¹ .811	.2	28 ¹ .367	28 ¹ .271

Tempe- rature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').	Tempe- rature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').	Tempe- rature.	Force in Inches of Mercury at 32°, at Sea Level at Equator.	Force in Inches of Merc. at 32°, at Sea Level at Dub- lin (Lat. 53° 21').
209°.3	28 ⁱ .424	28 ⁱ .328	210°.9	29 ⁱ .350	29 ⁱ .250	212°.5	30 ⁱ .300	30 ⁱ .196
.4	28.482	28.385	211°.0	29.408	29.308	.6	30.360	30.256
.5	28.539	28.441	.1	29.467	29.367	.7	30.420	30.316
.6	28.596	28.499	.2	29.526	29.425	.8	30.480	30.377
.7	28.653	28.556	.3	29.585	29.484	.9	30.541	30.437
.8	28.711	28.613	.4	29.644	29.543	213°.0	30.602	30.497
.9	28.768	28.670	.5	29.703	29.602	.1	30.662	30.558
210°.0	28.826	28.728	.6	29.762	29.661	.2	30.723	30.618
.1	28.884	28.786	.7	29.821	29.720	.3	30.784	30.679
.2	28.942	28.843	.8	29.881	29.779	.4	30.845	30.740
.3	29.000	28.901	.9	29.940	29.838	.5	30.906	30.801
.4	29.058	28.959	212°.0	30.000	29.898	.6	30.967	30.861
.5	29.116	29.017	.1	30.060	29.957	.7	31.029	30.923
.6	29.174	29.075	.2	30.120	30.017	.8	31.090	30.984
.7	29.233	29.133	.3	30.179	30.076	.9	31.152	31.045
.8	29.291	29.191	.4	30.239	30.136			

TABLE IV.

TABLE of the Values of $(1 + n \sin^2 \lambda)$ and its Reciprocal for every 5° of Latitude.

λ .	$(1 + n \sin^2 \lambda)$.	$\text{Log } (1 + n \sin^2 \lambda)$.	$\frac{1}{1 + n \sin^2 \lambda}$.	$\text{Log } \left(\frac{1}{1 + n \sin^2 \lambda} \right)$.
5°	1.000 040	0.000 017 52	0.999 959	$\overline{1.999\ 982\ 48}$
10	1.000 160	0.000 069 57	0.999 840	$\overline{1.999\ 930\ 43}$
15	1.000 356	0.000 154 55	0.999 644	$\overline{1.999\ 845\ 45}$
20	1.000 622	0.000 269 84	0.999 379	$\overline{1.999\ 730\ 16}$
25	1.000 949	0.000 411 94	0.999 052	$\overline{1.999\ 588\ 06}$
30	1.001 328	0.000 576 49	0.998 674	$\overline{1.999\ 423\ 51}$
35	1.001 748	0.000 758 48	0.998 255	$\overline{1.999\ 241\ 52}$
40	1.002 195	0.000 952 36	0.997 809	$\overline{1.999\ 047\ 64}$
45	1.002 657	0.001 152 22	0.997 350	$\overline{1.998\ 847\ 78}$
50	1.003 118	0.001 351 99	0.996 892	$\overline{1.998\ 648\ 01}$
55	1.003 565	0.001 545 60	0.996 447	$\overline{1.998\ 454\ 40}$
60	1.003 985	0.001 727 18	0.996 031	$\overline{1.998\ 272\ 82}$
65	1.004 364	0.001 891 24	0.995 655	$\overline{1.998\ 108\ 76}$
70	1.004 692	0.002 032 80	0.995 330	$\overline{1.997\ 967\ 20}$
75	1.004 957	0.002 147 60	0.995 067	$\overline{1.997\ 852\ 40}$
80	1.005 153	0.002 232 17	0.994 874	$\overline{1.997\ 767\ 83}$
85	1.005 273	0.002 283 95	0.994 755	$\overline{1.997\ 716\ 05}$
90	1.005 313	0.002 301 38	0.994 715	$\overline{1.997\ 698\ 78}$

TABLE V.

TABLE of the absolute Dilatation of Mercury from 32° to 652° F.

Temperature. <i>T.</i>	Dilatation from 32° to <i>T</i> °. $\delta T.$	Mean Coefficient of Dilatation for 1°, from 32° to <i>T</i> °. $\delta.$	Temperature. <i>T.</i>	Dilatation from 32° to <i>T</i> °. $\delta T.$	Mean Coefficient of Dilatation for 1°, from 32° to <i>T</i> °. $\delta.$
32°	0.000 000 000	0.000 000 000	352°	0.032 617 314	0.000 101 929
52	0.001 991 866	0.000 099 593	372	0.034 708 842	0.000 102 085
60	0.002 790 358	0.000 099 656	392	0.036 806 595	0.000 102 241
72	0.003 989 962	0.000 099 749	412	0.038 910 581	0.000 102 396
92	0.005 994 286	0.000 099 905	432	0.041 020 790	0.000 102 552
112	0.008 004 840	0.000 100 060	452	0.043 137 232	0.000 102 708
132	0.010 021 623	0.000 100 216	472	0.045 259 906	0.000 102 863
152	0.012 044 634	0.000 100 372	492	0.047 388 798	0.000 103 019
172	0.014 073 872	0.000 100 528	512	0.049 523 932	0.000 103 175
192	0.016 109 338	0.000 100 683	532	0.051 665 280	0.000 103 331
212	0.018 151 034	0.000 100 839	552	0.053 812 872	0.000 103 486
232	0.020 198 960	0.000 100 995	572	0.055 966 685	0.000 103 642
252	0.022 253 114	0.000 101 151	592	0.058 126 727	0.000 103 798
272	0.024 313 500	0.000 101 306	612	0.060 292 990	0.000 103 953
292	0.026 380 111	0.000 101 462	632	0.062 465 492	0.000 104 109
312	0.028 452 951	0.000 101 618	652	0.064 644 221	0.000 104 265
332	0.030 532 017	0.000 101 773			

TABLE VI.

TABLE of the Expansion of Flint and Crown Glass from 32° to 652° F.

Temperature. <i>T.</i>	FLINT GLASS.	CROWN GLASS.	Temperature. <i>T.</i>	FLINT GLASS.	CROWN GLASS.
	Mean Coefficient of Expansion for 1°, from 32° to <i>T</i> °.	Mean Coefficient of Expansion for 1°, from 32° to <i>T</i> °.		Mean Coefficient of Expansion for 1°, from 32° to <i>T</i> °.	Mean Coefficient of Expansion for 1°, from 32° to <i>T</i> °.
32°	0.000 000 000	0.000 000 000	352°	0.000 012 790	0.000 015 975
52	0.000 012 571	0.000 014 608	372	0.000 012 804	0.000 016 066
72	0.000 012 585	0.000 014 699	392	0.000 012 819	0.000 016 157
92	0.000 012 600	0.000 014 790	412	0.000 012 833	0.000 016 248
112	0.000 012 614	0.000 014 881	432	0.000 012 848	0.000 016 339
132	0.000 012 629	0.000 014 972	452	0.000 012 863	0.000 016 430
152	0.000 012 644	0.000 015 064	472	0.000 012 877	0.000 016 521
172	0.000 012 658	0.000 015 155	492	0.000 012 892	0.000 016 612
192	0.000 012 672	0.000 015 246	512	0.000 012 906	0.000 016 703
212	0.000 012 687	0.000 015 337	532	0.000 012 921	0.000 016 795
232	0.000 012 702	0.000 015 428	552	0.000 012 936	0.000 016 886
252	0.000 012 717	0.000 015 519	572	0.000 012 950	0.000 016 977
272	0.000 012 731	0.000 015 610	592	0.000 012 965	0.000 017 068
292	0.000 012 746	0.000 015 701	612	0.000 012 980	0.000 017 159
312	0.000 012 760	0.000 015 792	632	0.000 012 994	0.000 017 250
332	0.000 012 775	0.000 015 883	652	0.000 013 009	0.000 017 341

TABLE VII.

Comparison of the Indications of an Air Thermometer, corrected for the Expansion of its Envelope, with those of Mercurial Thermometers in different Envelopes.

Temperature as given by Air Thermometer. (T.)	Temperature given by Mercurial Thermometer.			
	Flint Glass. (t.)	Crown Glass. (t.)	Green Glass. (t.)	Swedish Glass. (t.)
212°	212°.00	212°.00	212°.00	212°.00
230	230.09	229.96	230.05	230.04
248	248.22	247.91	248.14	248.07
266	266.36	265.84	266.25	266.13
284	284.52	283.73	284.38	284.20
302	302.72	301.64	302.54	302.27
320	320.92	319.53	320.72	320.36
338	339.17	337.42	338.90	338.47
356	357.44	355.33	357.08	356.59
374	375.82	373.37	375.26	374.74
392	394.25	391.46	393.44	392.90
410	412.75	409.55	411.80	411.10
428	431.28	427.64	430.16	429.35
446	449.89	445.73	448.56	447.62
464	468.59	463.82	466.88	466.09
482	487.40	482.09	485.33	484.59
500	506.19	500.36	503.87	
518	525.02	518.68	522.50	
536	544.06	536.94	541.13	
554	563.18	555.44	559.94	
572	582.30	573.94		
590	601.61	592.61		
608	621.05	611.24		
626	640.80	630.32		
644	660.74	649.40		
662	680.90	669.20		

TABLE VIII.

*TABLE of the Density of various Gases and Vapours, as determined by Experiment, the Density of Air being 1.000.**

SIMPLE BODIES.				
Name of Substance.	Volume of Vapour corresponding to chemical Equiva- lent.	Symbol.	Density.	Obser- ver.
Hydrogen,	1	H.	0.0688	Bo.&D.
Ditto,	"	"	0.06926	Reg.
Carbon (hypothetical), . .	1	C.	0.4145	Reg.
Nitrogen,	"	N.	0.9720	Bo.&D.
Ditto,	"	"	0.97137	Reg.
Oxygen,	$\frac{1}{2}$	O.	1.1026	Bo.&D.
Ditto,	"	"	1.10563	Reg.
Chlorine,	1	Cl.	2.395	H. Da.
Ditto,	"	"	2.424	G.
Ditto,	"	"	2.470	G. & T.
Phosphorus,	$\frac{1}{2}$	P.	4.420	D.
Ditto,	"	"	4.580	M.
Bromine,	1	Br.	5.540	M.
Sulphur,	$\frac{1}{8}$	S.	6.5635	D.
Ditto,	"	"	6.9	M.
Mercury,	2	Hg.	6.976	D.
Iodine,	1	I.	8.716	D.
Arsenic,	$\frac{1}{2}$	As.	10.6	M.
BINARY COMPOUNDS.				
Name of Substance.	Composition by Volume.		Density.	Obser- ver.
Light carburetted hydrogen,	2H + 1C = H ₂ C.		0.5555	Th.
Ammoniacal Gas,	1N + 3H = 2NH ₃ .		0.5901	H. Da.
Water,	2H + 1O = 2HO.		0.6235	G. L.
Ditto,	" "		0.6250	Dz.

* For a complete list of all the gases and vapours whose densities had been examined up to the year 1840, see Poggendorff's *Annalen*,

tom. xlix. p. 417 (1840), and Dove's *Repertorium der Physik*, vii. p. 174.

Name of Substance.	Composition by Volume.	Density.	Observer.
Carbonic oxide,	$2C + 1O = 2CO.$	0.9678	
Olefiant gas,	$2H + 2C = H_2C_2.$	0.9709	Th.
Binoxide of nitrogen, . . .	$1N + 1O = 2NO_2.$	1.0388	Br.
Phosphuretted hydrogen, .	$1P + 6H = 4PH_3.$	1.1214	D.
Ditto,	" "	1.146	Ro.
Hydrosulphuric acid, . . .	$6H + 1S = 6HS.$	1.1912	G. & T.
Hydrochloric acid,	$1H + 1Cl = 2HCl.$	1.2474	B. & A.
Ditto,	" "	1.278	G.
Carbonic acid,	$1C + 1O = CO_2.$	1.5245	Bo. & D.
Ditto,	" "	1.52901	Reg.
Protoxide of nitrogen, . . .	$2N + 1O = 2NO.$	1.5204	C.
Cyanogen,	$1N + 2C = NC_2.$	1.8064	G.
Sulphurous acid,	$1S + 6O = 6SO_2.$	2.193	H. Da.
Ditto,	" "	2.2553	G. & T.
Fluoboric acid,	$1B + 3F = 2BF_3.$	2.3124	D.
Ditto,	" "	2.3709	J. Da.
Arseniuretted hydrogen, . .	$1As + 6H = 4AsH_3.$	2.695	D.
Sulphuric acid,	$1S + 9O = 6SO_3.$	3.000	M.
Fluosilicic acid,	$1Si + 2F = SiF_2.$	3.600	D.
Ditto,	" "	3.5735	J. Da.
Terchloride of boron, . . .	$1B + 3Cl = 2BCl_3.$	3.942	D.
Hydriodic acid,	$1H + 1I = 2HI.$	4.443	G.
Terchloride of phosphorus,	$1P + 6Cl = 4PCl_3.$	4.875	D.
Chloride of silicium, . . .	$1Si + 2Cl = SiCl_2.$	5.9390	D.
Terchloride of arsenic, . . .	$1As + 6Cl = AsCl_3.$	6.3006	D.
Bichloride of titanium, . .	$1Ti + 2Cl = TiCl_2.$	6.836	D.
Bichloride of tin,	$1Sn + 2Cl = SnCl_2.$	9.1997	D.
Chloride of mercury, . . .	$1Hg + 1Cl = HgCl.$	9.8	M.
Bromide of mercury,	$1Hg + 1Br = HgBr.$	12.16	M.
Arsenious acid,	$1As + 3O = AsO_3.$	13.85	M.
Iodide of mercury,	$1Hg + 1I = HgI.$	15.9	M.
Iodide of arsenic,	$1As + 6I = 4AsI_3.$	16.1	M.
TERNARY COMPOUNDS.			
Hydrocyanic acid,	$1H + 1N + 2C = 2HCN_2.$	0.9476	G.
Alcohol,	$8C + 12H + 2O = 4C_4H_6O_2.$	1.6133	G.
Formic acid (at 216° C.), .	$4C + 4H + 4O = 4C_2H_2O_4.$	1.61	Bin.
Acetic acid (at 250° C.), .	$8C + 8H + 4O = 4C_4H_4O_4.$	2.08	Ca.
Sulphuric ether,	$8C + 10H + O = 2C_4H_5O.$	2.586	G.
Butyric acid (at 261° C.), .	$16C + 16H + 4O = 4C_8H_8O_4.$	3.07	Ca.
Essence of anise (at 338° C.),	$40C + 24H + 2O = 4C_{20}H_{12}O_2.$	5.19	Ca.
Hydriodic ether,	$8C + 10H + I = 2C_4H_5I.$	5.4749	G.*

* *Explanation of Contractions*—Th., Thomson; Br., Berzelius; G. & T., Gay-Lussac and Thénard; B. & A., Biot and Arago; G., Gay-Lussac; Bo. & D., Boussingault and Dumas;

Reg., Regnault; C., Colin; H. Da., Sir Humphrey Davy; D., Dumas; J. Da., Dr. John Davy; M., Mitscherlich; Dz., Despretz; Bin., Bineau; Ca., Cahours; Ro., Rose.

TABLE IX.

TABLE of the Weight of 100 cubic Inches of Air, Hydrogen, Nitrogen, Oxygen, and Carbonic Acid Gas.

Name of Gas.	Density.	Weight at 32° F., and under the Pressure of 30 Inches of Mercury at the Equator.	Weight at 32° F., and under the Pressure of 30 Inches of Mercury at Dublin.
Air,	1.00000	32 ^{gr.} .690 541	32 ^{gr.} .802 342
Hydrogen,	0.06926	2 .264 147	2 .271 890
Nitrogen,	0.97137	31 .754 611	31 .863 211
Oxygen,	1.10563	36 .143 643	36 .267 253
Carbonic acid, . .	1.52901	49 .984 164	50 .155 109

TABLE X.

TABLE of the elastic Force of the Vapours of Alcohol, Ether, Oil of Turpentine, Petroleum or Naphtha, and Sulphuret of Carbon, at different Temperatures, expressed in Inches of Mercury.

Ether. (URK.)		Alcohol, Sp. gr. 0.813. (URE.)		Alcohol. Sp. gr. 0.813. (URE.)		Petroleum. (URE.)		Sulphuret of Carbon.* (MARX.)	
Temp.	Force of Vapour.	Temp.	Force of Vapour.	Temp.	Force of Vapour.	Temp.	Force of Vapour.	Temp.	Elastic Force.
34	6.20	32	0.40	193°.3	46.60	316°	30.00	17°	3 ¹ .487
44	8.10	40	0.56	196°.3	50.10	320	31.70	22	3.963
54	10.30	45	0.70	200	53.00	325	34.00	27	4.493
64	13.00	50	0.86	206	60.10	330	36.40	32	5.082
74	16.10	55	1.00	210	65.00	335	38.90	37	5.733
84	20.00	60	1.23	214	69.30	340	41.60	42	6.453
94	24.70	65	1.49	216	72.20	345	44.10	47	7.246

* Dove's Repertorium der Physik, i. p. 54.
This table has been calculated by means of the formula,

$$\text{Log } i = 2.6616681 + \log(479.25 + T) - \frac{2221.91}{479.25 + T},$$

in which i represents the elastic force in inches of mercury, and T the temperature in Fahrenheit degrees, counted from 32°.

Ether. (URE.)		Alcohol, Sp. gr. 0.813. (URE.)		Alcohol, Sp. gr. 0.813. (URE.)		Petroleum. (URE.)		Sulphuret of Carbon. (MARX.)	
Temp.	Force of Vapour.	Temp.	Force of Vapour.	Temp.	Force of Vapour.	Temp.	Force of Vapour.	Temp.	Elastic Force.
104	30.00	70	1.76	220°	78.50	350°	46.86	52°	8 ¹ .119
105	30.00	75	2.10	225	87.50	355	50.20	57	9.077
110	32.54	80	2.45	230	94.10	360	53.30	62	10.127
115	35.90	85	2.93	232	97.10	365	56.90	67	11.276
120	39.47	90	3.40	236	103.60	370	60.70	72	12.531
125	43.24	95	3.90	238	106.90	372	61.90	77	13.898
130	47.14	100	4.50	240	111.24	375	64.00	82	15.385
135	51.90	105	5.20	244	118.20	Oil of Turpentine. (URE.)		87	17.001
140	56.90	110	6.00	247	122.10			92	18.754
145	62.10	115	7.10	248	126.10			97	20.651
150	67.60	120	8.10	249.7	131.40	Temp.	Force of Vapour.	102	22.703
155	73.60	125	9.25	250	132.30			107	24.918
160	80.30	130	10.60	252	138.60	304°	30.00	112	27.306
165	86.40	135	12.15	254.3	143.70	307.6	32.60	117	29.876
170	92.80	140	13.90	258.6	151.60	310	33.50	122	32.640
175	99.10	145	15.95	260	155.20	315	35.20	127	35.607
180	108.30	150	18.00	262	161.40	320	37.06	132	38.788
185	116.10	155	20.30	264	166.10	322	37.80	137	42.194
190	124.80	160	22.60			326	40.20	142	45.838
195	133.70	165	25.40			330	42.10		
200	142.80	170	28.30			336	45.00		
205	151.30	173	30.00			340	47.30		
210	166.00	178.3	33.50			343	49.40		
		180	34.73			347	51.70		
		182.3	36.40			350	53.80		
		185.3	39.90			354	56.60		
		190	43.20			357	58.70		
						360	60.80		
						362	62.40		

SUPPLEMENT TO TABLE X.

*Elastic Force of Vapours generated in a very limited Space, according to M. Cagniard De la Tour.**

Ether (Series I.)†		Ether (Series II.)‡		Sulphuret of Carbon §	
Temp. (Centigr.)	Force in At- mospheres.	Temp. (Centigr.)	Force in At- mospheres.	Temp. (Centigr.)	Force in At- mospheres.
80°	5.6	80	4.2
90	7.9	90	5.5
100	10.6	100	14.0	100	7.9
110	12.9	110	17.5	110	10
120	18.0	120	22.5	120	13
130	22.2	130	28.5	130	16.5
140	28.3	140	35	140	20.2
150	37.5	150	42	150	24.2
160	48.5	160	50.5	160	28.8
170	59.7	170	58	170	33.6
180	68.8	180	63.5	180	40.2
190	78.0	190	66	190	47.5
200	86.3	200	70.5	200	57.2
210	92.3	210	74	210	66.5
220	104.1	220	78	220	77.8
230	112.7	230	81	230	89.2
240	119.4	240	85	240	98.9
250	123.7	250	89	250	114.3
260	130.9	260	94	260	129.6

* Ann. de Chim. et de Phys., tome xxii. p. 411.

† In this series the liquid originally occupied seven parts of the tube. When totally converted into vapour it filled twenty.

‡ In this case the liquid occupied three

and a half parts, the vapour, as before, twenty; in both cases the liquid passed completely into the vaporous state at 150°.

§ The liquid occupied eight parts, the vapour twenty. The transition to the state of vapour took place at 220°.

ADDENDA.

Page 104.—Add the following note to (68):

Not long after the passing of the Act of 1824 the fire which destroyed the houses of Parliament so far injured the legal standards as to render their restoration necessary, and a Committee was accordingly appointed in 1838 to consider the best means of effecting this object. This Committee reported* in 1841, that subsequent to the passing of the Act of 1824 it had been discovered that several elements of the reduction of pendulum-experiments, therein referred to, were doubtful or erroneous; and similarly, that the determination of the weight of a cubic inch of distilled water is also attended with considerable uncertainty. They were, therefore, of opinion that the method of restoring lost standards, recommended by the Act of 1824, could not be relied on, and expressed themselves “fully persuaded that, with reasonable precautions, it will always be possible to provide for the accurate restoration of the standards, by means of *material copies* which have been carefully compared with them, more securely than by reference to any experiments referring to natural constants.” In the present instance they availed themselves, for the restoration of the standards, of the Royal Astronomical Society’s scale, the Royal Society’s scale, the iron bars belonging to the Board of Ordnance, and several metallic weights, which had been most accurately compared with their respective standards.

— 157, last paragraph.—On the same principle M. Boutigny explains the remarkable fact, which he has recently verified, namely, that the human body may be brought with impunity into contact

* Parliamentary Papers, 1842, vol. xxv. p. 263.

with metals in a state of fusion, and at very high temperatures. Thus the naked hand may be passed through a stream of molten metal, or plunged into a bath of the same; it appears only necessary that the skin should be in its normal healthy, moist state, and that the operation should not be performed too rapidly. M. Boutigny supposes that the natural moisture of the skin assumes the spheroidal state, that accordingly there is no actual contact between it and the metal, and that the heat radiated by the latter is reflected from the surface of the moisture enveloping the skin. Of course if the skin were exposed to the action of the heat until all the moisture was evaporated, it would be no longer thus protected.

Page 194.—It is to be observed that the volume V_t of the glass balloon at the temperature t of the surrounding medium may also be obtained by the method of gauging with mercury, and its volume at the temperature of melting ice obtained from the former, by means of the known expansion of the glass of the balloon and of the gauging vessel.

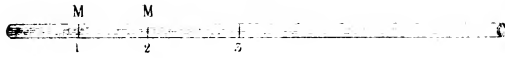
— 219.—This hypothesis of an imperfect vaporization, and of an intermediate or transition state between the liquid state and that of vapour, appears to derive confirmation from M. Cagniard De la Tour's experiments on the elastic force of vapours formed in a very limited space. For on reference to the Supplement to Table X., p. 280, it will be seen that the rate of increase of the elastic forces of ether-vapour above 150° , when it is no longer in contact with its liquid, is so rapid as to be quite inconsistent with the supposition that it is merely due to the effect of expansion by heat. It will also be seen that this rate diminishes with the temperature, and that it is much less rapid in the second series, where the particles of the vapour were less condensed than in the first. The same remarks apply to the vapour of sulphuret of carbon.

— 224.—Proposition IV. The probable value and probable error obtained by means of this proposition should be represented by the symbols \bar{A} , \bar{R} , to distinguish them from those from which they are derived.

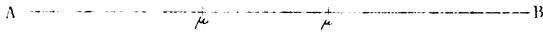
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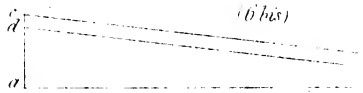
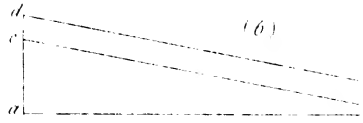
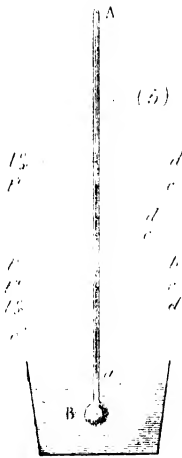
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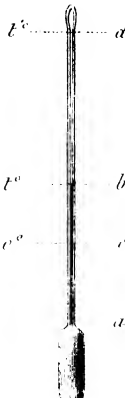
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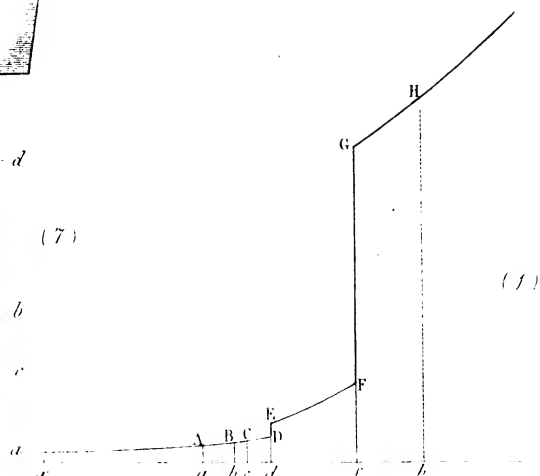
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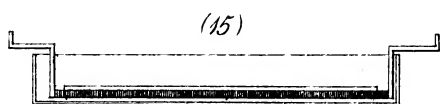
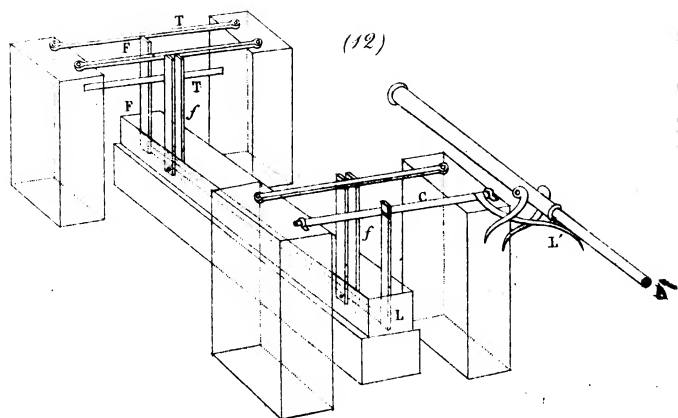


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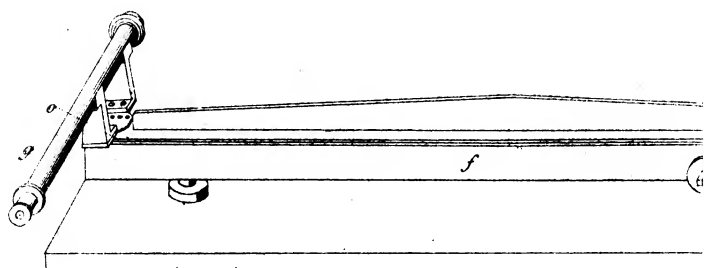
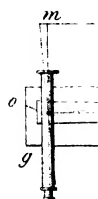


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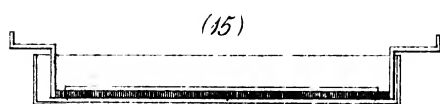
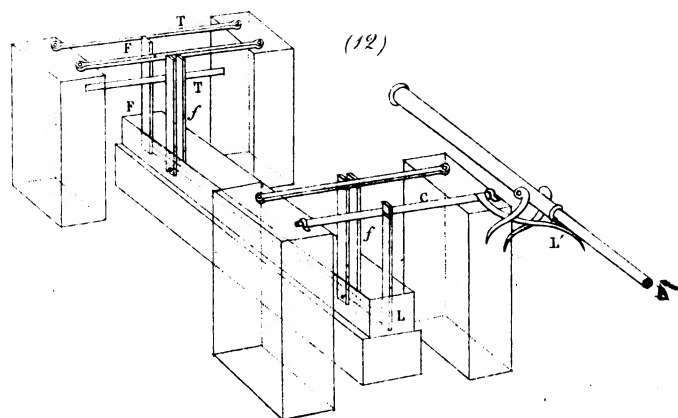




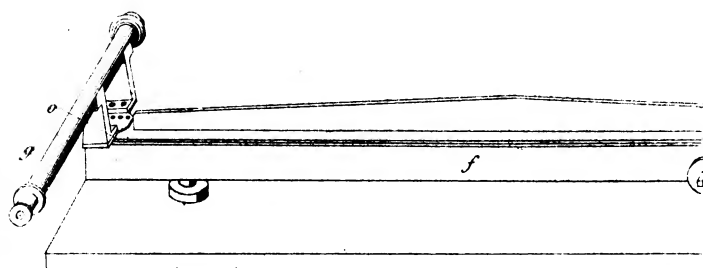
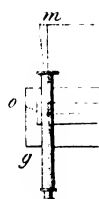
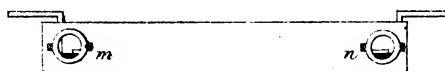
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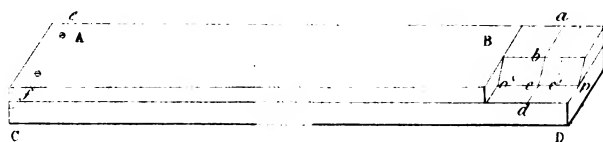


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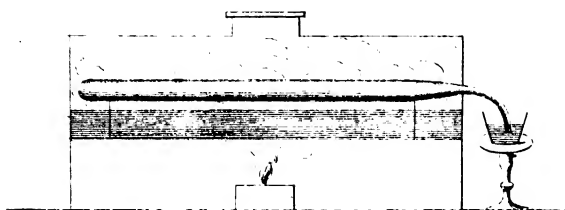
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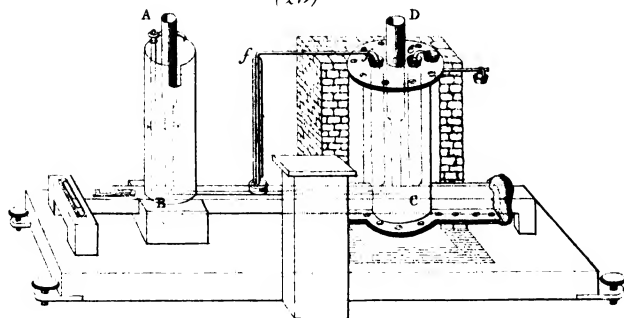
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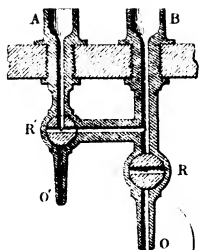
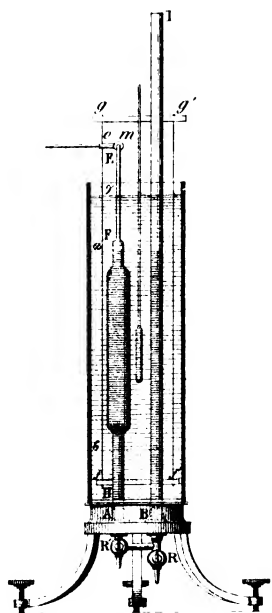
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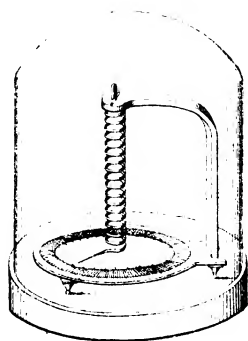
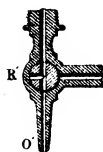
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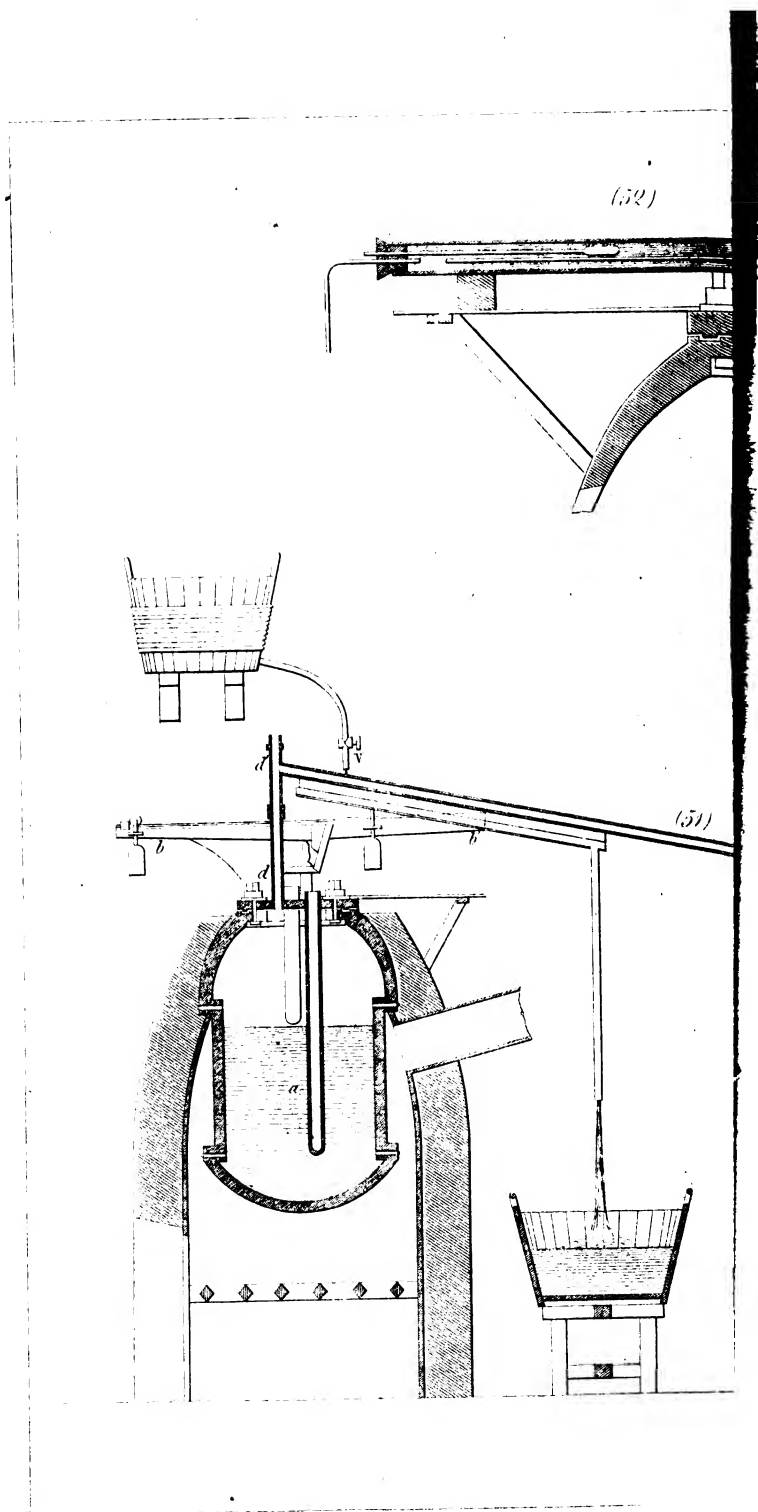
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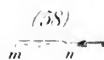
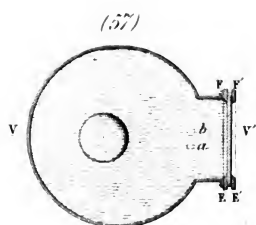
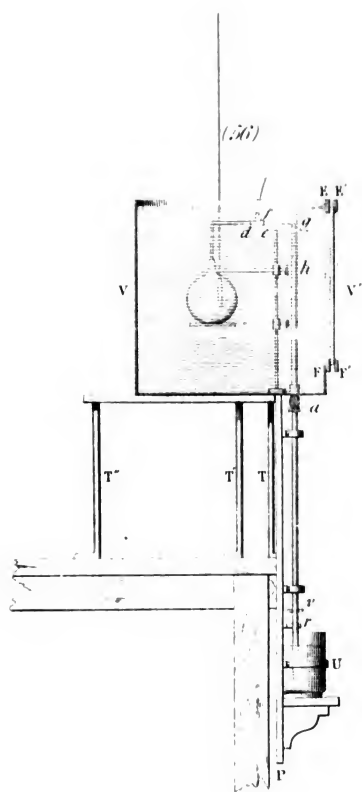


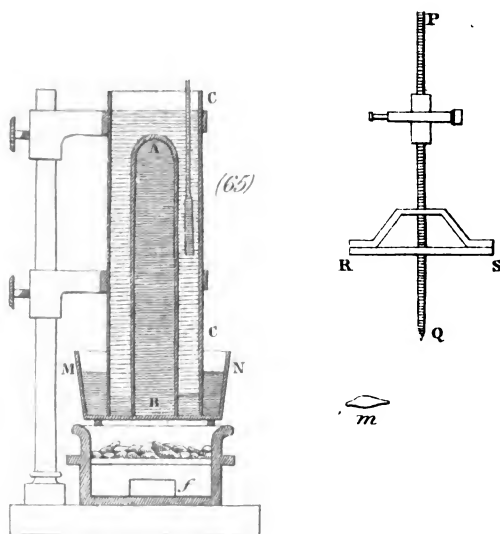
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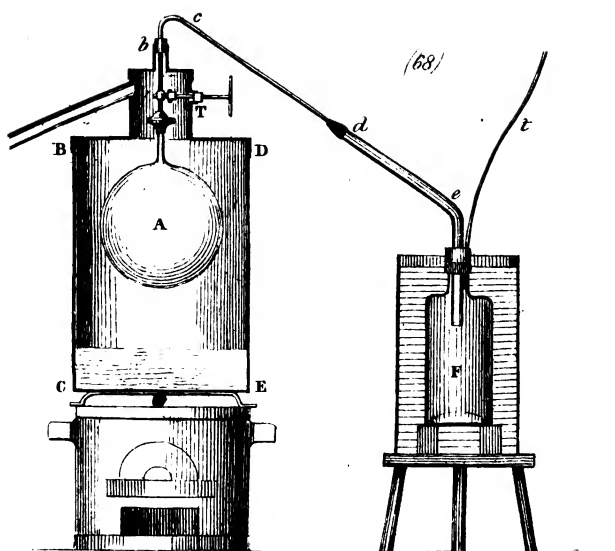
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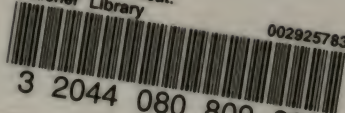
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